726

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From December 1911 *to October* 1912.

VOL. XXIV.

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PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY

OF LONDON,

SESSION 1911-1912.

October 27th, 1911.

Meeting held at the Imperial College of Science.

Prof. H. L. **Callendar**, F.R.S., President, in the Chair.

The following Papers were read :—

1. Further Observations on the Afterglow of Electric Discharge and Kindred Phenomena. By the Hon. R. J. **Strutt**, F.R.S.
2. Homogeneous Fluorescent X-Radiation of a Second Series. By Prof. C. G. **Barkla** and J. **Nicol**.

November 10th, 1911.

Meeting held at Finsbury Technical College by kind invitation of Prof. S. P. **Thompson**, F.R.S.

Prof. H. L. **Callendar**, F.R.S., President, in the Chair.

The Agenda was as follows :—

1. Exhibition of Reflecting Polariscopes for the Study of Optical Stress in Materials. By Prof. S. P. **Thompson** and Prof. E. G. **Coker**.

2. On the Effects of Holes and Semi-circular Notches in the Distribution of Stress in Tension Members. By Prof. E. G. Coker.

3. Exhibition of Some Surface Tension Phenomena. By C. R. Darling.

4. (*a*) Exhibition of a Large Harmonograph.

(*b*) Physiological Effect of an Alternating Field.

(*c*) Demonstrations of Acoustical Experiments, New and Old. By Prof. S. P. Thompson.

November 24th, 1911.

Meeting held at the Imperial College of Science.

Prof. H. L. Callendar, F.R.S., President, in the Chair.

The following Papers were read :—

1. The Maximum Value of the Electric Stress between Two Unequal Spherical Electrodes. By Dr. A. Russell.

2. On the Cubical Expansion of Fused Silica. By Mr. F. J. Harlow.

3. On the Temperature Coefficient of Diffusion. By Mr. B. W. Clack.

4. The α-Particles emitted by the Active Deposits of Thorium and Actinium. By Messrs. E. Marsden and T. Barratt.

5. The Magnetic Transition Temperature of Cementite. By Messrs. S. W. J. Smith, W. White and S. G. Barker.

January 26th, 1912.

Meeting held at the Imperial College of Science.

Prof. H. L. Callendar, F.R.S., President, in the Chair.

The Proceedings were as follows :—

1. Exhibition of a Direct-reading Instrument for Submarine and other Calculations. By Mr. R. Appleyard.

2. On the Vibration Galvanometer and its Applications to Inductance Bridges. By Mr. S. Butterworth.

3. On Sealing Metals. By Dr. P. E. Shaw.

4. On a Negative Result connected with Radio-activity. By Dr. J. H. Vincent and Mr. A. Bursill.

5. A Copper Zinc Uranium Oxide Cell and the Theory of Contact Electromotive Forces. By Prof. A. Anderson.

Annual General Meeting.

February 9th, 1912.

Meeting held at the Imperial College of Science.

Prof. H. L. CALLENDAR, F.R.S., President, in the Chair.

The Report of the Council was read by the Secretary.

Since the last Annual General Meeting there have been held eleven ordinary meetings and two informal meetings. Of these, ten were held at the Imperial College of Science, one at the City and Guilds (Engineering) College, one at the City and Guilds Technical College, Finsbury, and one at the National Physical Laboratory.

The average attendance, excluding the Exhibition and the informal meeting at the National Physical Laboratory, was 50 as compared with 55 the previous year.

The seventh Annual Exhibition of apparatus by manufacturers was held on December 19th in the afternoon and evening, the number of Fellows and visitors being about 650. An experimental discourse on "Electric Discharge and the Luminosity that Survives it" was kindly given by Prof. the Hon. R. J. Strutt, F.R.S. The number of exhibitors was 33, and would doubtless have been greater but for the fact that a prolonged electrical exhibition had been held about two months previously at Olympia.

Since the last Annual General Meeting the first complete volume of the "Proceedings" under the new arrangements adopted by the Council has been published. The arrangements have worked smoothly, the publication of the Papers has been prompt, and the Council are pleased to report that there has been no lack of Papers of high standard.

The number of Ordinary Fellows now on the roll, as distinct from Honorary Fellows, is 448, an increase of 15 on that of last year: 31 new Fellows have been elected, there have been 6 resignations, and 7 Fellows have been struck off the roll for non-payment of subscriptions. The Society has to mourn the loss by death of one Honorary Fellow, Prof. J. H. van't Hoff, who was elected at the last Annual General Meeting, and three ordinary Fellows, namely: F. Hovenden, F. J. Jervis-Smith and G. Johnstone Stoney.

The Report of the Council was adopted.

The Report of the Treasurer was read by the Treasurer.

The total income of the Society shows a slight diminution over that of 1910. This is due to the fact that no life compositions have been received during the year and to the fact that in the accounts for 1910 the sum of £26. 12s. 8d. was included, being income tax recovered.

A very satisfactory feature is an increase in the subscriptions, which are nearly £16 up. Further, the sales of the publications of the Society continue to increase, showing that the Society's publications are appreciated by outside readers.

The expenditure side of the accounts is of especial interest this year owing to the fact that they contain for the first time the cost of printing the publications of the Society at the new printers.

During the two previous years the cost of the Society's publications was unduly high, mainly due to the effort that was being made to work off the arrears that had accumulated. These arrears are now completely wiped out, and the figure that appears in the balance-sheet, namely, £283. 4s. 6d., is the cost of a complete year of the Society's work.

The last payment, namely, £86. 13s. 9d., in clearing up the arrears in the publications when changing over from Taylor & Francis to "The Electrician" Printing & Publishing Company, appears in this year's accounts.

By comparing the balance brought into the accounts on the 1st January, 1911, with the balance carried forward to 1912, it will be seen that the Society is £38. 3s. 5d. to the good on the year.

If it had not been for the expenditure of over £86 for arrears in publications the Society would have made a profit of approximately £125 on the year.

The property account of the Society shows but little change. The investments have kindly been valued by the manager of Parr's Bank, and the figures appearing in the accounts are the values of the securities at market prices on December 31st, 1911.

The liabilities on account of the life composition fund have decreased owing to the death of several life members and consequently the value available to the general fund is slightly increased.

It is to be noted that the publications in stock still appear in the property account at £200. In my last report I mentioned that these publications were being revalued. Unfortunately the work of sorting and getting in order the old publications has proved more lengthy than was anticipated, much of the stock being in sheet form and not stitched up in parts. As soon as the stocktaking is completed this figure will be revised.

The Report of the Treasurer and the Balance-sheet, which was also presented to the meeting, were adopted.

THE TREASURER IN ACCOUNT WITH THE PHYSICAL SOCIETY, FROM JANUARY, 1911, TO DECEMBER 31ST, 1911.

Dr.	£	*s.*	*d.*	£	*s.*	*d.*
Entrance Fees	24	3	0			
Subscriptions	486	1	0			
Extra Subscriptions	25	4	0			
Arrears	17	1	0			
Subscriptions paid in Advance	10	10	0			
Students' Subscriptions	4	4	0			
Subscriptions for "Science Abstracts" and Advance Proofs	9	17	6			
				577	0	6
Composition Fees					nil	
Dividends :—						
Furness Debenture Stock	15	1	2			
Midland Railway	37	13	4			
Metropolitan Board of Works	6	12	0			
Lancaster Corporation Stock	11	6	0			
New South Wales	8	7	6			
London, Brighton & South Coast Railway	26	9	9			
Great Eastern Railway	18	16	8			
				124	6	5
Interest on £400 deposit account				7	17	7
Sales of Publications (Taylor & Francis)				1	12	5
Sales of Publications ("Electrician" Printing & Publishing Company)				60	17	7
				771	14	6
January 1, 1911, balance at Bank	164	8	0			
Less two cheques	104	13	10			
				59	14	2
				£831	8	8

Cr.	£	*s.*	*d.*	£	*s.*	*d.*
"Science Abstracts"				280	13	0
Printing and Distributing Publications of the Society :—						
Messrs. Taylor & Francis				86	13	9
"The Electrician" Printing & Publishing Co.—						
Proceedings	215	17	2			
Bulletin	15	12	7			
Distribution	33	3	10			
General	20	11	8			
	285	5	3			
Less credit for use of type and insurance rebate	2	0	9			
				283	4	6
Periodicals				4	4	0
Reporting				35	0	0
Refreshments and attendance at meetings				24	17	8
Petty cash				16	14	7
Bank commissions				0	1	7
Royal Asiatic Society				2	2	0
				733	11	1
Balance at Bank carried forward to 1912				97	17	7
				£831	8	8

WILLIAM DUDDELL, *Honorary Treasurer.*

Audited and found correct,

C. W. S. CRAWLEY.
CHARLES V. DRYSDALE.

PROPERTY ACCOUNT OF THE PHYSICAL SOCIETY, DECEMBER 31ST, 1911.

ASSETS.	£	s.	d.
Subscriptions due, Treasurer's estimate...	29	8	0
£533 Furness Ry. Co.3 per cent. Debenture Stock	400	0	0
£1,600 Midland Railway 2½ per cent. Preference Stock	1,040	0	0
£200 Metropolitan Board of Works 3½ per cent. Consolidated Stock	198	0	0
£400 Lancaster Corporation 3 per cent. Redeemable Stock	324	0	0
£254. 2s. 9d. New South Wales 3½ per cent. Inscribed Stock	248	0	0
£500 London, Brighton & South Coast Railway Ordinary Stock	580	0	0
£500 Great Eastern Railway 4 per cent. Debenture Stock	520	0	0
Balance at Bank	97	17	7
Ditto on deposit	400	0	0
Outstanding credit (since paid)	18	7	5
Publications	200	0	0
	£4,055	13	0

LIABILITIES.	£	s.	d.
Outstanding liability (since paid)	18	7	5
Life Composition Fund	2,159	10	0
Balance General Fund	1,877	15	7
	£4,055	13	0

WILLIAM DUDDELL, *Honorary Treasurer.*

Audited and found correct,

C. W. S. CRAWLEY.
CHARLES V. DRYSDALE.

LIFE COMPOSITION FUND

		£	s.	d.
181	Fellows paid £10	1,810	0	0
3	Fellows paid £15	45	0	0
4	Fellows paid £21	84	0	0
7	Fellows paid £31. 10s.	220	10	0
195		£2,159	10	0

WILLIAM DUDDELL, *Honorary Treasurer.*

Audited and found correct,
C. W. S. CRAWLEY.
CHARLES V. DRYSDALE.

The election of Officers and other Members of the Council then took place, the new Council being constituted as follows :—

President.—Prof. A. Schuster, Ph.D., F.R.S.

Vice-Presidents, who have filled the Office of President.—Prof. G. C. Foster, F.R.S. ; Prof. W. G. Adams, M.A., F.R.S. ; Prof. R. B. Clifton, M.A., F.R.S. ; Prof. A. W. Reinold, M.A., F.R.S. ; Prof. Sir Arthur W. Rücker, M.A., D.Sc., F.R.S. ; Sir W. de W. Abney, R.E., K.C.B., D.C.L., F.R.S. ; Prin. Sir Oliver J. Lodge, D.Sc., F.R.S. ; Prof. S. P. Thompson, D.Sc., F.R.S. ; R. T. Glazebrook, D.Sc., F.R.S. ; Prof. J. H. Poynting, M.A., F.R.S. ; Prof. J. Perry, F.R.S. ; C. Chree, Sc.D., F.R.S. ; Prof. H. L. Callendar, M.A., F.R.S.

Vice-Presidents.—A. Campbell, B.A. ; Prof. C. H. Lees, D.Sc., F.R.S. ; Prof. T. Mather, F.R.S. ; A. Russell, M.A., D.Sc.

Secretaries.—W. R. Cooper, M.A.; S. W. J. Smith, M.A., D.Sc.

Foreign Secretary.—Prof. S. P. Thompson, D.Sc., F.R.S.

Treasurer.—W. Duddell, F.R.S.

Librarian.—S. W. J. Smith, M.A., D.Sc.

Other Members of Council.—Prof. C. G. Barkla, D.Sc. ; Prof. P. V. Bevan, M.A. ; Major W. A. J. O'Meara, C.M.G. ; S. Skinner, M.A. ; F. E. Smith ; Prof. the Hon. R. J. Strutt, F.R.S. ; W. E. Sumpner, D.Sc. ; F. Twyman ; R. S. Whipple ; R. S. Willows, M.A., D.Sc.

Prof. H. Nagaoka was elected an Honorary Fellow of the Society.

Votes of thanks were passed to the Auditors, to the Officers and Council and to the Governors of the Imperial College of Science and Technology.

Prof. A. Schuster then took the Chair, and delivered the President's Address.

PRESIDENT'S ADDRESS.

This will be found printed in the " Proceedings," pp. 121-138.

February 23rd, 1912.

Meeting held at the Imperial College of Science.

Prof. A. Schuster, F.R.S., President, in the Chair.

The following Papers were read :—

1. A Balance Method for the Accurate Comparison of Quantities of Radium and Some of its Applications. By Prof. E. Rutherford and Mr. J. Chadwick.

2. The Absorption of γ-Rays by Gases and Light Substances. By Mr. J. Chadwick.

3. Wave-form Sifters for Alternating Currents. By Mr. A. Campbell.

March 8th, 1912.

Meeting held at the Imperial College of Science.

Prof. A. Schuster, F.R.S., President, in the Chair.

The Proceedings were as follows :—

1. Exhibition of a Method of making Capillary Filaments. By H. S. Souttar.

2. The Intensity at Points near the Principal Focus of an Object Glass with Symmetrical Aberration. By Mr. James Walker.

3. Description of the Equipment of the Spectroscopic Laboratory of the Imperial College of Science. By Prof. A. Fowler.

The meeting afterwards adjourned to the laboratory to inspect it and witness demonstrations of experiments.

March 22nd, 1912.

Meeting held at the Imperial College of Science.

Prof. C. H. Lees, F.R.S., Vice-President, in the Chair.

The following Papers were read :—

1. A 2,000-Frequency Alternator. By Mr. W. Duddell.

2. A Method of comparing Capacities at Various Frequencies. By Mr. A. Campbell.

3. The Coefficients of Expansion of Fused Silica and Mercury. By Mr. H. Donaldson.

April 26th, 1912.

Meeting held at the Imperial College of Science.

Mr. A. Campbell, Vice-President, in the Chair.

The following Papers were read:—

1. On the Expansion of Vitreous Silica. By Prof. H. L. Callendar, F.R.S.
2. The Solution of Network Problems by Determinants. By Mr. R. Appleyard.

May 10th, 1912.

Meeting held at the Imperial College of Science.

Prof. A. Schuster, F.R.S., President, in the Chair.

The Proceedings were as follows:—

1. Demonstration of the Generation of Electricity by Carbon at High Temperatures. By Dr. J. A. Harker and Dr. G. W. C. Kaye.
2. A Method of Measuring Small Inductances. By Mr. S. Butterworth.
3. On the Conversion of Starch into Dextrin by X-Rays. By H. A. Colwell and Dr. S. Russ.

May 31st, 1912.

Meeting held at the Imperial College of Science.

Prof. C. H. Lees, F.R.S., Vice-President, in the Chair.

The Proceedings were as follows:—

1. On the Calibration of Wave-Meters for Radio-telegraphy. By Prof. G. W. O. Howe.
2. Applications of Heaviside's Resistance Operators to the Theory of the Air-core Transformer and Coupled Circuits in General. By Dr. W. H. Eccles.
3. Exhibition of the Movements of Semi-oily Liquids on a Water Surface. By Mr. C. R. Darling.
4. Surface Leakage with Alternating Currents. By Mr. G. L. Addenbrooke.

June 14th, 1912.

Meeting held at the Imperial College of Science.

Prof. A. SCHUSTER, F.R.S., President, in the Chair.

The Proceedings were as follows :—

1. A Demonstration of the Use of Specific Gravity Balls for Determining very Small Differences of Density. By Mr. T. H. BLAKESLEY.
2. The Maximum Sensibility of a Duddell Vibration Galvanometer. By Dr. H. F. HAWORTH.
3. An Accurate Examination of the Steinmetz Index for Transformer Iron, Stalloy and Cast Iron. By Mr. F. STROUDE.

June 21st, 1912.

Joint Meetings with the Optical Convention.

Morning Meeting at 10 a.m.

The President, on behalf of the Physical Society, welcomed the opportunity the present meeting afforded of co-operation with the Optical Convention.

A Demonstration of Apparatus for the Teaching of Optics was given by Dr. G. F. C. SEARLE, and discussed by Prof. S. P. THOMPSON.

A Paper on "Diffraction Patterns" was read by Mr. J. W. GORDON, and discussed by Prof. S. P. THOMPSON and Mr. C. P. BUTLER.

A Paper on "The Shape of Scales required for Reflecting Instruments with Concave Mirrors" was read by Mr. E. H. RAYNER, and discussed by Messrs. CHALMERS, MILNE and TROTTER.

A Paper on "The Visibility of Distant Lights" was read by Messrs. C. C. PATTERSON and B. P. DUDDING, and discussed by Prof. S. P. THOMPSON and Messrs. CHALMERS, CLAY, MEES, MILNE, RAYNER, ROSENHAIN and TROTTER.

A Paper on "Some Points in the Use and Design of Refractometers" was read by Dr. T. M. LOWRY, and discussed by the PRESIDENT and Messrs. CHALMERS, MEES, ROSENHAIN and VON ROHR.

A Paper on "Some Recent Advances in the Measurement of Light and Illumination" was read by Messrs. J. S. DOW and V. H. MACKINNEY, and discussed by the PRESIDENT, Prof. S. P. THOMPSON and Messrs. CHALMERS, PATERSON, TROTTER and WHIPPLE.

Evening Meeting at 8 p.m.

Some Models illustrating Geometrical Optics were exhibited by Prof. S. P. Thompson, and discussed by Messrs. Beck, Blakesley, Chalmers, Davis, T. Smith, Von Rohr and Wilmott.

A Paper "On the Measurement of Colour" was read by Mr. A. C. Jolley and Mr. A. J. Bull, and discussed by Prof. S. P. Thompson and Messrs. Edridge Green, Trotter and Twyman.

The Paper by Dr. J. A. Harker on "Optical Pyrometry" was postponed to a later meeting of the Convention.

June 28th, 1912.

Meeting held at the Imperial College of Science.

Mr. A. Campbell, Vice-President, in the Chair.

The following Papers were read :—

1. Hysteresis Loss as affected by Previous Magnetic History. By Prof. E. Wilson and Messrs. B. C. Clayton and A. E. Power.
2. Dielectric Hysteresis at Low Frequencies. By Prof. W. M. Thornton.
3. The Efficiency of Generation of High-Frequency Oscillations by Means of an Induction Coil and Ordinary Spark-Gap. By Prof. G. W. O. Howe and Mr. J. D. Peattie.
4. The Resistance to the Flow of Water along a Capillary Sodaglass Tube at Low Rates of Shear. By Dr. A. Griffiths and Miss C. H. Knowles.
5. On the Self-Demagnetisation of Steel. By Messrs. S. W. J. Smith and J. Guild.

I. *Further Observations on the Afterglow of Electric Discharge, and Kindred Phenomena. By the* Hon. R. J. Strutt, *F.R.S., Professor of Physics, Imperial College of Science, S. Kensington.*

Received July 7, 1911. Read October 27, 1911.

The present Paper is supplementary to two others published by the Society. (1) " The Afterflow of Electric Discharge " (" Proc." Phys. Soc., XXIII., p. 66, Dec., 1910). (2) " Flames of Low Temperature Supported by Ozone " (XXIII., p. 147, Feb. 15, 1911). Acquaintance with the contents of these Papers is assumed.

§ 1. *The Glow from Ozone Produced at Atmospheric Pressure.*

In the first Paper it was mentioned that ozone from the Siemens tube used in the ordinary way at atmospheric pressure failed to give the afterglow when mixed with nitric oxide. It seemed unsatisfactory to leave the matter there, and the question pressed for answer: Why does ozone produced in the low-pressure discharge possess this property when the other does not? This question gained in interest from the fact that a polymer more complex than O_3 has been suspected to be produced by the low-pressure discharge.*

It was found that ozone from the Siemens tube, generated at atmospheric pressure, was not made capable of giving the effect by mere rarefaction, after it was produced. The gas was allowed to issue through a fine glass jet into a tube connected to the air pump, so that the pressure of the stream was reduced to a few millimetres of mercury. On the low-pressure side a stream of nitric oxide was led in through a side tube, so as to mix with it. No glow resulted. As mentioned in the first Paper, when the ozone was generated in the Siemens tube at low pressure the glow *was* produced, though not well.

The only remaining possibilities seemed to be (1) that the ozone generated at low pressure possessed a more complex molecule than 0_3 or (2) that its power of producing the glow depended on high concentration, the other kind being too much diluted with oxygen.

It was shown in the first Paper that the glow could be produced when ozone from the vacuum tube condensed in liquid air was allowed to evaporate, and then mixed with nitric oxide.*

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* Ladenburg and Lehman, " Ann. d." Phys., XXI., p. 302, 1906.

The same method was used to concentrate ozone produced at atmospheric pressure. Oxygen passed through sulphuric acid in A (*see* Fig. 1) into the Siemens tube B, and thence into the U tube C, which was cooled in liquid air, and in which oxygen and ozone liquified together to a pale-blue liquid. The current of gas was regulated by the depth to which C was immersed in liquid air. After about one minute's run,† D was closed, and connection opened to an air pump through E. The oxygen readily boiled off, as its vapour pressure is considerable at —180°C., and the colour of the liquid became deeper as its volume diminished, and the ozone became more concentrated. Finally the latter was left pure as a deep-blue solid. E was then closed, the liquid air removed, and a little nitric oxide admitted from the space between the taps F and G. The characteristic greenish-yellow flash was produced. A similar

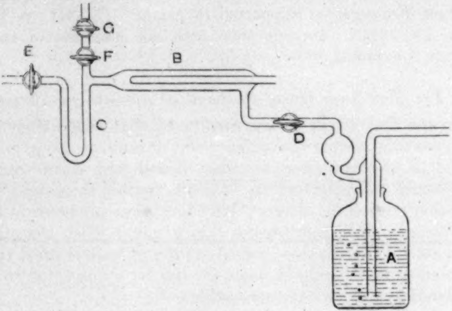

Fig. 1.

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* The following form of the experiment is easier than that originally described. A straight vacuum tube is immersed in liquid air, and oxygen admitted to a pressure of a few millimetres. The discharge is passed, ozone forms and is condensed, and the pressure runs down. The ozone may be kept as long as desired in liquid air. On removing the latter, and at once admitting a little nitric oxide, the yellow flash is seen. This method of condensing ozone was first employed by Goldstein.

† It is unsafe to accumulate large quantities of ozone, which is very explosive.

experiment when sulphuretted hydrogen was admitted gave a blue flash.

I conclude therefore that ozone made in the common way fails to give the glow because of the large admixture of oxygen, and that the low-pressure glow discharge yields ozone of much higher concentration. The great ozonising efficiency of this kind of discharge has been pointed out by Goldstein.*

§ 2. *Effect of Sulphur Compounds in Improving the Glow.*

The older experimenters observed a great improvement in the afterglow when ordinary or fuming sulphuric acid, or sulphur dioxide, were present along with air in the vessel in which the discharge was taken; and they not unnaturally concluded that the sulphur compounds must in this case play some essential part in its production. Morren, for instance,† regarded the glow as due to the formation of a compound containing oxygen, nitrogen and sulphur.

There certainly is in some cases a great improvement in the brightness of the glow when sulphur compounds are added. But its colour and general character are not altered. The following experiments throw light on the subject.

1. The electrodeless discharge was produced in a bulb containing a mixture of oxygen and nitrogen. No afterglow was obtained. Then the bulb was washed out with concentrated sulphuric acid, and afterwards repeatedly with water, and re-exhausted. A brilliant afterglow was now obtained, although no sulphuric acid remained. This experiment indicates that the function of the sulphuric acid is to remove some substance prejudicial to the afterglow—probably organic matter which destroys the ozone necessary to its development.

2. Using a continuous flow of the oxygen and nitrogen mixture through the discharge vessel, it is often noticed that the afterglow takes time to develop particularly if the apparatus has been out of use. Here again there is probably a film of greasy matter which has to be got rid of before the ozone is available for producing the glow. When the glow is well established, it is not at all improved by admitting a small proportion of sulphur dioxide into the gas stream, either before it enters the discharge tube or afterwards.

Taking these experiments together, there seems to be little doubt that the function of sulphur compounds is merely to destroy prejudicial organic matter, and that when this is accomplished they have no further effect. These remarks have of course no application to the blue variety of afterglow* which I was able to reproduce by the action of ozone on sulphur or sulphuretted hydrogen.†

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* "Ber. Deutch. chem. Gesellsch." XXXVI., p. 3043, 1903.

† "Ann. chim. et Phys." (4) IV., p. 293 (1865).

§ 3. *Does Pure Oxygen give an Afterglow?*

It is found ("Proc. Phys. Soc.," XXIII., Dec. 15, 1910) that when oxygen perfectly free from nitrogen is run through a vacuum tube the afterglow is very faint compared with that observed when nitrogen is present. Ordinarily, however, there is some trace of afterglow, which is considerably enhanced by the use of a powerful jar discharge with spark-gap, instead of the direct coil discharge. This glow is white, or, at all events, much less yellow than the nitric oxide glow, and its spectrum appears continuous as far as I have been able to observe it. What is the origin of this glow? Is it due

(*a*) To the reversion of ozone into oxygen; or

(*b*) To the interaction of ozone with some impurity?

When solid ozone, obtained as in § 1, is allowed to evaporate into an exhausted space it does not last long, quickly reverting into oxygen. But no trace of luminosity is seen when the tube is watched in the dark by a well rested eye. This is unfavourable to hypothesis (*a*).

The white afterglow is completely stopped by silver gauze, which destroys ozone. When, however, more ozone is supplied to the gas thus quenched, from a tributary stream which has been through a second discharge tube, a marked revival of luminosity is observed at the confluence. The experiment is best made passing a jar discharge in the first tube and a simple coil discharge in the second. Thus the ozone feed from the latter does not possess much disturbing luminosity of its own. This experiment, similar to that which elucidated the nature of the air afterglow, proves that two substances are involved—ozone and some other that reacts with it.

The spectrum of a discharge giving the white afterglow was photographed. The water vapour bands in the ultra-violet were the only conspicuous impurity in the photographed spectrum. The red hydrogen line was also fairly strong to visual observation.

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* *See* Newall, "Proc. Camb. Phil. Soc.," Vol. IX., p. 295, 1897.

† "Proc. Phys. Soc.," Vol. XXIII., p. 149, Feb., 1911.

It appeared, therefore, that water vapour was present to an appreciable extent, in spite of the fact that the gas had been passed through a phosphorus pentoxide drying tube. To secure better drying, some oxygen was stored for 24 hours in a glass reservoir containing phosphorus pentoxide. This specimen of gas, allowed to flow through a discharge tube, showed no signs of the red hydrogen line, and no trace whatever of afterglow. It was concluded, therefore, that the white afterglow is bound up with the presence of water vapour, and that pure oxygen does not give it.

The following experiments are confirmatory. Electrodeless discharge bulbs containing rarefied oxygen ordinarily give the white afterglow much more conspicuously than it can be got in the above flow experiments, which are necessarily carried out at higher gas pressures. Such bulbs, if well exhausted and heated, and then charged with oxygen which has been passed through a phosphorus pentoxide drying tube, still show the red line of hydrogen, often in an intensity comparable with that of the red line of oxygen. Under these circumstances the white afterglow is strong. If, however, some phosphorus pentoxide is sealed up in the bulb along with the oxygen, the red hydrogen line disappears from the discharge a few minutes after sealing off, and the afterglow disappears with it.*

The yellow glow seen in an electrodeless bulb when nitrogen is present along with oxygen is not at all hindered by prolonged drying with phosphorus pentoxide. There is, therefore, no reason to think that this substance is prejudicial to ozone.

An alternative method of proving the necessity of moisture to the white glow is by freezing out with liquid air. A tail of quill tubing is sealed to the oxygen bulb. On cooling this the hydrogen line disappears in a few minutes, and the glow with it, a perfectly pure oxygen spectrum remaining. On warming up, the hydrogen line and the glow reappear almost simultaneously. Too much water vapour is unfavourable. The effect seems to be at its best when the red line of hydrogen has about the same intensity as the neighbouring red line of oxygen.

The freezing experiment, too, might be thought ambiguous, on the ground that ozone might be supposed to be frozen out before it could produce the glow. The disappearance of the *capacity to glow*, however, takes a much longer time than the

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* There is apt to be some phosphorescence of the phosphorus pentoxide, easily seen to come from the solid, not from the gas.

30 seconds or so that the glow itself lasts when produced.* Thus, when the annexe has been fully cooled down to −180°, the afterglow can be produced repeatedly before the water vapour has had time to condense. This shows that rapid removal of ozone is not a cause to be reckoned with in explaining the experimental result.

It is difficult to get an oxygen bulb quite free of bands due to carbon dioxide, which is frozen out along with water vapour. I have, however, one bulb quite free from the carbon bands, which shows the experiment admirably. It is not necessary, therefore, to suspect the intervention of carbon.

The above experiments conclusively prove that the white glow is not obtained from pure oxygen, but that it requires the presence of water vapour, and that ozone is concerned in its formation. Indeed, the experiment in which the glow is quenched by silver gauze, and revived by the re-introduction of ozone, is held to prove that some substance is manufactured in the discharge from the ingredients oxygen and water vapour, and that this substance, which we may call X, reacts with ozone to give the glow. The obvious supposition to make is that X is hydrogen peroxide, which is known to react energetically with ozone. Experiment has compelled me to abandon this idea.

It was found that the vapour from Merck's 33 per cent. hydrogen peroxide generated no glow when fed into a stream of ozone from a vacuum discharge tube. Nor could any other result be got by completely distilling the hydrogen peroxide preparation away in the ozone stream, although in this case ozone must, at one stage, have mingled with nearly pure hydrogen peroxide.

X, then, is not water nor hydrogen peroxide. It is not hydrogen, for hydrogen does not glow when fed into ozone. I think there is no alternative left but to assume that X is a hitherto unrecognised compound of oxygen and hydrogen, which reacts with ozone to give the white glow.

Arguments of this kind, pointing to the existence of a new substance, do not constitute a satisfactory contribution to chemistry. My attempts to collect any of it for examination have not been successful, a failure which is attributed to the extremely small quantities involved. The white glow is not produced with any brilliancy except at quite low pressures. and the difficulty of producing any quantity of the material responsible for it is great.

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* That is, when the cooled annexe communicates with the bulb by a channel of quill tubing. With large tubing the disappearance is almost immediate.

I hope, however, that the hypothesis will not be dismissed without due consideration of the perfectly definite experimental results which have led to it.

§ 4. *Luminosity of Water Shaken with Ozone.*

In reading up the literature of the subject in connection with the earlier investigations, a Paper by E. Fahrig (" Chemical News," LXII., p. 39, 1890) was consulted, in which he describes a luminosity observed on shaking ozone with ordinary tap water. He was not able to arrive at any definite view of the cause of this phenomenon, but was inclined on the whole to connect it with the presence of bacteria, alive or dead.

It was at first thought that the luminosity might be due to the oxidation of nitric oxide contained in the water, being in fact the same as the afterglow in air. But distilled water saturated with nitric oxide failed to give it. Although the phenomenon was thus shown not to be directly relevant, a few experiments were made which have thrown light on its nature.

Several litres of tap water were evaporated to a small volume, and placed in a flask which was about half filled. A similar flask was half filled with water which had not been evaporated down. Equal quantities of ozone were delivered by displacement into the two flasks, which were then closed with watch glasses. On shaking the two flasks simultaneously, one in each hand, in a dark room, luminosity was observed, feeble in the tap water, much stronger in that which remained after evaporation. Further, the concentrate was able to glow again with more ozone, whereas the ordinary water was exhausted.

This proves that the constituent necessary for giving light can be concentrated by evaporation. The concentrate was observed to be brown in colour, and ozone bleached it. This at once suggested that peaty matter was present in the water, and that the luminosity was evolved in the process of oxidising and decolourising it.

Some brown peaty water was obtained from a bog in Ireland. This was found to give very much more light than London tap water. An artificial decoction made by boiling peat with water and then filtering did the same.

These experiments were considered to prove that the luminosity of ordinary water is due to the oxidation of peaty matter present in it.

ABSTRACT.

It is shown that ozone prepared by means of the Siemens ozone tube used at atmospheric pressure is able, when mixed with nitric oxide, to give the greenish-yellow afterflow flame. This result is only attained, however, when the ozone has been concentrated by fractional distillation. A blue glow is obtained under the same conditions with sulphuretted hydrogen and ozone.

The effect of sulphur compounds in improving the air glow noticed by the older experimenters is shown to be due not to any direct intervention of these bodies in the reaction, but to their power of destroying organic matter prejudicial to ozone. When once this is got rid of, the sulphur compounds are of no further advantage.

Really pure oxygen gives no afterglow whatever. The white afterglow seen in oxygen purified with ordinary precautions is due to reaction of ozone with some substance produced by the action of the discharge on water vapour.

The luminosity given out when ordinary spring water is shaken with ozone is shown to be due to oxidation of peaty matter contained in it. Brown peat water gives greatly enhanced luminosity.

DISCUSSION.

Mr. A. **Campbell** inquired whether quartz would not be better than glass in vacuum tube work for preventing occlusion of moisture and gases.

Mr. **Egerton** asked whether the afterglow might not be due to ozone decomposing into oxygen. In this case both liquid air and phosphoric pentoxide would prevent the glow by removing the ozone.

Mr. H. **Borns** asked Prof. Strutt, whether he had—in view of the difficulties of avoiding contamination of his ozone with nitrogen oxides when using ozonisers—also prepared his ozone by electrolysis of sulphuric acid.

The **Author** replied that quartz would undoubtedly be superior to glass, but pointed out the difficulties of working with it, and the impossibility of making sealed joints. In reply to Mr. Egerton he stated that there was some evidence that moisture was necessary for the afterglow in oxygen as it was found that the afterglow disappeared at the same time as the red hydrogen line. He had not prepared ozone electrolytically, but he had used the purest oxygen.

II. *Homogeneous Fluorescent X-radiations of a Second Series.** *By Professor* C. G. BARKLA, *M.A., D.Sc., and* J. NICOL, *M.A., B.Sc.*

RECEIVED SEPTEMBER 25, 1911. READ OCTOBER 27, 1911.

IT was shown by Barkla and Sadler † that many elements when subject to a suitable beam of X-rays, emit a homogeneous beam of secondary X-rays—fluorescent X-rays—of penetrating power characteristic of the radiating element. One of the writers ‡ later showed that more than one characteristic fluorescent X-radiation is emitted, and that each element in all probability emits radiations, giving what might be called a line spectrum in X-rays. It was also shown that these spectral lines could be arranged in series, each line in a given series moving towards the more penetrating end of the spectrum with an increase in the atomic weight of the radiating element. From no single element, however, was the homogeneity of more than one radiation proved, or the penetrating power accurately determined—indeed, the homogeneity of the radiation from only one series—the K series §—had been demonstrated. In a preliminary letter to "Nature,"¶ however, the writers announced the discovery of the homogeneity of the radiations in a second series—the L series—showing that the fluorescent radiations emitted by the elements give, as far as observed, line spectra. In other words, each element when exposed to Röntgen radiation emits a number of homogeneous beams of Röntgen radiation, the penetrating power of each of which is characteristic of the element. Each radiation is widely separated in penetrating power from the other radiations emitted by the same element. The total fluorescent radiation can, however, be analysed into these constituents.

It is intended in this short Paper to give more detailed information of the work establishing the homogeneity of the radiations of series L.

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* The expenses of these researches were partially covered by a Government grant through the Royal Society.

† "Phil. Mag.," September, 1907, pp. 408-422; October, 1908, pp. 550-584.

‡ Barkla, "Proc. Camb. Phil. Soc.," May, 1909, pp. 257-268.

§ *See* "Phil. Mag.," September, 1911, pp. 396-412. Series K and L were orginally denoted by letters B and A.

¶ August 4, 1910.

From a knowledge of the penetrating powers of the fluorescent X-radiations from many elements, it was evident that the most favourable conditions for investigating the character of two fluorescent radiations from one element would be obtained by experiments on elements of atomic weight in the neighbourhood of 110 to 140. From elements of lower atomic weight the radiation of series L becomes too absorbable and from those of higher atomic weight radiation of series K becomes too penetrating to permit of accurate work without the adoption of special methods. The method adopted is illustrated by the experiments on barium (atomic weight 137·37).

The substance used in these experiments was not the element itself but barium hydroxide, the only effect of the combination with hydrogen and oxygen possibly observable being a slight admixture of a scattered Röntgen radiation from these elements.* No fluorescent radiation has been found to be emitted by them. Their characteristic fluorescent radiations are probably absorbed in a few millimetres of air and do not appreciably affect an electroscope at a distance of several centimetres.

When a *penetrating* primary beam was used, a mixture of the secondary radiations characteristic of barium was emitted by the barium hydroxide, in addition to a weak scattered radiation from each of the elements, barium, hydrogen, oxygen. A sheet of aluminium, 2 mm. thick, was, however, sufficient to absorb all the soft fluorescent radiation of series L and much of the scattered radiation. The effect of this transmitted scattered radiation was inappreciable in comparison with the fluorescent radiation of series K. This radiation was then found to be homogeneous, for successive similar sheets of aluminium absorbed equal fractions; that is, the ionisation produced by the radiation in an electroscope was an exponential function of the thickness of aluminium traversed.

This is shown by the results tabulated below. Column 1 gives the thickness of aluminium traversed × times the density of aluminium; column 2 the relative deflections of the electroscope receiving the secondary radiation; column 3 the logarithm of the deflection.

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* "Phil. Mag.," June, 1906. Later experiments on many compounds have shown that these emit the radiations characteristic of the constituent elements.

TABLE I.

Thickness of aluminium absorbing sheets × density of Al.	Deflection of electroscope due to secondary rays.	Log$_{10}$ of deflection.
0·00 gr. per sq. cm.	100·00	2·000
0·48 ,, ,,	44·00	1·644
0·96 ,, ,,	30·40	1·483
1·44 ,, ,,	21·15	1·325
1·92 ,, ,,	14·72	1·168
2·40 ,, ,,	10·00	1·000

$\frac{\lambda}{\rho}$ in Al. = 0·8 for flourescent radiation (series K) from Ba.

The homogeneity is seen after the initial absorption of soft fluorescent X-rays and some scattered X-rays, by the arithmetic progression of the numbers in column 3.

The relation between the thickness of the absorbing aluminium and the logarithm of the percentage of the original ionisation which was produced by the transmitted radiation is shown in the upper curve of Fig. 1, the straight portion of K denoting the homogeneity of the fluorescent radiation K.

In order to isolate and study the more absorbable fluorescent radiation from barium—that of series L—a very soft primary beam was used—too soft to excite the more penetrating fluorescent radiation of series K. The radiation which then entered the electroscope, placed only a few centimetres from the barium hydroxide, consisted of the radiation of series L and a heterogeneous scattered radiation. In this case the latter was not negligible, in comparison with the other, partly because being much more penetrating it emerged from a much greater depth of radiating substance. It was necessary, therefore, to determine the effect of the scattered radiation alone, and to subtract this from the total effect in the electroscope. To do this, the fluorescent radiation, in this case very absorbable, was entirely cut off by a sheet of aluminium, which transmitted much of the scattered radiation. The amount of scattered radiation transmitted was then compared with the amount transmitted from a substance like aluminium, from which the radiation received by an electroscope in such a position is purely a scattered radiation. Thus the ionisation produced by the scattered radiation from barium hydroxide after transmission through 0·163 gramme per square centimetre of aluminium was 106, while that from a certain sheet of aluminium under similar conditions was 396 (*see* Table II.). But the effect of the scattered radiation from aluminium after transmission through various thicknesses of aluminium was directly determined by experiment and the values given in the third series of readings in Table II. were obtained. From these then by simple proportion the effect of the scattered radiation from barium hydroxide under similar conditions was obtained.* Thus its direct effect without any absorbing plates is approximately $\frac{106}{396} \times 1{,}367$, for 1367 is the direct effect from the aluminium radiator. The numbers, then, in column 3 denoting the effect of the scattered radiation from

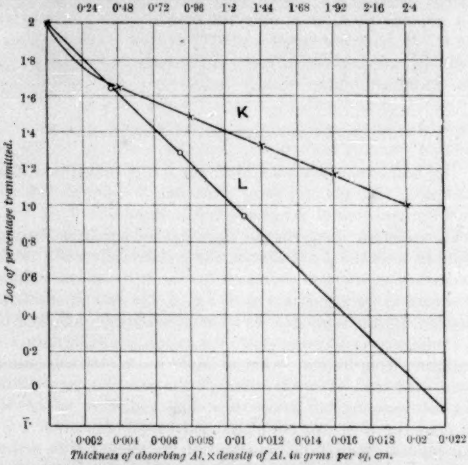

FIG. 1.—SHOWING HOMOGENEITY OF THE TWO OBSERVED FLUORESCENT X-RADIATIONS CHARACTERISTIC OF BARIUM.

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* This is justified by the following laws, (*a*) X-rays of all penetrating powers are scattered to the same extent in passing through matter ("Phil. Mag.," May, 1904, pp. 543-560, and later papers); (*b*) the ratio of the absorption coefficients for a given radiation in two substances is approximately independent of the penetrating power of the radiation provided the range of penetrating power does not include or approach on the more penetrating side a fluorescent X-radiation characteristic of either substance ("Phil. Mag.," May 1909, pp. 739-760). Limits to the first law may yet be found, but it certainly holds over the range of penetrating power here dealt with.

barium hydroxide are proportional to those giving the effect from aluminium.

After subtraction of the numbers in column 3 from those in column 2 representing the total ionisation due to the fluorescent and scattered radiations combined, we get the values of column 4, representing the ionisations due to the fluorescent rays alone after transmission through the thickness of aluminium given in column 1. If the radiation is homogeneous the intensity I after transmission of a radiation of initial intensity I_0, through thickness x, is given by $I = I_0 e^{-\lambda x}$, where λ is a constant. Thus $\frac{\log_e I_0/I}{\rho x} = \frac{\lambda}{\rho}$ [ρ is density of aluminium]. Column 5 gives the quantity $\frac{\log_{10} I_0/I}{\rho x'}$ for the *fluorescent X-radiation* from barium hydroxide and for the *scattered X-radiation* from aluminium. It should be noticed that x' equal to $1{\cdot}115x$, the effective thickness of absorbing aluminium, replaces x the actual thickness of the absorbing sheet (*see* below).

The agreement between the values given in column 5 shows the constancy of λ, and thus the homogeneity of the fluorescent radiation from barium. The reading for the fluorescent radiation effect (column 4) after transmission through 0·01966 gramme per square centimetre of aluminium is so small, and consequently so dependent on the accuracy of the scattered radiation effect, that it is useless as an accurate measure of the absorbability of the radiation. Seeing from the values in column 5 that the fluorescent radiation was homogeneous, this reading was used to more accurately estimate the scattered radiation effect. Thus, taking 107·5 instead of 106 as the deflection due to scattered radiation after transmission through 0·163 gramme per square centimetre of aluminium, and the other effects in proportion, the small deflection due to the fluorescent radiation through 0·01966 gramme per square centimetre becomes perfectly consistent. Although the value so obtained may not be used to determine the absorbability of the fluorescent radiation, it is useful as further confirmation of the homogeneity shown by the first three successive absorptions, for the difference between 106 and 107·5 is less than the possible error of experiment.*

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* The scattered radiation effects proportional to those from aluminium might have been got entirely by trial, so as to leave a homogeneous fluorescent radiation. The possibility of this would have shown the homogeneity.

The variations between the values 101·4, 94·5, 96·1, 96·0, given in the first series in column 5, and better still 98·8, 97·2, 96·1, 96·9 in the second series are within the possible errors of experiment. The contrast between these and those of the third series (Table II.) for the scattered radiation from aluminium is very pronounced. Again, the homogeneity is shown graphically in Fig. 1, the straight line L denoting the linear relationship between the logarithm of percentage transmitted and the thickness of absorbing aluminium. It should be noticed that the scale of abscissæ has been increased to 240 times that used for the K radiation.

We have thus established the homogeneity of both the observed fluorescent X-radiations emitted by barium.

TABLE II.—*Radiator : Barium Hydroxide.*

1. Thickness of Al. absorbing sheets × density of Al.	2. Deflection of electroscope due to secondary rays.	3. Deflection of electroscope due to scattered X-rays.	4. Deflection of electroscope due to fluorescent X-rays.	5. $\frac{\log I_0/I}{\rho x'}$.
gr. per sq. cm.				
0·0	1,648	371	1,277	...
0·0032	894	339	555	101·4
0·00662	572	316	256	94·5
0·00982	414	301	113	96·1
0·01966	278	268	10	96·0
0·163	106	[106] 107·5	...	...
0·0	1,708	377	1,331	...
0·0032	935	344	591	98·8
0·00662	576	321	255	97·2
0·00982	424	306	118	96·1
0·01966	282	272	10	96·9
		Radiator : Aluminium.		
0·0	1,367	1,367	...	...
0·0032	1,247	1,247	...	11·18
0·00662	1,165	1,165	...	9·41
0·00982	1,108	1,108	...	8·33
0·01966	987	987	...	6·45
0·163	396	396	...	2·96

$\frac{\lambda}{\rho}$ in Al. = 224 for fluorescent radiation (series L) from Ba.

The correction of the thickness of absorbing aluminium from that of the plates as given in column 1 to 1·115 times that value was necessary owing to the proximity of the radiating substance to the detecting electroscope. Much of the radiation traversed the absorbing plate at a considerable angle to the normal, and thus the effective thickness of the absorbing substance was greater than that given in column 1. To obtain the necessary constant connecting the thickness of the aluminium plate and the effective thickness, experiments were made upon the well-known radiations from copper and iron. Taking the absorption coefficients for these radiations in aluminium as known,* the effective thickness of the aluminium absorbing sheets was found to be about 1·115 times the actual thickness. The correction appeared somewhat large for obliquity alone, but a possible error of 3 or 4 per cent. in the accepted values for either copper or iron in these results has no special importance. We have, therefore, made use of the full correction factor on the assumption that the values for copper and iron are correct. Thus, in column 5 of Table II., giving $\frac{\log_{10} I_0/I}{\rho x'}$, the effective thickness x' is taken as 1·115 times the thickness of the plate given in column 1.

Taking the average of the values in column 5, for the two series of experiments, there is excellent agreement, and dividing the result by $\log_{10} e$, we get for the value of $\frac{\lambda}{\rho}$ in aluminium 224.

The value of $\frac{\lambda}{\rho}$ obtained from the results of Table I. for the penetrating fluorescent X-radiation K is 0·8.

From similar experiments with iodoform as the radiating substance, both the hard and soft fluorescent radiations characteristic of iodine were isolated and proved homogeneous. The values of $\frac{\log I_0/I}{\rho x'}$ were slightly more variable than in the case of barium, one series of values, for instance, being 131·2, 128·5, 117·3, 126·0, and giving a value of λ in Al. of about 290. From a number of experiments the values found were (series K) $\frac{\lambda}{\rho}=0{\cdot}92$; (series L) $\frac{\lambda}{\rho}=300$ approximately.

Similarly, from antimony, the radiations corresponding to these were found to have absorbabilities $\left(\frac{\lambda}{\rho}\text{ in Al.}\right)$ of 1·21 and 435. The latter value cannot be regarded as satisfactory, the possible error of experiment being unusually large. A re-determination is desirable.

The absorbability of the K radiation from silver has been previously published, the value of $\frac{\lambda}{\rho}$ being 2·5.[*] The soft radiation (L series) was investigated. The thinnest rolled aluminium used in these experiments was found to absorb too much of this radiation to permit of accurate work. The absorption by thin sheets of paper was found instead, and the thickness of aluminium equivalent to these was determined by absorbing copper and iron K radiations, by both paper and aluminium.[†] Again the radiation was found to be homogeneous. Two series of readings gave values for $\frac{\lambda}{\rho}$ of 719 and 696—values as consistent as can be expected from the nature of the experiments. In one of these experiments the K radiation was not experimentally eliminated, so the ionisation produced both by the scattered radiation, and by the K radiation was determined from observations of the mixture of these two after absorption of the L radiation. We may take the true value of $\frac{\lambda}{\rho}$ as approximately 700. Thus the two known fluorescent radiations from silver give values 2·5 and 700.

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* $\frac{\lambda}{\rho}$ in Al. for the fluorescent X-radiations (series K) from Fe. and Cu. are taken as 88·5 and 47·7 respectively.

The substance of lowest atomic weight from which the homogeneity of a fluorescent radiation had previously been proved was chromium. The K radiation from calcium was therefore investigated in a manner similar to that from barium and the other elements mentioned. After correction for the scattered radiation, the fluorescent radiation proved to be homogeneous. After 79 per cent. absorption the transmitted radiation was found to possess the same absorbability within about 1 per cent.—less than the possible error of experiment. The value of $\frac{\lambda}{\rho}$ was found to be 435.

Summary.

This Paper thus shows the homogeneity of some of the fluorescent X-radiations in a series (the L series) in which homogeneity had not previously been proved. Details are given of experiments showing the homogeneity of both the K and L radiations from barium; and similar results are recorded in the case of radiations from iodine, antimony and silver. The absorbability of the fluorescent radiations from these elements is given by the following values of $\frac{\lambda}{\rho}$ for absorption in aluminium of density ρ.

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[*] Barkla and Sadler, "Phil. Mag.," May, 1909, pp. 739-760.

[†] This is justified by the laws of absorption. The method has also been applied to radiations whose absorbability in aluminium was directly determined, and very good agreement was found between the two.

Ag	radiator :	(series K)	[2·5] ;	(Series L)	700
Sb	,,	,,	1·21	,,	435
I	,,	,,	0·92	,,	300
Ba	,,	,,	0·8	,,	224

The series of K radiations has also been extended to include calcium, and the value of $\frac{\lambda}{\rho}$ found to be about 435.

It has also been found that the fluorescent radiations of both series are emitted simultaneously as distinct homogeneous beams.

ABSTRACT.

It was shown by one of the writers that the fluorescent X-radiations emitted by elements exposed to primary Röntgen radiation can be arranged in series, one radiation in a particular series being emitted by each element, and the radiation belonging to that series becoming more penetrating with an increase in the atomic weight of the radiating element ("Proc." Camb. Phil. Soc., May, 1909). The homogeneity of radiations of only one series (the K series) was shown by Barkla and Sadler. This Paper describes experiments showing the homogeneity of fluorescent X-radiations in the second series (the L series), and exhibiting the homogeneity of radiations of the two series from a number of elements. Details are given of the observations on the radiations from barium. Similar results are recorded in the case of the radiations from iodine, antimony and silver.

The homogeneous fluorescent radiations of different series are emitted simultaneously by an element exposed to Röntgen radiation of more penetrating type than either. The fluorescent X-radiations can thus be analysed into well defined radiations widely different in penetrating power, and may be said to give a line spectrum in X-rays.

The absorbability of the fluorescent radiations is given by the following values of $\frac{\lambda}{\rho}$, where λ is the coefficient of absorption of these radiations in aluminium of density ρ.

Ag	radiator :	(Series K)	[2·5] ;	(Series L)	700.
Sb	,,	,,	1·21	,,	435
I	,,	,,	0·92	,,	300
Ba	,,	,,	0·8	,,	224

DISCUSSION.

Dr. S. **Russ** inquired as to the mechanism of the production of scattered radiations, and asked whether scattered radiation would be produced by homogeneous radiations.

Prof. C. G. **Barkla**, in reply, said that scattered radiation was produced by a pulse passing over an electron disturbing it, and so causing it to radiate a pulse of the same wave length or hardness. No satisfactory explanation of either X-ray fluoresence or ordinary fluoresence had been given, but he pointed out that after the pulse had passed over an electron the latter would continue to vibrate with its natural frequency.

III. *Demonstrations given before the Society by Prof.* S. P. THOMPSON, *F.R.S., at the Finsbury Technical College.*

NOVEMBER 10, 1911.

Prof. Silvanus P. Thompson and Prof. E. G. Coker exhibited two reflecting polariscopes for the study of optical stress in materials. The polariser in each is a sheet of blackened glass at the polarising angle. One has a field 6 in. square, the other 10 in. square. They were designed for the purpose of studying the stresses in objects cut out in sheets of xylonite (celluloid) subjected to mechanical tension. To eliminate the black-cross effect the objects are placed between two quarter-wave plates of mica, having their axes set at 45 deg. Some larger quarter-wave plates of mica were shown, one of them 24 in. in diameter.

Prof. Silvanus P. Thompson exhibited a large harmonograph for inscribing on a moving ribbon of paper the harmonic movements produced by compounding together mechanically the oscillations of two heavy pendulums. The recording pencil consists of a glass tube drawn to a fine aperture, in which tube is inserted an ordinary sable-hair paint brush, dipped in ink, so that the mere point of the brush projects through the orifice. He also described his apparatus (shown in operation in the laboratory) for demonstrating the physiological effect of an alternating magnetic field. An alternating electric current with a frequency of 50 periods per second is led into two large copper coils, producing an alternating magnetic field of an intensity of about 10,000 C.G.S. units in the space between the coils. If the head is placed in this space a singular sensation of a flickering blueish illumination is perceived. The effect is greatest if the direction of the lines of force of the field is through the temples.

Prof. Thompson also demonstrated several experiments in acoustics. These were: (*a*) A Rubens' tube 15 ft. in length, with a row of small gas flames along it, exhibiting the positions of the nodes and ventral segments when the internal gas column was thrown into partial modes of vibration by shrill tones; (*b*) a Rayleigh flame of peculiar sensitiveness; (*c*) a dynamical model in glass of the three semicircular canals of the ear. The canals being filled with coloured liquid, and containing model otoliths (made of glass bubbles filled with air), movement of rotation about any one of the three axes sets the liquid in the corresponding canal into relative motion. (*d*) A set of steel bars for demonstrating the superior limit of audition; (*e*) a *ton-variator* of Dr. Stern, consisting of a resonant cavity of adjustable volume, excited by a gentle air-blast, and graduated to produce any tone between the frequencies of 200 and 400 per second; (*f*) an adjustable resonator, in which the size of the front opening is altered by use of an iris diaphragm; (*g*) a new phenomenon in resonance. If a tuning fork is introduced into a tubular resonator having a front orifice just wide enough to admit the two prongs, resonance is observed when the prongs are placed just in the opening. On inserting them deeper the sound ceases, but is again heard if they are inserted still further.

IV. *Note on the Effect of a Narrow Saw-cut in the Edge of a Conducting Strip on the Resistance of the Strip. By Prof.* CHARLES H. LEES, *D.Sc., F.R.S.*

IN connection with the Paper on the above subject which appeared in Vol. XXIII., p. 361 (1911), of the "Proceedings," Lord Rayleigh has called my attention to a Paper by Prof. Lamb, on "The Reflection and Transmission of Electric Waves by a Metallic Grating," which appeared in the "Proceedings" of the London Mathematical Society, Vol. XXIX., p. 523 (1898). In the first part of this Paper Prof. Lamb deals with certain problems in electrostatics, one of which is that of the lines of electrostatic force in the neighbourhood of a grating of parallel conducting strips. The solution of this problem is obtained by the aid of the Schwarzian transformation, $\cosh w = a \cosh z$, used by me in the above Paper, and Prof. Lamb points out that the solution also applies to the case of a conducting strip with a saw-cut in its edge. The use of the transformation for the solution of this problem had, therefore, been anticipated by Prof. Lamb. In a footnote to his Paper, Prof. Lamb points out that the transformation was already known to Prof. Sir J. Larmor in 1895, as he made it the subject of a question in the Mathematical Tripos, Part II.

V. *Some Surface Tension Phenomena exhibited by* MR. C. R. DARLING *before the Society.*

NOVEMBER 10, 1911.

IN addition to aniline, which has the same density as water at 64 deg., the author has found the following liquids to possess equidensity temperatures: Anisol at 15, orthotoluidine at 24, butyl benzoate at 48 and aceto-acetic ether at 53. These results have been applied in the production of large drops of liquids, by forming the same in water, when, owing to the slight difference in density between the drop and its surroundings the process is much slower than in the case of a drop falling in air, and may be followed clearly by the eye.

Spherical drops of large size may be produced by taking water in a beaker and one of the above liquids at the temperature of equal density. The liquid is admitted through the stem of a funnel, about 1 cm. diameter, furnished with a tap, the stem terminating below the surface of the water. The liquid sphere forms on the end of the stem, and, if the temperatures be correctly adjusted, may be made to possess a volume of 100 c.c. or more.

Large breaking drops, falling vertically, may be obtained if a liquid slightly denser than water is taken. The liquid is allowed to run into water through a tube terminating in a widened portion about 2 cm. in diameter, the flow being controlled by a tap. A pendent drop forms on the end of the tube, which gradually elongates and finally breaks away from the tube, drawing out a neck of liquid, which is afterwards resolved into two or more secondary drops. In the case of small drops breaking from the end of a tube only one small droplet is observed to follow the major portion, and in some cases is not formed. With large drops, however, three droplets of diminishing diameter are generally formed. When the water used is drawn from the tap, orthotoluidine at the temperature of the room may generally be used with satisfactory results; if observed to be of insufficient density, a little aniline may be added.

Large breaking drops rising vertically are obtained when the liquid is less dense than the water, the delivery tube in this case being bent upwards; the drop rises and finally breaks away, usually giving rise to a single droplet. Orthotoluidine at 30 deg., discharged into water at 30 deg., is suitable for this experiment.

Supported drops, or liquid columns, may be obtained by taking a large test tube containing a shallow layer of water and pouring into it a quantity of aceto-acetic ether sufficient to form a hanging drop which rests on the bottom of the vessel. The liquid then remains in equilibrium in the form of a column of curved outline surrounded by water, the upper end clinging to the water surface. On adding more water gradually the column becomes thinner and finally breaks, leaving a large drop at the bottom of the vessel and a smaller drop hanging from the surface of the water. Butyl benzoate is also suitable for this experiment.

A quantity of aniline in water at temperatures above 70 deg. alternately rises to the surface and breaks off in the form of a drop which sinks to the bottom of the vessel, the two processes being repeated indefinitely. This is due to the liquid cooling on the surface until denser than the water beneath, when a drop is detached; but in the lower part of the vessel the drop is re-heated until less dense than the water, when it rises to the surface. An investigation of the temperatures of the water and aniline made by means of a thermal junction showed that the temperature of the floating aniline was always about 6 deg. below that of the surrounding water, probably on account of the lower specific heat of aniline (0·52).

When a small quantity of impure orthotoluidine is poured upon the surface of water the floating mass is set into violent agitation, finally breaking up into numerous small spheres. On adding a further quantity a number of globules of circular section are formed, which periodically become indented on one side, so as to resemble a kidney in shape; after which they dart rapidly across the surface. A period of repose follows, after which the globules again become indented and are projected across the surface as before. This procedure is repeated, with decreasing frequency, for two hours or more. When a tranquil state is reached a single large globule is usually observed in the centre of the surface.

VI. *The Maximum Value of the Electric Stress between Two Unequal Spherical Electrodes. By* ALEXANDER RUSSELL, *M.A., D.Sc., M.I.E.E.*

RECEIVED OCTOBER 10, 1911. READ NOVEMBER 24, 1911.

WHEN equal spherical electrodes are used the maximum value of the potential gradient between them can be found at once from tables.* We only need to know their size, their distance apart and the P.D. between them. In this case, however, the sparking distances do not show how the value of the disruptive stress varies with the polarity of the applied voltage, as the sparking distances are the same whichever electrode is positive. To obtain this information it is necessary to find the sparking distances with unequal sized electrodes, and to calculate the maximum electric stress between them at the instant of the discharge.

Unfortunately, the computation of the electric stress when unequal electrodes are used is very laborious. From a knowledge of their radii, their distance apart and the P.D. between them, we have first to calculate the potentials of each. We then compute the electric stress from a complicated series formula which in many cases converges very slowly. The Author has, therefore, been led to attempt to simplify the formulæ so as to shorten the time required for the computation which to many is at present prohibitive. In the general case he has made them quite manageable. In the particular case, when the distance between them is small compared with the diameter of either, he has found an extremely simple formula for the maximum electric stress.

Computation of the Potentials of the Electrodes.

When making experiments on sparking distances the apparatus is practically always so arranged that the charges on the two electrodes are equal and of opposite sign at the instant of the discharge. Let us suppose that q, $-q$, v_1 and v_2 are the charges and potentials of the spheres, and let $k_{1.1}$, $k_{2.2}$ and $k_{1.2}$ be their capacity coefficients. We have

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* "Phil. Mag." [6], Vol. II., p. 258, 1906.

$q=k_{1,1}v_1+k_{1,2}v_2$, and $-q=k_{2,2}v_2+k_{1,2}v_1$. Hence, we easily deduce that

$$v_1=\frac{k_{2,2}+k_{1,2}}{k_{1,1}+k_{2,2}+2k_{1,2}}\mathrm{V}, \quad \ldots \ldots \quad (1)$$

$$v_2=-\frac{k_{1,1}+k_{1,2}}{k_{1,1}+k_{2,2}+2k_{1,2}}\mathrm{V}, \quad \ldots \ldots \quad (2)$$

where $\mathrm{V}=v_1-v_2=$ the P.D. between the spheres. The Author has recently* given formulæ by means of which these coefficients can be fairly readily computed.

We shall now find formulæ from which the maximum value $\mathrm{R}_{\mathrm{max.}}$ of the electric stress between the electrodes can be found.

Formula for the Maximum Electric Stress.

Let a be the radius of the smaller sphere, b the radius of the larger and c the distance between their centres. The potential gradient is a maximum at that point of the smaller sphere where the line joining their centres cuts its surface. Let us first suppose that the potential of the sphere whose radius is a is unity, and that the potential of the other is zero. Then, following Jeans† and using Kelvin's method of images, we get

$$\mathrm{R}_{\mathrm{max.}}=\frac{e_0}{a^2}+\frac{e_1}{(a-a_1)^2}+\frac{e_2}{(a-a_2)^2}+\ldots$$
$$-\frac{e'_1}{(c-b_1+a)^2}-\frac{e'_2}{(c-b_2+a)^2}-\ldots,$$

where $\dfrac{a^2}{a_s}=c-\dfrac{b^2}{c-a_{s-1}}$, $\dfrac{b^2}{b_s}=c-\dfrac{a^2}{c-b}$,

$e_s=-ae'_s/(c-b_s)$, $e'_s=-be_{s-1}/(c-a_{s-1})$, $a_0=0$, $b_1=b^2/c$
$e_0=a$, and $e'_1=-ab/c$.

Hence we find that

$$\mathrm{R}_{\mathrm{max.}}=\sum_0^\infty\frac{a+a_s}{(a-a_s)^2}\cdot\frac{e_s}{a}$$

and

$$\mathrm{R}'_{\mathrm{max.}}=-\sum_0^\infty\frac{b+b_{s+1}}{(b-b_{s+1})^2}\cdot\frac{e'_{s+1}}{b},$$

where $\mathrm{R}'_{\mathrm{max.}}$ is the electric stress at the point where the line joining the centres cuts the larger sphere. It, therefore, follows by the principle of superposition that when the potentials of the spheres are v_1 and v_2 respectively, the maximum value of the electric stress, which is at the point where the central line cuts the smaller sphere, is given by

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* "Proc." of the Phys. Soc. Vol. XXIII., p. 352, 1911.

† "Electricity and Magnetism," p. 193.

$$R_{\text{max.}}=v_1\Sigma\frac{a+a_s}{(a-a_s)^2}\cdot\frac{e_s}{a}+v_2\Sigma\frac{a+a'_{s+1}}{(a-a'_{s+1})^2}\cdot\frac{e''_{s+1}}{a}$$

and it is easy to prove by the Calculus of Finite Differences, or by simple verification, that

$$a_s=\frac{a\xi(1-\alpha^{2s})}{1-\xi^2\alpha^{2s}}\,;\quad e_s=\frac{a\alpha^s(1-\xi^2)}{1-\xi^2\alpha^{2s}}\,;$$

$$a'_{s+1}=a\alpha\frac{1-\eta^2\alpha^{2s}}{\eta(1-\alpha^{2s+2})}\text{ and }e''_{s+1}=-\frac{ab}{c}(1-\alpha^2)\frac{\alpha^s}{1-\alpha^{2s+2}},$$

where $\xi=\dfrac{a+b\alpha}{c}$, $\eta=\dfrac{b+a\alpha}{c}$, $\xi\eta=\alpha$, and α is the root of the equation $(a\alpha+b)(b\alpha+a)=c^2\alpha$, which is less than unity.

It will be noticed that e''_s is the charge at the sth image in the smaller sphere when its potential is zero and the potential of the larger is unity, corresponding to the charge at the sth image in the larger sphere when the potentials are reversed.

Thus, we get

$$R_{\text{max.}}=\frac{(1+\xi)^2}{a(1-\xi)}\Bigg[v_1\left\{\frac{1-\xi}{(1+\xi)^2}+\alpha\frac{1-\xi\alpha^2}{(1+\xi\alpha^2)^2}+\alpha^2\frac{1-\xi\alpha^4}{(1+\xi\alpha^4)^2}+\ldots\right\}$$
$$-v_2\left\{\eta\frac{1-\eta\alpha}{(1+\eta\alpha)^2}+\alpha\eta\frac{1-\eta\alpha^3}{(1+\eta\alpha^3)^2}+\alpha^2\eta\frac{1-\eta\alpha^5}{(1+\eta\alpha^5)^2}+\ldots\right\}\Bigg].\quad(3)$$

Except in the case when the spheres are far apart, several terms of both series in (3) have to be calculated, and so the computation is laborious.

If we write* $\varepsilon^{\alpha_1}=1/\xi$, $\varepsilon^{\beta_1}=1/\eta$ and $\varepsilon^{\omega_1}=1/\alpha$, (3) becomes

$$R_{\text{max.}}=\frac{\cosh^2(\alpha_1/2)}{a\sinh(\alpha_1/2)}$$
$$\Bigg[v_1\left\{\frac{\sinh(\alpha_1/2)}{\cosh^2(\alpha_1/2)}+\frac{\sinh(\alpha_1/2+\omega_1)}{\cosh^2(\alpha_1/2+\omega_1)}+\frac{\sinh(\alpha_1/2+2\omega_1)}{\cosh^2(\alpha_1/2+2\omega_1)}+\ldots\right\}$$
$$-v_2\left\{\frac{\sinh(\omega_1-\alpha_1/2)}{\cosh^2(\omega_1-\alpha_1/2)}+\frac{\sinh(2\omega_1-\alpha_1/2)}{\cosh^2(2\omega_1-\alpha_1/2)}+\ldots\right\}\Bigg],\quad\ldots\quad(4)$$

where $\sinh\alpha_1=\lambda/a$, $\sinh\omega_1=\lambda c/ab$,
and $4c^2\lambda^2=(c+a+b)(c+a-b)(c-a+b)(c-a-b)$.

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* "Proc." of the Phys. Soc. Vol. XXIII., p. 353, 1911.

When more elaborate tables of hyperbolic sines and cosines are available, (4) will be useful, but even in this case the labour will be great when the spheres are close together.

Improved Formulæ.

Formula (3) only converges rapidly when the spheres are far apart. It can, however, be easily converted into one in which the series portion converges with any required rapidity. To do this we use a method similar to that employed in modifying Kirchhoff's formulæ for $k_{1,1}$ and $k_{1.2}$ (*see* "Proceedings," Vol. XXIII., p. 354). Keeping the first four terms, for instance, and expressing ξ and α in terms of a, b and c, we get

$$\begin{aligned}R_{\text{max.}}=\frac{v_1}{a}\Bigg[1+\frac{ab\{c(c+a)-b^2\}}{\{c(c-a)-b^2\}^2}+\alpha\frac{(1+\xi)^2}{1-\xi}\{&A_1-3A_3(\xi\alpha^2)\\&+5A_5(\xi\alpha^2)^2-\ldots\}\Bigg]\\-\frac{v_2}{a}\Bigg[\frac{b(c+a)}{(c-a)^2}+\frac{ab^2\{(c+a)^2(c-a)-cb^2\}}{\{(c-a)^2(c+a)-cb^2\}^2}&\\+\eta\alpha\frac{(1+\xi)^2}{1-\xi}\{A_1-3A_3(\eta\alpha^3)+5A_5(\eta\alpha^3)^2-\ldots\}\Bigg],&\quad (5)\end{aligned}$$

where $A_n=\alpha^n/(1-\alpha^n)$, and noticing that

$$\frac{1-x}{(1+x)^2}=1-3x+5x^2-7x^3+\ldots.$$

Except when x is nearly unity—that is, except when the spheres are very close together, the series in (5) converge rapidly.

Spheres Very Close Together.

To get a manageable formula for the case when the spheres are very close together, we apply the Euler-Maclaurin sum formula to (4). We have

$$\frac{\sinh x}{\cosh^2 x}+\frac{\sinh(x+\omega)}{\cosh^2(x+\omega)}+\frac{\sinh(x+2\omega)}{\cosh^2(x+2\omega)}+\ldots,$$

$$=\frac{\operatorname{sech} x}{\omega}-\tfrac{1}{2}\frac{\partial}{\partial x}\operatorname{sech} x\mp\frac{B_1}{\underline{|2}}\omega\frac{\partial^2}{\partial x^2}\operatorname{sech} x-\frac{B_3}{\underline{|4}}\omega^3\frac{\partial^4}{\partial x^4}\operatorname{sech} x+\ldots$$

where B_1, B_3, B_5 . . . are Bernoulli's numbers. Using this theorem in (4), and developing approximate formulæ for the various functions on the assumption that $c-a-b$ is small, we find after very heavy algebraical work that

$$R_{\text{max.}}=\frac{v_1-v_2}{x}\left\{1+\frac{2b-a}{3ab}x+\frac{4(a-b)^2+ab}{45a^2b^2}x^2+\ldots\right\}, \qquad (6)$$

where $x=c-a-b$=the distance between the spheres. In this case, therefore, it is unnecessary to compute the values of v_1 and v_2, as their difference only is required.

Numerical Examples.

I. Let us suppose that $a=2\cdot1r$, $b=9\cdot6r$, and $c=12\cdot5r$, so that $x=0\cdot8r$, $\xi=7/15$, $\eta=5/6$ and $\alpha=7/18$.

Hence, by the formulæ given in a previous Paper (*l.c.*, *ante*), we find that $k_{1,1}=3\cdot168\,414\,7\ldots$, $k_{2,2}=11\cdot608\,133\,8\ldots$ and $-k_{1\,2}=2\cdot497\,135\,7\ldots$

We thus find by (1) and (2) that

$$v_1=0\cdot931\,378\,2\ldots\text{V} \quad\text{and}\quad v_2=-0\cdot068\,621\,8\ldots\text{V}.$$

By (5) also we have

$$\begin{aligned}R_{\text{max.}}&=\frac{v_1}{2\cdot1r}\{1+1\cdot271\,945+0\cdot998\,148\\&\qquad-0\cdot020\,754+0\cdot000\,351-0\cdot000\,004+\ldots\}\\&\quad-\frac{v_2}{2\cdot1r}\{1\cdot295\,758+1\cdot129\,597+0\cdot831\,790\\&\qquad-0\cdot012\,010+0\cdot000\,141-0\cdot000\,002+\ldots\}\\&=\frac{v_1}{2\cdot1r}\times3\cdot249\,685-\frac{v_2}{2\cdot1r}\times3\cdot245\,274\\&=\frac{\text{V}}{0\cdot8r}\times1\cdot237\,870.\end{aligned}$$

By formula (6) we get at once

$$R_{\text{max.}}=\frac{\text{V}}{0\cdot8r}\times1\cdot235.$$

It will be seen that for most practical purposes the formula (6) will suffice in this case when the distance between the spheres is not greater than the third of the radius of the smaller sphere.

II. As a further example, let us suppose that $a=r$, $b=7r$ and $c=10r$, so that $x=2r$=twice the radius of the smaller sphere. We have $\xi=1/5$, $\eta=5/7$ and $\alpha=1/7$, and hence we get $k_{1\,1}=1\cdot160\,112\,4r$, $k_{2,2}=7\cdot576\,597\,0r$ and $-k_{1\,2}=0\cdot814\,326r$.

Hence $v_1=0{\cdot}951\,353\,0\text{V}$ and $v_2=-0{\cdot}048\,647\,0\text{V}$. We thus find by (5) that

$$\text{R}_{\text{max.}}=\frac{v_1}{r}\left[1+\frac{427}{1681}+\frac{3}{70}-\frac{27}{35\times 342\times 245}+\ .\ .\ .\ \right]$$

$$-\frac{v_2}{r}\left[\frac{77}{81}+\frac{49\times 599}{401^2}+\frac{3}{98}-\frac{135}{99^3\times 342}+\ .\ .\ .\ \right]$$

$$=\frac{v_1}{r}\times 1{\cdot}296\,863\,4-\frac{v_2}{r}\times 1{\cdot}163\,756\,0$$

$$=\frac{\text{V}}{2r}\times 2{\cdot}580\,775\,6.$$

If we use formula (6) we get

$$\text{R}_{\text{max.}}=\frac{\text{V}}{2r}\times 2{\cdot}512.$$

It is interesting to notice that even in this case—where the distance between the spheres equals the diameter of the smaller one and the numerical value of the potential of the large sphere is only about the twentieth part of the potential of the small one—the error introduced by using the approximate formula (6) is less than 3 per cent.

Equal Spheres.

When the spheres are equal, so that $a=b$, and $v_1=-v_2=\text{V}/2$, formula (4) becomes

$$\text{R}_{\text{max.}}=\frac{\text{V}}{2a}\cdot\frac{\cosh^2(a_1/2)}{\sinh(a_1/2)}\left\{\frac{\sinh(a_1/2)}{\cosh^2(a_1/2)}+\frac{\sinh(3a_1/2)}{\cosh^2(3a_1/2)}+\frac{\sinh(5a_1/2)}{\cosh^2(5a_1/2)}+\ .\ .\ .\ \right\}, \quad (7)$$

where $\cosh a_1=c/2a$.

The formula (5) also becomes

$$\text{R}_{\text{max.}}=\frac{\text{V}}{2a}\left[1+\frac{a(c+a)}{(c-a)^2}+\frac{a^2\{c(c+a)-a^2\}}{\{c(c-a)-a^2\}^2}+\frac{a^3\{(c+a)^2(c-a)-ca^2\}}{\{(c-a)^2(c+a)-ca^2\}^2}+\frac{(\xi^2+\xi^3)^2}{1-\xi}\left\{\frac{1}{1-\xi}-\frac{3\xi^9}{1-\xi^3}+\frac{5\xi^{18}}{1-\xi^5}\ .\ .\ .\ \right\}\right], \quad (8)$$

where $\xi=\{c-\sqrt{c^2-4a^2}\}/2a=\varepsilon^{-a_1}$

Equal Spheres Very Close Together.

Applying the Euler-Maclaurin sum formula to (7) we get

$$R_{\max.}=\frac{V}{x}\left\{1+\frac{1}{3}\cdot\frac{x}{a}+\frac{1}{45}\cdot\frac{x^2}{a^2}+\frac{2}{945}\cdot\frac{x^3}{a^3}+\frac{17}{14175}\cdot\frac{x^4}{a^4}+\ .\ .\ .\right\}. \quad (9)$$

The first three terms of this series agree with Schuster's well-known formula*; but Schuster gives 73/53760 as the coefficient of the fourth term. The first three terms may be deduced by putting $a=b$ in (6). In using the formula (9) it is convenient to put $2/945=0{\cdot}002116$ and $17/14175=0{\cdot}001199$.

Numerical Example.

Let us suppose that $c=2{\cdot}5a$, so that $x=a/2$ and $\xi=0{\cdot}5$. Substituting in (8) we get

$$R_{\max.}=\frac{V}{2a}\left\{1+\frac{14}{9}+\frac{124}{121}+\frac{15{\cdot}875}{(5{\cdot}375)^2},\right.$$
$$\left.+\frac{9}{16}\left(1-\frac{3}{7}\cdot\frac{1}{128}+\frac{5}{31}\cdot\frac{1}{128^2}-\frac{7}{127}\cdot\frac{1}{128^3}+\ .\ .\ .\right)\right\}$$
$$=\frac{V}{a/2}\times 1{\cdot}172\,614\,8.$$

By formula (9) we get

$$R_{\max.}=\frac{V}{a/2}\times 1{\cdot}172\,5\,6.$$

Hence when the distance between the spheres is not greater than half the radius of either, the inaccuracy of (9) is less than 1 in 20,000.

ABSTRACT.

The experiments carried out by F. W. Peek ("Journal" Am. Inst. of Electrical Engineers, 1911) for the General Electric Company of America prove conclusively the value in practical work of a knowledge of how to compute the maximum value of the electric stress between high-pressure conductors. With equal spherical electrodes the electric stress between them can easily be computed from known tables. When, however, they are unequal the calculation becomes so laborious that it is prohibitive to nearly every experimeter. The Author developes formulæ for this case, by means of which, and of the formulæ for the capacity coefficients given in a recent Paper to the Society, the calculation is very appreciably shortened. When the distance between the spheres is very small compared with the diameter of either, the following

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* "Phil. Mag." February, 1890.

approximate formula for $R_{max.}$ (the maximum value of the electric stress) can be used

$$R_{max.}=(V/x)[1+(2b-a)x/(3ab)+\{4(a-b)^2+ab\}x^2/(45a^2b^2)],$$

where V is the maximum P.D. between the electrodes, x their distance apart, a the radius of the smaller and b the radius of the larger sphere. In this case a knowledge of the values of the capacity coefficients is not required.

DISCUSSION.

Mr. E. H. **Rayner** asked for what ratio of diameters the formula held. He also stated that the values of $R_{max.}$ found by the General Electric Co. were 10 per cent. lower than those given by some experiments of his own, and pointed out what he thought might lead to errors in the General Electric Co.'s tests.

Prof. C. H. **Lees** said that it was for experimentalists to try to verify Dr. Russell's formula, and asked whether the presence of ions between the spheres affected the value of $R_{max.}$ appreciably.

Dr. R. S. **Willows** asked if the value of $R_{max.}$ agreed with that obtained from the theory of ionisation by collision. He also asked if the results accounted for the difference in the values obtained according as the small electrode was positive or negative. That the fields differed in some way was shown by the appearance of the discharge; a long brush and sensitiveness to radium radiations when the small sphere was positive, no brush and insensitiveness to radium when it was negative.

Mr. R. **Appleyard** pointed out the simplification when the two spheres were equal.

Mr. W. **Duddell** observed that now that continuous current at from 50,000 to 100,000 volts, with plenty of power behind it, was available in London, further experiments on sparking potentials should be made.

The **Author**, in reply, stated that he had read Mr. Peek's Paper, and that his results agreed with Whitehead's ionic experiments to within 2 or 3 per cent. With a small Wimshurst machine there was a difference of 3 or 4 per cent. in the sparking potential according as to whether the small sphere was positive or negative. When the small sphere was positive the electric stress of the air was weaker. This difference, he thought, would disappear if there were more power available. High-tension continuous-current could be now obtained at Mr. Hyfield's station, where a huge Wimshurst, under compressed air, was driven by a motor.

VII. *The Cubical Expansion of Fused Silica, with Note on the Variation of the Boiling Point of Aniline with Pressure.*

By F. J. Harlow, *A.R.C.S., B.Sc., Assistant Lecturer in Physics, Sir John Cass Technical Institute.*

Received October 16, 1911. Read November 24, 1911.

The properties of fused silica have been investigated considerably of late years, and the results have shown that it can be used with advantage for many scientific purposes. In particular it seems to be a suitable substance for the envelope in both liquid and gas thermometers, as many of the disadvantages of the use of glass for this purpose do not exist in the case of fused silica.

The following is an account of measurements which have been made of the cubical expansion of fused silica by the weight-thermometer method. Hitherto all measurements of the thermal expansion have been measurements of the linear coefficient, and as the possibility of calculating the cubical coefficient of expansion of a substance in the form of a bulb from the linear coefficient, with any approach to accuracy is doubtful, the direct measurements given in the present paper should prove useful for thermometric and other purposes.

Weight Thermometers and Method of Filling.

The three types of weight thermometer used are indicated in Fig. 1. They were made of clear transparent fused silica, and varied in capacity from 70 c.c. to 140 c.c. approximately. Observations were first of all made with thermometer *a*, which was filled in the ordinary way by alternate heating and cooling with the open end under mercury. As a larger value of the apparent coefficient of expansion of mercury than was expected was obtained with this filling, it was thought necessary to fill the bulb under a vacuum, in order to exclude the possibility of the presence of any air bubbles in the bulb. This was done, but on account of the difficulty experienced in thoroughly cleaning and filling the bulb, one of a different type, Fig. 1 (*b*), having tubes at both ends, was subsequently used. Fig. 1 (*c*) shows a similar bulb having in addition a number of concentric tubes sealed inside; the use of these will be explained later.

The bulb *b* was first of all dried and any volatile matter removed from its inner surface by heating it while a current of dry air was being aspirated through. Marks were now made upon the tubes as indicated in the figure, and the quantity of mercury filling them from their ends to the marks obtained.

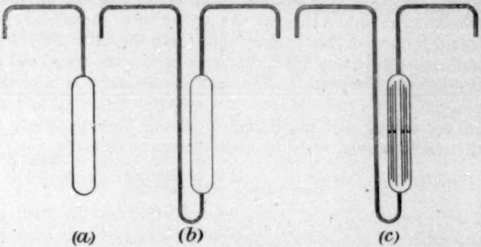

FIG. 1.

The bulb was then connected by airtight rubber joints to the apparatus shown in Fig. 2. By means of this, mercury, which had been dropped through a vacuum to rid it of all traces of

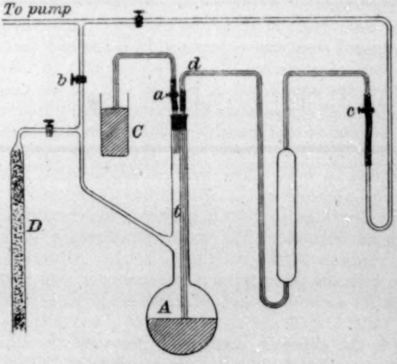

FIG. 2.

air, could be made to fill the bulb after the latter had been evacuated. The apparatus was first carefully dried by repeatedly evacuating and admitting dry air through the drying tube D, and was finally evacuated to a high degree of exhaustion. Mercury from the reservoir C was now allowed to flow into the bulb A slowly by an adjustment of the pinch cock *a*, and any air given off from it, as it fell through the vacuous space or as it was boiled in A, was pumped out. By closing the tap *b* and admitting air into A through the drying tube, the mercury was forced by way of the capillary tube *t* into the bulb. When the bulb was completely filled, the pinch cock *c* was closed and the thermometer removed. The end of the tube *d* was then plugged with a piece of iron wire sealed in with wax, and, the rubber tubing and pinch cock *c* having been removed, the thermometer was ready for observations to be made.

Observations and Results.

For convenience in taking the observations the bulb was supported at the bottom by a holder attached to a brass rod parallel to the bulb. To the top of this rod was attached an arm carrying a small removable shelf and a small weighing bottle containing mercury placed upon this caught the overflow mercury. The bottle and its contents could thus easily be removed and weighed.

Fundamental Interval.—Readings of the fundamental interval were made in the usual manner. The temperature of the exposed stem was ascertained by a mercury thermometer, so that the expansion of the known quantity of mercury contained in the stem could be allowed for. It was necessary to get the rise of temperature of the stem merely approximately, as it was found that an error of 5 deg. would produce an ultimate error of 1 in 20,000 only, the accuracy to which one attempted to work. The smallest fundamental interval was about 16 grammes, so that since it was found possible to repeat similar readings to 0·0005 gramme, the required accuracy could be obtained. The steam temperature was obtained from readings of the barometric height. Observations were made with the three types of thermometer with two or three fillings for each, the results of which are given in Tables I. and II. Table I. gives the complete set of observations taken with bulb *b*, the apparent coefficients given in the last column being calculated by dividing the overflow by the mass left in the bulb at the steam temperature and by the range of temperature. The third set of readings given in this table was obtained after the bulb had been heated in aniline, and shows good agreement with the first two sets. The ice readings at the foot of the table were taken both before and after the steam readings in each case, and the very close agreement obtained shows that no permanent alteration of volume of the bulb had occurred through heating. This was also found to be the case when the bulb was heated to the temperature of boiling aniline.

TABLE I.

	Date.	Steam temperature in degrees Centigrade.	Weight of mercury filling to marks at steam temp., in grammes.	Corrected overflow in grammes.	Apparent coefficient of expansion of mercury × 10^9.
	1910.				
1st filling	May 27 ...	100·096	1,134·10	20·5555	181,076
	,, ...	100·083	1,134·10	20·5523	181,071
	,, ...	100·076	1,134·10	20·5490	181,055
	June 2	99·871	1,134·14	20·5086	181,064
	,, ...	99·869	1,134·14	20·5092	181,073
	June 3	99·889	1,134·14	20·5141	181,079
	,, ...	99·874	1,134·14	20·5113	181,081
	June 4	99·713	1,134·17	20·4754	181,052
	,, ...	99·722	1,134·17	20·4780	181,058
	June 5	99·935	1,134·13	20·5197	181,047
	,, ...	99·898	1,134·14	20·5132	181,054
				Average	181,065
	1910.				
2nd filling	June 25 ...	99·547	1,134·16	20·4421	181,061
	June 28 ...	99·722	1,134·12	20·4787	181,064
	,, ...	99·721	1,134·12	20·4777	181,065
	June 29 ...	99·748	1,134·12	20·4805	181,041
	July 6......	99·833	1,134·12	20·4963	181,026
				Average	181,051
	1911.				
3rd filling	January 27	100·492	1,134·02	20·6345	181,056
	,,	100·476	1,134·02	20·6290	181,049
	January 28	100·531	1,134·01	20·6414	181,056
	,,	100·522	1,134·03	20·6394	181,056
	February 1	100·776	1,133·97	20·6904	181,056
	February 2	100·745	1,133·98	20·6844	181,056
				Average	181,056

Ice Readings.—

First filling	May 26........................	140·5004
	June 4........................	140·4979
	June 9........................	140·4979
Second filling	June 25	140·0510
	June 28	140·0505
	July 6	140·0500
Third filling	January 26..................	168·4622
	February 15	168·4592

TABLE II.

Bulb.	Date.	Filling.	Apparent coeff. of expansion of mercury $\times 10^9$.	Cubical coeff. of expansion of silica ${}_0S_{100}$.
a	Oct., 1909	(1) Ordinary...	181,017	
	"	(2) Vacuum....	181,042	
		Mean	181,030	$100{\cdot}6\times10^{-8}$
b	May-June, 1910	(1) Vacuum....	181,065	
	June, 1910....	(2) Vacuum....	181,051	
	Jan.-Feb., 1911	(3) Vacuum....	181,056	
		Mean	181,057	$97{\cdot}9\times10^{-8}$
c	July, 1910......	(1) Vacuum....	181,026	$101{\cdot}0\times10^{-8}$
			Mean	$99{\cdot}8\times10^{-8}$

In Table II. the fourth column gives the average values of the apparent coefficients of expansion of mercury obtained from each series of readings taken. The first filling of *a* was carried out by the ordinary method of heating and cooling; the second after evacuating by a modification of the apparatus described. The approximate agreement between the results obtained shows that the ordinary method of filling a weight thermometer is fairly satisfactory. The values of the cubical coefficients of expansion of the bulbs given in the fifth column were calculated from the means of the results for each thermometer, by the well-known formula :—

$$a=\frac{m}{Mt}+\frac{M+m}{M}S,$$

where a=absolute coefficient of expansion of mercury.
m=overflow.
M=mass of mercury left in at the higher temperature.
t=higher temperature in degrees Centigrade.
S=coefficient of cubical expansion of the bulb.

The value of the absolute coefficient of expansion of mercury used was that given by Callendar and Moss * in their recent paper—viz., 0·000182054.

Surface Effect.—It was thought at one time during the investigation that possibly the expansion of the mercury near the surface was in some way affected by the surface, and that this might explain the discrepancies existing in the values of the coefficient of expansion of mercury obtained by the weight-thermometer and absolute methods. For the purpose of testing this, the bulb *c* was used, in which the ratio of the surface to the volume was considerably increased by the concentric silica tubes sealed inside. The results with the three bulbs show that a surface effect, if it exists at all, is extremely small.

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* "Phil. Trans.," January, 1911.

Pressure Effect.—The possibility of an error arising from the pressure on the bulb was also considered. Such an error may arise either from the change in the effect of the pressure in the bulb due to an alteration of the elastic properties of the silica with the rise of temperature, or from the change in the pressure itself due to the change in the density of the mercury. To test the first, the contraction which took place, when a pressure of an atmosphere was applied, was measured at 0°C. and at 100°C. These were found to differ by 0·004 of a gramme of mercury only, so that since the average internal pressure on the bulb was only about $\frac{1}{10}$th of an atmosphere, the effect was quite negligible. A calculation of the effect due to the alteration in the density of the mercury showed this also to be quite negligible.

Ice-Aniline Readings.—It was thought desirable to extend the observations to a higher temperature, and for this purpose an aniline vapour bath was used. The bulb was enclosed in a large beaker containing aniline, the beaker being carefully lagged with asbestos cloth and closed at the top with sheets of tin and uralite. To prevent cooling by radiation, the bulb was surrounded by a cylinder of tin, and between the lid of the beaker and the bulb were placed two discs of tin separated by a distance of about 1 cm. In this way a vapour bath of uniform temperature was obtained. The temperature was measured by a platinum resistance thermometer of about 5 ohms fundamental interval, which enabled one to measure the temperature to $\frac{1}{100}$th of a degree. It was thus possible to work to an accuracy of about 1 in 20,000, since the boiling point of aniline is 184°C. or thereabouts. A small coil of nickel wire surrounding the bulb was used for heating up the bulb slightly after a weighing of the overflow vessel. This was found necessary in order to join on the thread of mercury to the mercury in the vessel. The results of the observations are given in Table III., and it will be observed that a good agreement exists between them. With two sets of observations also the ice readings are given. The agreement between those taken before the aniline readings and those taken after shows that here again no permanent charge in the volume of the bulb took place through heating.

The value of the absolute coefficient of expansion of mercury used for the calculation of the expansion of the bulb was 0·000183705, obtained from the formula given by Callendar and Moss.

TABLE III.

Bulb.	Date.	Aniline bath temperature in degrees Centigrade.	Weight of mercury filling to marks at bath temperature, in grammes.	Corrected overflow in grammes.	Apparent coefficient of expansion of mercury $\times 10^9$.	Cubical coefficient of expansion of silica ${}_0S_{184}$.
	1911.					
c	March 4...	184·168	1,849·67	62·0712	182,214	
	„ 6...	184·026	1,849·72	62·0212	182,203	
	„ 8...	184·292	1,849·62	62·1169	182,230	
	„ 9...	184·072	1,849·70	62·0374	182,207	
	„ 13...	183·483	1,849·90	61·8376	182,184	
	„ 16...	183·683	1,849·85	61·9034	182,184	
	„ 17...	183·908	1,849·75	61·9875	182,218	
				Average	182,205	$145·1 \times 10^{-8}$
	1911.					
b	July 10...	182·518	1,116·84	37·6163	182,186	
	„ 10...	182·556	1,116·83	37·6242	182,187	
	„ 12...	182·372	1,116·85	37·6019	182,264	
	„ 12...	182·432	1,116·85	37·6124	182,253	
	„ 13...	182·336	1,116·88	37·5834	182,207	
	„ 13...	182·324	1,116·88	37·5835	182,221	
	„ 14...	182·315	1,116·88	37·5834	182,230	
	„ 14...	182·292	1,116·89	37·5742	182,205	
				Average	182,219	$143·8 \times 10^{-8}$
	1911.					
c After alteration necessitated by breakage	July 19...	184·397	1,850·55	62·1699	182,190	
	„ 20...	184·426	1,850·54	62·1837	182,203	
	„ 20...	184·446	1,850·53	62·1926	182,210	
	„ 20...	184·505	1,850·51	62·2168	182,225	
	„ 25...	183·870	1,850·73	62·0001	182,196	
				Average	182,205	$145·1 \times 10^{-8}$
					Mean	$144·7 \times 10^{-8}$

Ice Readings.—

c.	First filling	March 3	121·3624
		March 18	121·3593*
b.	„	July 10	152·2663
		July 15	152·2660

* A smaller weight was expected here, as a tiny drop of mercury was known to have been lost.

Conclusion.

The mean results for the cubical coefficient of expansion of the three bulbs are :—

$$S_{100} = 99{\cdot}8 \times 10^{-8}.$$
$${}_0S_{184} = 144{\cdot}7 \times 10^{-8}.$$

The values of the various measurements of the linear coefficient of expansion of fused silica which have been made have been collected in a paper* by Kaye, of the National Physical Laboratory, and the values of the cubical coefficients of expansion calculated from the mean linear coefficients given in this paper are :—

$${}_0S_{100} = 147 \times 10^{-8}.$$
$${}_0S_{184} = 154 \times 10^{-8}.$$

Although the legitimacy of this calculation, as already mentioned, is doubtful, the wide discrepancy between the two values for the fundamental coefficient can scarcely be attributed to this. The result given in the present paper thus leads one to think that the observations, on which the value given in Kaye's paper,—viz., 49×10^{-8} for the linear coefficient between 0°C. and 100°C., depend, are too large. This seems all the more likely, since the same observations show that the coefficient changes sign at about −80°C. It does not, therefore, seem likely that it will attain the above value between 0°C. and 100°C, since this value is not very much less than the maximum value.

Most of the observations on the linear expansion at ordinary temperatures have been made by the well-known Fizeau interference method, in which specimens about 1 cm. long are used. The expansion to be measured is thus of the order of the wavelength of light only, and it seems that, although most of the observations have been carried out with great care, and fairly consistent readings obtained, the results may be inaccurate from possible systematic errors. The difficulty which was first met with in the application of the Fizeau method to the measurement of the expansion of silica was that inconsistent readings were obtained by reason of the existence of traces of air between the surfaces of contact. This difficulty was avoided eventually, as was also the very large and uncertain correction due to the change in the refractive index of the air with the temperature and pressure changes, by evacuating the envelope containing the apparatus. As, however, it is extremely difficult to remove every trace of air from the surface of silica it seems quite probable that an appreciable error is introduced on this account, if one considers the extremely small expansion which is measured. It thus seems desirable that accurate measurements of the linear expansion should be made with longer specimens.

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* Kaye, "Thermal Expansion of Fused Silica," "Phil. Mag.," October, 1910.

It is well known that the accuracy with which one can work with a mercury in glass thermometer is limited by the temporary changes of zero which occur on account of the bulb not returning quickly to its original volume after heating. The observations made above show that with fused silica this would not occur, the reason no doubt being that at ordinary temperatures it is so far removed from its melting point. On this account, then, the accuracy attainable with mercury thermometers should be considerably increased by the use of fused silica as the containing envelope.

Variation of the Boiling Point of Aniline with Pressure.

During the course of the experiments with the aniline vapour bath the barometer varied over a fairly wide range. A table (Table IV.) of boiling points at pressures ranging from 74 cm. to 78 cm. of mercury is given, as it may prove useful to other observers.

TABLE IV.

Barometric readings in centimetres of mercury.	Boiling point of aniline in degrees Centigrade.
74·0	183·09
75·0	183·58
76·0	184·07
77·0	184·56
78·0	185·05

These values were obtained from the curve in which temperature was plotted against pressure. Most of the points were found to be well on a straight line, the slope of which gave as the rate of change of temperature with pressure 0·49°C. per centimetre of mercury. Care, however, must be taken in applying these figures to the boiling liquid, since, if pure aniline is boiled for some time, partial decomposition takes place. This does not alter the temperature of the vapour at some distance from the liquid unless the aniline has decomposed considerably. The use of copper vessels should be avoided, as the copper seems in some way to accelerate decomposition.

In conclusion, I desire to thank Prof. Callendar for suggesting this research, and for the help he has given me from time to time during the performance of the work, which has been carried out at the Imperial College of Science and Technology.

ABSTRACT.

The author describes experiments in which measurements of the cubical coefficient of expansion of fused silica from 0°C. to 100°C., and from 0°C. to 184°C., were made by the weight thermometer method. The values obtained were ${}_0S_{100}=99{\cdot}8\times10^{-8}$, and ${}_0S_{184}=144{\cdot}7\times10^{-8}$. The fundamental coefficient is considerably less than that calculated from previous linear measurements, whereas ${}_0S_{184}$ is only slightly less. A low value of the fundamental coefficient is to be expected, since the coefficient has been shown to change sign at about −80°C. Observations of the ice-point before and after heating showed that no permanent change in the volume of the bulb occurred through heating, thus confirming the utility of fused silica for thermometric purposes.

DISCUSSION.

Dr. G. W. C. KAYE said that Mr. Harlow's results should prove useful now that mercury in silica thermometers were an accomplished fact. These bid fair to revolutionise mercurical thermometry. He pointed out that Mr. Harlow's results did not agree with the linear expansion of silica found by Chappuis, Holborn and Henning or Randall and Scheel. He thought for fused silica it should be possible to deduce the linear expansion from the cubical. He would like Mr. Harlow to obtain some results at a higher temperature. Some preliminary results obtained with a silica meter bar at the National Physical Laboratory confirmed these other workers.

The PRESIDENT remarked that his own experiments on the linear expansion of a rod of silica 30 cm. long (described in a Paper read before the Physical Society on March 22, 1901) showed no appreciable change in the mean coefficient 0·00000054 from 15°. to *t*, when *t* was varied between the limits 340° and 1,400°C. Recent experiments at temperatures below 0° appeared to indicate a lower value, and it was possible that the linear coefficient between 0 and 100° might be sensibly smaller. He did not, however, consider that the cubical coefficient could be deduced satisfactorily from the linear, because it was impossible to anneal a drawn specimen or bulb satisfactorily, owing to the rapid devitrification of amorphous silica at 1,000°C. If Mr. Harlow's value of the cubical coefficient between 0° and 100° were employed in the reduction of gas-thermometer observations with silica bulbs, in place of that obtained by assuming the above constant value for the linear coefficient, the temperature scale would be raised by a tenth of a degree at 445°C. A correction of this order was far from negligible at the present time.

Mr. T. BARRATT asked whether the Author had made any observations of the linear expansion of the thermometer bulbs used by him.

The AUTHOR thought it was quite possible that there might be a difference between the cubical coefficient of expansion and three times the linear. He was endeavouring to extend his results to higher temperatures. He had not made any observations on the linear expansion of his bulbs as the Fizeau method was the only practicable one.

VIII. *On the Temperature Coefficient of Diffusion.* *By* Basil W. Clack, *B.Sc., Lecturer in Physics at Birkbeck College.*

Received October 17, 1911. Read November 24, 1911.

§ 1. Introduction.
§ 2. Theory.
§ 3. Apparatus.
§ 4. Method.
§ 5. Results.
§ 6. The temperature coefficient.
§ 7. Comparison with other workers.

1. *Introduction.*

In 1908 the Author read a Paper before this Society describing a method he had employed for determining the coefficient of diffusion of various salts through water at 0°C. (" Proc." Phys. Soc., XXI., p. 374. *See also* " Phil. Mag.," December, 1908.) In these experiments a flask of special construction was filled with a solution of the salt under investigation and suspended in a bath of distilled water kept constantly at 0°C. by a surrounding ice jacket. The gradual decrease in weight of the flask as the salt diffused out and water simultaneously entered was noted from day to day, and from these figures it was shown that the value of the coefficient of diffusion could be obtained.

The objects of the present Paper are :—

1. To describe experiments extending this method to other temperatures than 0°C.

2. To find how the value of the coefficient varies with the concentration at these selected temperatures.

3. To find how the value of the coefficient for given solutions varies with the temperature.

In employing the method at higher temperatures than 0°C. it is obvious that special care must be taken to avoid any but minute temperature variations, for the size of the flask is such as to render even a small change in the relative density of the solution and the water in which it is suspended of such great effect as to vitiate the result entirely. It is, therefore, advisable to compensate as far as possible for any variation in density due to change in temperature or to addition of diffused salt to the water by suspending similar flasks from each arm of the Sartorius balance. These flasks were of the same construction as already employed in the earlier investigation, and both were of the same size and were filled with the same solution and suspended in the same bath of distilled water. Both were fitted with diffusion tubes of about the same length, but of considerably different cross-section.

As the salt diffuses out of each at different rates on account of the difference in the section of the diffusion tubes, the balance records the difference in their weights, and it is easy to apply the theory already given in the previous Paper to find the coefficient of diffusion of the salt.

2. *Theory.*

Let L=length of a diffusion tube.

A=its area of cross-section.

N=concentration of solution in grammes per cubic centimetre of solution at the lower end of the tube.

D=density of the solution at the same point.

K=coefficient of diffusion in C.G.S. units.

δ=the ratio of the increment in volume produced to the increment in the mass of salt dissolved for a solution of concentration N.

Then it was shown that the final steady rate of decrease in weight of the flask (i) could be expressed in the form

$$i=\frac{KAN}{L}\left\{1-\frac{N\delta}{2}(1-bN)\right\}\{1-\delta(1-bN)\}.$$

Here the last two factors are corrections depending on (I.) the fact that a mass of water enters the flask due to the change in volume of the solution as the salt diffuses out, and (II.) the velocity with which the water enters. That this is so may be proved by assuming that no water enters the flask at all, but that, the water remaining perfectly stationary, the salt simply diffuses out. The expression then obtained for the final steady rate of change in weight of the flask is the much simpler one

$$i=\frac{KAN}{L}.$$

In reality, however, the water is not stationary, and if the theory given in the earlier Paper be inspected it will be seen that the factor $1-\delta(1-bN)$ corrects for the mass of water entering the flask, and the factor $1-\frac{N\delta}{2}(1-bN)$ corrects for the effect of the velocity on the rate of the diffusion.

The Author now prefers to write the above equation in the form

$$i=\frac{KAN}{L}\left\{1-\frac{N\delta}{2}(D-N)\right\}\{1-\delta(D-N)\},$$

as this emphasises the fact that it is unnecessary to know the relation between the density (d) of the solution and its concentration (n), where (d) and (n) are variables. In the former Paper we assumed $d=1+an$, and, from the value found for a, $b=1-a$ was obtained; but, as a matter of fact, there is no need to make any such assumption, and we do not require the values of either a or b.

If, then, we have two diffusion flasks, one fitted with a tube of length L_1 and area of cross-section A_1, the other having a tube of length L_2 and cross-section A_2, we have

$$i_1=\frac{KA_1N}{L_1}\left\{1-\frac{N\delta}{2}(D-N)\right\}\{1-\delta(D-N)\}$$

for the final rate of decrease in weight of the first flask, and a similar expression can be written down for the rate of decrease in weight of the other flask.

The difference in these rates is

$$i_1-i_2=K\left\{\frac{A_1}{L_1}-\frac{A_2}{L_2}\right\}N\left\{1-\frac{N\delta}{2}(D-N)\right\}\{1-\delta(D-N)\}$$

or

$$K=\frac{(i_1-i_2)L_1L_2}{(A_1L_2-A_2L_1)N\left\{1-\frac{N\delta}{2}(D-N)\right\}\{1-\delta(D-N)\}}.$$

3. *Apparatus.*

A and B are the two specially constructed flasks of about 450 c.c. each, one suspended from each arm of the delicate Sartorius balance by means of the fine wires *ww* of No. 33 platinum wire.

As the design of the flasks already used had been found by experience to be quite satisfactory, it was adhered to in the present investigation, and a full description of the construction need not be repeated here; it will be found in the Paper previously referred to.

Both flasks, after being filled with the solution under investigation, were suspended in a copper vessel, C, containing about 25 litres of distilled water.

This was surrounded by a bath, I, which contained either ice or water whose temperature was maintained as constant as possible in the following manner:—

The whole of the apparatus was set up in a special room in the cellar of the college, which was provided with double doors and only lit artificially, and whose temperature could be kept constant for many weeks to within a small fraction of a degree Centigrade by means of a small gas stove regulated by a Toluene thermostat bulb.

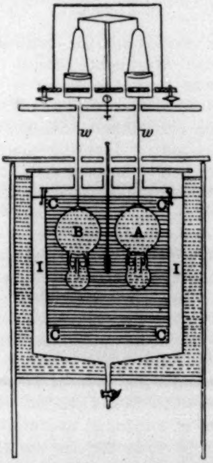

Fig. 1.

The flask A was fitted with the diffusion tube 4·001 cm. long and 1·694 sq. cm. in cross-section, which has already been shown to have a negligible end correction; the other flask, B, had a diffusion tube 3·978 cm. long and of cross-section

0·5033 sq. cm., for which the end correction would be still smaller. The lengths were obtained from a series of readings with a screw-gauge reading to $\frac{1}{100}$ mm. The cross-section was determined from a number of experiments on the weight of the tubes entirely filled with distilled water. This method has been found by the Author to give more consistent results for tubes of the size here employed than any other method to which he had access.

4. *Method.*

In preparing the solution and the distilled water, especially in the case of those experiments carried out at the higher temperatures, it was found necessary to heat the liquids considerably above the temperature at which they were to be employed, and then immediately to use them, after cooling down as much as necessary. If this precaution were not taken difficulties were frequently encountered from bubbles of air coming out of solution in the liquids and adhering to the walls of the diffusion flask.

In commencing an experiment, the 25 litres of distilled water were run into the copper vessel and the flasks filled with the solution, and having the upper open ends of the diffusion tubes temporarily closed by small thin glass plates, were gently lowered into the water and suspended, one from each arm of the balance by the fine platinum wire, as already mentioned, with the diffusion tubes vertical. The glass plates were then removed and a thin layer of Fleuss pump oil run on to the surface of the water from a pipette. It may be recalled that the use of this oil was found to increase the accuracy of weighing, probably by reducing the surface tension of the water.

The movement of the balance beam was limited by means of stops, so that the pointer could not move over more than one scale division on each side of the zero. A microscope, having a scale in the eye-piece, was focussed on a very fine wire fixed to the pointer, and the sensitiveness of the balance was such that 1 m.g. caused the pointer to move over 20 divisions of this microscope scale.

The method of carrying out the experiment was to weigh the flasks day by day, adding weights to one side of the balance to restore equilibrium, the final figure being obtained by means of the microscope.

The graph obtained by plotting the change in weight as recorded by the balance against the time becomes practically straight after about a week, and the slope of this final portion of the curve gives the quantity (i_1-i_2) referred to in § 2.

The time at which the steady state is attained may be calculated if we assume that the water is stationary, and we find that if q is the quantity of salt which leaves unit area of the upper end of the tube per second

$$q=\frac{\mathrm{KN}}{\mathrm{L}}+\frac{2\mathrm{KN}}{\mathrm{L}}(e^{-x}+e^{-4x}+e^{-9x}\ldots),$$

where $$x=\frac{\pi^2\mathrm{K}t}{\mathrm{L}^2}$$

Take L as 4 cm. and $\mathrm{N}=0{\cdot}2$ (20 per cent. solution), and effect the summation of the series by the aid of Newman's tables of the descending exponential ("Trans." Camb. Ph. Soc., Vol. XIII.), and it can be shown that q has become constant to within 0·5 per cent. in eight days for a solution whose diffusivity has the value $1{\cdot}4\times10^{-5}$ C.G.S., the error decreasing as time goes on.

The steady state would be attained rather more rapidly than this for solutions of higher diffusivity, and the figure $1{\cdot}4\times10^{-5}$ has been selected as near the mean value actually found for the various solutions employed in this research. It is probable that the fact that the water is not stationary will not much affect the time required for the steady state to be attained, but the problem is complicated by another consideration: that as the salt diffuses out of the flasks the concentration at the lower end of the tube may be slightly disturbed.

Experiment shows that this effect must be small, and approximate calculations show that for a solution such as that just mentioned the error involved cannot possibly be as great as 0·5 per cent. in 10 days, increasing as time goes on to less than 1 per cent. in 20 days. As these two errors tend to counterbalance one another, it is certain that the slope of the curve over the selected period must be correct to within a fraction of 1 per cent. as far as these influences are concerned.

5. *Results.*

The results obtained with solutions of potassium chloride and nitrate at various temperatures are given in the table.

The figures obtained in the former Paper at 0°C. are added for comparison.

Experiment.	Concentration gms./c.c. soln.	Temp.	$(i_1-i_2)\times 10^7$ = mean slope.	$K\times 10^5$.
24 Old series	0·1975 KCl	0°	..	0·966
25 „ „	0·1975	0°	..	0·978
4	0·2008	0°	3·419	0·976
5	0·1934	13°	4·412	1·304
6	0·1973	14°	4·548	1·317
12	0·1993	23·7°	5·752	1·648
13	0·1993	23·8°	5·755	1·648
26 Old series	0·1014 KCl	0°	..	0·953
27 „ „	0·1016	0°	..	0·956
7	0·0985	14·6°	2·414	1·373
8	0·0995	16·8°	2·553	1·432
9	0·1006	16·9°	2·627	1·457
14	0·1003	23·8°	3·101	1·726
15	0·1014	23·9°	3·130	1·722
16 Old series	0·1001 KNO_3	0°	..	0·843
17 „ „	0·1001	0°	..	0·845
10	0·0991	15·1°	2·169	1·222
18	0·0968	16·3°	2·168	1·249
16	0·1034	23·4°	2·713	1·463
17	0·1016	23·6°	2·668	1·465
22 Old series	0·0521 KNO_3	0°	..	0·870
23 „ „	0·0521	0°	..	0·872
19	0·0513	16·2°	1·180	1·265
20	0·0513	16·3°	1·195	1·281
21	0·0496	25·0°	1·460	1·520
22	0·0529	25·6°	1·486	1·547

The accompanying graphs (Fig. 2), which are marked in accordance with the number of the experiments, show the above results for KCl, represented by plotting the change in weight of the suspended system as ordinates against the time as abscissæ.

6. *The Temperature Coefficient.*

If we write $K_t=K_0(1+at+bt^2)$,

where K_t is the value of the coefficient of diffusion at t°C.

K_0 „ „ „ „ „ 0°C.

a and b are constants,

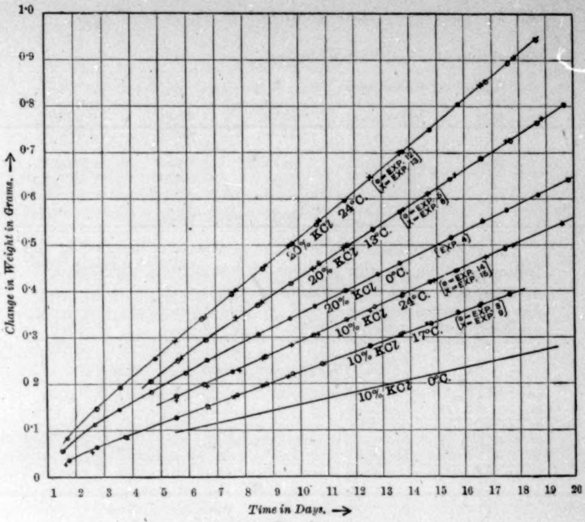
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0
Change in Weight in Grams. →
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Time in Days. →
20% KCl 24°C.
○ = EXP. 12
× = EXP. 13
20% KCl 13°C.
○ = EXP. 5
× = EXP. 6
20% KCl 0°C.
EXP. 4
10% KCl 24°C.
○ = EXP. 14
× = EXP. 15
10% KCl 17°C.
○ = EXP. 8
× = EXP. 9
10% KCl 0°C.

FIG. 2.

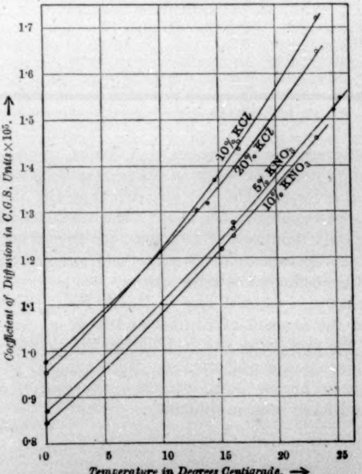
1·7
1·6
1·5
1·4
1·3
1·2
1·1
1·0
0·9
0·8
Coefficient of Diffusion in C.G.S. Units × 10⁵. →
0 5 10 15 20 25
Temperature in Degrees Centigrade. →
10% KCl
20% KCl
5% KNO₃
10% KNO₃

FIG. 3.

the values of *a* and *b* are as follows :—

Salt.	Concentration.	$K_0 \times 10^5$.	*a*.	*b*.
KCl	20 per cent.	0·974	0·021	0·00035
,,	10 ,,	0·955	0·024	0·00040
KNO_3	10 per cent.	0·844	0·026	0·00022
,,	5 ,,	0·871	0·025	0·00018

7. *Comparison With Other Workers.*

The results obtained by other observers are included in the following table for the purpose of comparison, and the substantial agreement is satisfactory.

Temperature.	Concentration.	$K \times 10^5$.	Observer.
	Per cent.		
7·0°	1 KNO_3	1·06	Scheffer.
7·0°	3	0·98	Scheffer.
7·0°	5	1·03	Clack.
10·0°	15	0·92	Schuhmeister.
10·0°	10	1·08	Clack.
17·6°	14	1·27	Thovert.
17·6°	10	1·28	Clack.
17·6°	5	1·30	Clack.
17·6°	3	1·46	Thovert.
17·6°	2	1·48	Thovert.
10·0°	10 KCl	1·27	Schuhmeister.
10·0°	10	1·22	Clack.
18·0°	20	1·53	Öholm.
18·0°	20	1·46	Clack.
18·0°	10	1·53	Öholm.
18·0°	10	1·49	Clack.

In some cases the coefficient of diffusion increases as the concentration increases, while in other cases (*e.g.*, solutions of KNO_3) the reverse holds. It appears that the temperature affects the phenomenon, for at 0°C. the coefficient is greater for 20 per cent. solutions of KCl than for 10 per cent., but at the higher temperatures the weaker solution has the greater value for the coefficient of diffusion.

In conclusion, I should like to thank Principal Armitage-Smith and the Council of Birkbeck College for assistance in supplying the apparatus and in building the thermostat room specially for experimental work on diffusion, and particularly Dr. Griffiths for his continued interest in the research and ready help when I have been in difficulty.

ABSTRACT.

The Paper describes further experiments carried out by the Author with an improved form of the apparatus previously described (" Proc." Ph. Soc., XXI., p. 374), by means of which the value of the coefficient of diffusion of salts through water can be found at various temperatures.

Special flasks, similar to those already employed, were filled with the solution under investigation, and one was suspended from each arm of the balance in a large bath of distilled water maintained at constant temperatures in a thermostat room. The diffusion tubes of both flasks were of equal length, but their cross-sections differed considerably, and a method of differential weighing was used to compensate for any small changes in temperature.

From the rate at which the flasks change in weight the value of the coefficient of diffusion of the salts is deduced. Figures are given for this value in the case of KCl and KNO_3 at various concentrations and at different temperatures, and from these figures the temperature coefficient of diffusion is found.

DISCUSSION.

Dr. A. **Griffiths** said that some calculations made by Miss C. H. Knowles prove that by simple modifications of the initial conditions the rate of attainment of the steady state may be hastened considerably. Instead of filling the whole of the tube with solution of strength N, let the tube be filled with a solution of strength qN to a height pL, then the rate at which salt is diffused through the lower end of the tube, assuming the liquid to be stationary, is given by

$$\frac{kN}{L}\left(1 - \sum A_i i\pi e^{-i^2\pi^2 kt/L^2}\right),$$

where $A_i = \frac{2}{i\pi}(q-1-q\cos ip\pi)$ and i has all integral values from unity to infinity. If $p=0{\cdot}5$ and $q=1$, both the time taken for the attainment of the steady state and the total quantity (obtained by integration) diffused by the time of attainment are reduced to one-quarter. If $p=q=0{\cdot}\dot{6}$ the time is reduced to one-ninth. Quite apart from these calculations Mr. Clack would do well to consider the advisability of replacing the single diffusion tube by half-a-dozen or more short tubes, each of which would have to be narrow in order to keep down the end correction. Reduction of the length to one-quarter would reduce the time of attainment of the steady state to one-sixteenth, and moreover the quantity diffused by the time of attainment would be diminished considerably. Mr. Clack's results compare most favourably so far as their consistency is concerned with those of other workers; but there seem to be variations in his valuable results for which there is no apparent reason, and I suggest that experiments should be made with liquids which have been sterilised and are free from colloidal particles.

Prof. C. H. **Lees** said that Dr. Griffiths' suggestion of using several narrow tubes instead of one wide one was important. We were much in need of further experimental data regarding diffusion. We do not yet know just what is involved in the process.

Mr. H. **Borns** asked whether the apparatus consisted entirely of glass.

The **Author**, in reply, attached great importance to Dr. Griffiths' suggestions, but he considered the 1½ per cent. variations which he sometimes got as fairly small. Possibly sterilising the solutions might be advantageous as he found that there was a trace of fungoid growth after the solutions had been left for three weeks in the dark. The apparatus was almost entirely made of glass.

IX. *The a Particles emitted by the Active Deposits of Thorium and Actinium.* *By* E. MARSDEN, *M.Sc., John Harling Fellow, University of Manchester, and* T. BARRATT, *B.Sc., A.R.C.S., East London College, University of London.*

RECEIVED NOVEMBER 6, 1911. READ NOVEMBER 24, 1911.

THE present Paper deals mainly with the application of probability to the emission of α particles from thorium and actinium active deposits. The most important result of the investigation will, however, be found in the latter part of the Paper, where it is shown that the numbers of α particles emitted by thorium C_1 and thorium C_2, when the active deposit is in equilibrium, are not equal, thus proving that these products are not directly genetically connected. The nomenclature of the various products proposed by Rutherford and Geiger ("Phil. Mag.," XXII., p. 621, 1911) will be adopted throughout. The Paper may conveniently be divided as follows :—

1. Discussion of probability laws as applied to the emission of α particles.
2. Experiments with actinium active deposit.
3. Experiments with thorium active deposit.

1. In a previous Paper* we showed that if α particles are counted on a zinc sulphide screen at a mean rate of μ per second, then the probability of occurrence of a time interval of length between t and $t+\delta t$ is $\mu e^{-\mu t}\delta t$. This law of intervals may be employed in all experiments in which α particles are counted to give some indication as to whether the results are reliable. Thus, when counting α particles of short range by the scintillation method, the scintillations are so faint that with an untrained observer they are often missed, and generally in such a way that the eye fails to recognise, not merely a single scintillation here and there, but frequently a whole group of successive scintillations. An analysis of the intervals between successive records of scintillations would reveal such errors and give some idea of the correction necessary. Again, as has been pointed out by Mme. Curie,† in counting α particles by the electrical method, the damping of the electrometer needle,

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* Phys. Soc. "Proc.," XXIII., V., p. 367, 1911.

† "Le Radium," VIII., p. 354, 1911.

and the effect of the Bronson resistance, is generally such that two α particles coming within a small interval would scarcely produce separate effects, and from an examination of the numbers given by Rutherford and Geiger[*] the magnitude of the throws does not always give reliable indication as to whether one or two α particles have entered the ionisation chamber. In the experiments of Rutherford and Geiger the rate at which the α particles were counted was very small (3·5 per minute), and, assuming that intervals of $\frac{1}{4}$ second would not be recognised, the correction would be 1·4 per cent. The shape of the curve of intervals given in their Paper, in which there are too few small intervals, is due to this cause.

The formula given above is derived initially from the assumption that the probability of a scintillation occurring in any small time interval, δt, is $\mu\delta t$, and it is, therefore, analogous to the derivation of the exponential law of decay in radio-active disintegration, the only difference being the exact meaning of μ in the two cases. If, then, we have in our source of α particles a product present of very short life, it will reveal itself by there being too many short intervals, and thus, by observing the intervals between successive α particles, we can hope to determine the presence of such products. Thus, by the experiments of Geiger and Marsden,[†] a product was discovered after thorium emanation of mean life $\frac{1}{5}$ second, while our own experiments[‡] on uranium and polonium showed that no short life products were present in these substances.

2. *Experiments with Actinium Active Deposit.*

The active deposit of actinium is known to consist of three successive products—actinium B (β), C (α) and D (β). The double-screen experiments of Geiger and Marsden,[§] however, indicated certain irregularities in the emission of the α particles, in that 10 per cent. of them seemed to be emitted in pairs. Irregularities indicating the presence of a product of short life (not act. A) were also obtained in the experiments of Mlle. Blanquies.[||] On the other hand, Moseley and Fajans,[¶] by a special method, failed to recognise any such product. The question as to the existence of such a product is very interesting, for it would indicate a lateral disintegration, such as Fajans* found in radium C and Antonoff† in uranium. We have, therefore, repeated the experiments of Geiger and Marsden to find out whether the large number of connected particles is explained by probability, which we have shown gives what at first sight appears to be an unexpectedly large number of small intervals. The apparatus used is the same as was employed in the above experiments, and for convenience is drawn in Fig. 1.

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[*] Roy. Soc. "Proc.," A, LXXXI., p. 141, 1908.

[†] "Phys. Zeitschr.," XII., p. 7, 1910.

[‡] *Loc. cit.*

[§] *Loc. cit.*

[||] "Le Radium." VI., p. 230, 1909.

[¶] "Phil. Mag.," XXII., p. 629, October, 1911.

Two microscopes, M_1 and M_2, are focussed on exactly opposite areas of two zinc sulphide screens, S_2 and S_1. Aluminium foils are placed between the screens in such a way as to allow emanation from the actinium source A to diffuse between them, at the same time preventing the screen S_2, say, being visible through the microscope M_2. The emanation source A was placed in position and left there for 36 minutes, so that the active deposit between S_1 and S_2 attained half its equilibrium value. The source was then removed, and, after allowing about 25 seconds for the emanation to decay, scintillations were observed and recorded by both observers on a travelling chronograph tape until the expiration of five minutes. The source A was then replaced for five minutes, and the observations repeated until a sufficient number of scintillations had been recorded. The results of several experiments are given in Table I., the sums of all the observations being placed in the last column. The calculated values given are obtained from the formula

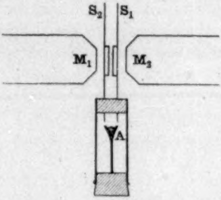

FIG. 1.

$$N\mu\int_{t_1}^{t_2} e^{-\mu t}dt = N(e^{-\mu t_1} - e^{-\mu t_2}).$$

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* "Phys. Zeitschr.," XII., p. 369, 1911.

† "Phil. Mag.," XXII., p. 419, 1911.

TABLE I.

Duration of interval between successive scintillations.	Number of Intervals.					
	Experiment V.		Experiment IX.		Totals of all experiments.	
Seconds.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
0·0 to 0·5	51	49·3	9	7·6	219	207·3
0·5 to 1·0	46	44·2	7	7·5	200	189·6
1·0 to 2·0	77	76·8	17	14·3	340	332·4
2·0 to 3·0	74	62·5	19	13·4	285	278·0
3·0 to 4·0	43	51·4	15	12·7	220	232·9
4·0 to 5·0	37	41·6	8	11·9	184	193·9
5·0 to 7·0	59	62·4	23	21·7	290	301·7
7·0 to 10·0	44	56·8	24	27·9	279	296·5
10·0 to 15·0	61	43·7	29	36·5	263	262·4
15·0 to 20·0	14	16·1	26	26·8	128	122·4
20·0 to 30·0	6	7·8	32	34·3	97	93·4
30 to infinity	2	1·2	46	40·4	70	64·5
$\mu =$	0·201		0·061		Average. 0·149	

The agreement between the observed and calculated values is on the whole good, though for intervals up to three seconds the observed numbers are slightly in excess.* If this effect is real, we have no certain explanation for it, but the excess only corresponds to about 1 per cent. of the whole number of scintillations, indicating that if there is a product present of period about one second it must be a lateral one, and consist of only about 1 per cent. of the main line of descent. It is of interest to note that, assuming a period of one second for such a hypothetical product, its range, according to the empirical formula given by Geiger and Nuttall,† would be only 4 mm. in excess of that of actinium C., and in such circumstances it would be very difficult to detect. Experiments are, however, in progress on this point.

We, therefore, conclude that the 10 per cent. of connected particles found in the experiments of Geiger and Marsden, referred to above, are due to probability, and that no certain evidence of the existence of a short life product can be drawn from them.

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* The question as to the probability difference between the observed and calculated values will be examined in a later Paper.

† "Phil. Mag.," XXII., p. 613, October, 1911.

3. *Experiments with Thorium Active Deposit.*

In the thorium family, after the emanation, there are known to be three α-ray products. The first, thorium A, was discovered by the double-screen experiments of Geiger and Marsden, and has a period of only 0·14 second, so that it practically accompanies the emanation. The other α-ray products have been investigated chiefly by Hahn, who was, however, unable to separate them by recoil or by chemical methods. They are only recognised by their different ranges, for which Hahn found the values 5·0 cm. and 8·6 cm. Hahn explained his results by assuming that the products are genetically derived from one another, and that the second has an extremely short period.

Geiger and Marsden, in some preliminary experiments,* found that the number of pairs of scintillations, using a double-screen arrangement, is smaller and of a different order to the number obtained with thorium and actinium emanations. Previous results had also shown that thorium emanation +A give twice as many α particles as the active deposit. From these results Rutherford and Geiger† have suggested that the changes occurring in thorium active deposit are irregular, and have proposed the following scheme of disintegration :—

$$\begin{array}{lccccc} & & & & C_1+C_2 & \\ & & & & \| & \\ \text{Product:} & \text{Th. Em.}\longrightarrow & \text{Th. A}\longrightarrow & \text{Th. B}\longrightarrow & \text{Th. C}\cdots\rightarrow & \text{Th. D} \\ \text{Radiation:} & \alpha & \alpha & \beta & \alpha(5{\cdot}0 \text{ and } 8{\cdot}6) & \beta \end{array}$$

Assuming that for each α particle given out by thorium emanation and A, thorium C_1 and C_2 give one particle between them, this scheme explains all the above observed facts. However, Miss Leslie,‡ by comparing the numbers of α particles from radio-thorium and thorium emanation has come to the conclusion that the latter gives (+A) four α particles, in which case no irregularities of the nature of lateral disintegration need be assumed. To throw light on this interesting question we have, therefore, made double-screen experiments on thorium active deposit, and have also, by counting the number of α particles at different distances from a source of thorium active deposit, determined the relative number of α particles emitted by thorium C_1 and C_2. The apparatus used in this latter experiment is shown in Fig. 2.

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* See note Rutherford and Geiger, "Phil. Mag.," XXII., p. 621, October, 1911.

† Rutherford and Geiger, *loc. cit.*

‡ "Le Radium," VIII., p. 356, 1911.

A plate, A, was made active by exposure to the emanation from a preparation of mesothorium,* and placed at a distance greater than 8·6 cm. from the zinc sulphide screen S sealed on to the end of a wide tube. Scintillations were counted on the screen for different pressures of the air in the tube, and thus a curve was drawn showing the number of α particles corresponding to different " ranges " from the source, the latter being obtained by multiplying the distance from S to A by the number of centimetres of pressure in the tube and dividing by 76, correction being also made for the temperature to reduce the observations to 15°C. The results of a particular experiment are shown in Fig. 3.

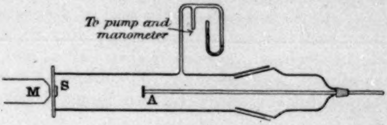

FIG. 2.

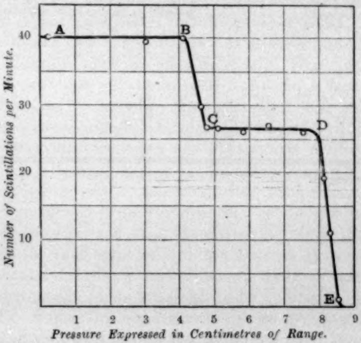

FIG. 3.

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* We are deeply indebted to Mr. F. H. Glew, who very kindly loaned us this preparation.

In the figure the part AB corresponds to the α particles from both thorium C_1 and C_2. The part BC shows that the α particles from thorium C_1 are gradually being reduced in number, C denoting the end of the range. The part CDE corresponds to the α particles from thorium C_2 only. The curve is similar to those given by Geiger* for actinium, &c. It will be noticed (1) The ranges of thorium C_1 and C_2 are 4·8 cm. and 8·6 cm. respectively; (2) the ordinates of the part of the curve CD are greater than half the ordinates corresponding to AB, thus showing that thorium C_2 gives more than half as many α particles as thorium C_1+thorium C_2. The weighted mean of 12 different experiments in which the ratio found varied between 0·60 and 0·70 is

$$\frac{\text{No. of } \alpha \text{ particles from thorium } C_2}{\text{No. from thorium } C_1 + \text{No. from } C_2} = 0{\cdot}646.$$

That is to say, thorium C_2 gives 1·83 times as many α particles as thorium C_1.

This ratio is not due to an admixture of radium C with the thorium active deposit, as might, perhaps, be expected, since mesothorium preparations contain generally a large percentage of radium. Observations of the ratio were made in several cases for 30 hours after the active plate was taken from the mesothorium, and although the active deposit decayed accurately with a period of 10·6 hours, the ratio was unaltered, showing that the decay of both thorium C_1 and thorium C_2 is governed by that of thorium B. Again, with a short exposure, the ratio was within the experimental error independent of the rise in activity of the active deposit.

We will withhold a full discussion of the deductions to be derived from the above experiments until the figures referring to the double-screen experiments have been considered. These experiments were carried out in the same way as those with actinium, except that mesothorium was used instead of the actinium preparation, while in some cases the active deposit was first deposited on an aluminium foil which was subsequently placed between two other aluminium foils between the two screens. In the experiments countings were generally made for two hours, the decay of the active deposit in this period being such that the calculated values of the various intervals are influenced to something less than 1 per cent. by the fact that the source was not constant, and consequently the value of μ variable. The results are given in Table II., where the columns have the same significance as in Table I. The last two columns refer to an experiment in which the active source was thorium emanation plus about 20 per cent. of its equilibrium amount of active deposit. The calculated values in this case are obtained from the above formula, with the additional assumption that 59 per cent. (found by trial) of the scintillations are such that half of them are produced by a particles from emanation atoms, and the other half by the thorium A produced by the disintegration of these emanation atoms, the period of thorium A being taken as 0·145 second*. This experiment gives an idea of the efficiency of the screens, as the total possible number would be about 90 per cent.

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* "Phil. Mag.," XXII., p. 201, 1911.

TABLE II.

Duration of interval between successive scintillations.	Number of Intervals.							
	Experiment VII. Active deposit obtained by diffusion of emanation.		Experiment VIII. Active deposit on separate aluminium foil.		Totals. Active deposit.		Thorium emanation.	
Seconds.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
0·0 to 0·5	91	68·5	101	109·1	353	314·2	438	435·0
0·5 to 1·0	55	60·3	98	93·5	311	278·9	178	179·0
1·0 to 2·0	92	99·0	159	149·0	438	466·2	212	203·2
2·0 to 3·0	63	76·5	114	109·5	354	362·8	96	107·5
3·0 to 4·0	59	59·0	74	80·4	250	289·4	55	56·6
4·0 to 5·0	44	45·5	48	59·4	214	228·3	27	30·2
5·0 to 7·0	57	61·7	75	75·8	323	327·6	26	24·7
7·0 to 10·0	57	49·1	59	53·6	295	290·1	12	8·3
10·0 to 15·0	26	30·3	32	28·0	226	225·3	1	1·5
15·0 to 20·0	15	8·2	4	6·0	102	93·4		
20·0 to 30·0	2	2·8	2	1·6	76	64·6		
30·0 to infinity	0	0·2	0	0·1	24	25·1		
μ=	0·2606		0·3069		Average. 0·196		...	

The observed and calculated values are, on the whole, in good agreement, except that there is a small excess of intervals between 0 and 1 second, corresponding to about 2 per cent. of the whole number of scintillations. This may be partly or wholly due to probability, and this latter question will be examined in a later theoretical Paper. However, the excess is only shown in the experiments in which the active deposit was obtained by diffusion of the emanation between the two screens, and is not pronounced in the experiments in which the active deposit was first obtained on a separate aluminium foil. Again, the excess of small intervals is not noticeable in the analysed observations of each separate observer. An explanation may be that the emanation diffuses through unavoidable small holes in the aluminium foils, causing the active deposit to be deposited directly on the zinc sulphide screens themselves, the pairs of scintillations being the result of either (1) a single α particle producing a scintillation on both screens, or (2) the recoil atom or δ-rays produced on the explosion of an α particle. In the former case we should expect the whole of the excess of intervals to be from 0 to 0·5 second, while in regard to the latter the experiments of one of us* have shown that the luminescent effect of β-rays is not altogether simultaneous with their impact on the screen. Diffused patches, or " sploshes " of light, were occasionally seen, and sometimes the observer might tap these as scintillations without being able to decide at the exact moment as to whether or not they are real scintillations. A special experiment, in which the whole of the active deposit was on one of the zinc sulphide screens, gave 255 scintillations on the active screen and 144 on the other, with an excess of 4 per cent. of intervals 0 to 1 second, the difference in the numbers being largely explained by the uneven surface of the crystals on the screen.

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* Moseley and Fajans, "Phil. Mag.," XXII., p. 629, October, 1911.

It is also improbable that the excess of small intervals mentioned above is due to the missing of scintillations by either observer. Evidence on this question was obtained as follows : The tapes were examined with regard to the number of times either observer recorded 1, 2, 3, &c., scintillations without the sequence being interrupted by the other observer recording a scintillation. The total numbers so obtained are given in Table III., and it will be seen that they are in excellent agreement with the numbers calculated from simple probability—*i.e.*, the probability of 1's=$\frac{1}{2}$, 2's=$\frac{1}{4}$, &c. These results are also interesting from the point of view of the question as to the application of probability to radio-active disintegration.

Independently of the possible small excess of small intervals the above double-screen experiments show conclusively that if thorium C_2 comes directly after thorium C_1, its period must be greater than a few seconds. In that case, however, Hahn would have been able to separate the products by recoil experiments, while similar experiments by Geiger and one of us, in which an α-ray recoil product of a fraction of a second period would have been detected, failed to reveal any. Thus we may conclude that thorium C_2 is not produced by direct disintegration of thorium C_1, and the above experiments on the relative number of α particles from these products show that they must be on different lines of descent, otherwise the numbers would be equal. It is, however, just about within the limit of experimental error that the ratio C_2/C_1+C_2 above should be 0·667. In that case we should have thorium C_2 with twice as many α particles as thorium C_1. To explain this without bringing in the idea of lateral disintegration we should have to assume two products of range 8·6 cm., and also that thorium emanation (+A) gives 6 α particles. These assumptions seem somewhat out of the question.

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* E. Marsden, " Proc." Roy. Soc., A 83, p. 548, 1910.

TABLE III.—*Analysis of Scintillations with respect to the number of times either observer records a sequence of Scintillations uninterrupted by the other observer recording a Scintillation.*

—	—	Number of Scintillations in Sequence.								
		1.	2.	3.	4.	5.	6.	7.	8.	9 and over.
Records of actinium active deposit	Observed	533	297	125	63	23	9	6	5	1
	Calculated	531	266	133	66	33	16	8	4	4
Records of thorium active deposit	Observed	836	423	215	109	57	26	13	7	9
	Calculated	848	424	212	106	53	26	13	7	7
Totals	Observed	1,369	720	340	172	80	35	19	12	10
	Calculated	1,379	690	345	172	86	43	21	11	11

We have tried another method to throw light on the question. Thorium X was obtained on a plate of 3 cm. diameter by recoil from radio-thorium, and an attempt was made by sucking air over it to determine the ratio of the number of α particles emitted from thorium emanation to the number from thorium X. The results were, unfortunately, not decisive, for assuming one α particle per disintegrating atom of thorium X, it was calculated that the film retained more than 25 per cent. of its emanation, even when a strong current of air was passed over it continuously.

If the logarithms of the transformation constants be plotted against the logarithms of the ranges according to the method of Geiger and Nuttall,* we find that the product of 4·8 cm. range corresponds to the 60 minutes' period, previously known as that of thorium C; while the period of a product of 8·6 cm. range should be of the order 10^{-12} second.

However, as noted above, with a short exposure of a plate to thorium emanation the rise in activity with time of both the products is practically the same. If thorium C_2 has a short life and comes directly after thorium B such a plate would not rise in activity with time, but would decay along with the thorium B to half value in 10·6 hours. Thus thorium C_2 must have a parent of period about the same as that of thorium C_1 (60 mins.) It therefore appears that the lateral disintegration begins with the atoms of thorium C_1, 35 per cent. giving rise to α particles of 4·8 cm. range, and the remaining 65 per cent. giving rise to α particles of 8·6 cm. range with probably an intermediate emission of slow β particles.

The experiments of Geiger and Kovarik† are interesting in regard to the above questions. Assuming that, with the active deposit in equilibrium, thorium C_1+C_2 give two α particles per disintegrating atom of thorium D, they found results which suggest that the latter product gives two β particles per disintegrating atom. If we take our above result that thorium C_1+C_2 give only one α particle between them, we need only assume one β particle from thorium D, provided we also assume a scheme of disintegration somewhat as follows:—

$$\begin{tikzcd}
 & & & \text{Th. } C_1 \arrow[drr] & & \\
\text{Th. Em.} \arrow[r] & \text{Th. A} \arrow[r] & \text{Th. B} \arrow[ur] \arrow[dr] & \alpha\ (4{\cdot}8) & & \text{Th. D.} \\
\alpha & \alpha & \beta & \text{Th. } C_1' \arrow[r] & \text{Th } C_2 \arrow[ur] & \beta \\
 & & & ? & \alpha\ (8{\cdot}6) &
\end{tikzcd}$$

However, the various assumptions involved are such that this latter question cannot be taken as settled, and further consideration must be delayed until more experimental evidence is available.

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* "Phil. Mag.," XXII., p. 613, October, 1911.

† "Phil." Mag., XXII., p. 604, October, 1910.

Summary.

1. A discussion is given of the magnitude and nature of an error which sometimes affects experiments on the counting of α particles.

2. No irregularities in the emission of α particles from actinium active deposit are shown by the experiments.

3. The range of thorium C_1 has been determined and found to be 4·8 cm.

4. The experiments show that there is a lateral disintegration in thorium active deposit, thorium C_1 and thorium C_2, being on different branches in the ratio 1 : 1·83.

This research was carried out at the East London College, and we are much indebted to the Research Grant Committee of that institution and to Prof. Lees for placing the resources of the laboratory at our disposal.

ABSTRACT.

In a previous Paper ("Proc." Phys. Soc., Aug., 1911) the authors showed that if α-particles are counted on a zinc sulphide screen at a mean rate of μ per second, then the probability of occurrence of a time interval, of length between t and $t+\partial t$, is $\mu e^{-\mu t}$. This formula may be applied, to test whether two α-particles are given off simultaneously from a disintegrating atom or whether in any source of α-particles there exist two successive α-ray products, the latter being of short life.

In the previous Paper Uranium and Polonium were shown not to give such irregularities, and in the present Paper the same result has been found for Actinium and Thorium active deposits, although experiments of various investigators pointed to the probability of positive results.

The experiments further suggest a lateral disintegration in Thorium active deposit, and this is proved to be the case by results which show that the two α-ray products in Thor. Act. Dep. (Th. C_1 and C_2) do not give an equal number of α-particles when the active deposit is in equilibrium, which is required by the ordinary disintegration theory. Thus it is concluded that of the atoms Th. C, 35 per cent. give rise to α-particles of 4·8 cm. range and 65 per cent. to α-particles of 8·5 cm. range, with probably the intermediate emission of β-particles. Various cognate questions are also discussed in the Paper.

X. *The Magnetic Transition Temperature of Cementite.* By S. W. J. Smith, W. White *and* S. G. Barker.

RECEIVED NOVEMBER 9, 1911. READ NOVEMBER 24, 1911.

THE experiments described below were made upon a sample of steel kindly given to one of us, in 1908, by Prof. J. O. Arnold, of Sheffield. The steel had the following percentage composition:—

C=0·85, Si=0·05, Mn=0·06, S=0·03, P=0·02 and Fe (by difference)=98·99.

It was supplied in the form of rods, ½ in. in diameter, which had been carefully annealed and were thus in a condition suitable for the extraction of the carbide of iron (cementite) from them by the method described by Arnold,[*] which we have followed.

Very little was known about the magnetism of cementite at the time when the material was obtained, and it was thought desirable to obtain further knowledge. It seemed obvious, for example, that a determination of the temperature at which iron carbide ceases to be magnetisable might prove useful in the attempts being made to analyse the effects of thermal treatment upon the properties of carbon steels.

An opportunity to carry out the intention of examining the thermomagnetic properties of cementite, by isolating it from the steel described above, did not present itself until last session, when the measurements given below were made in the Imperial College of Science.

In the interval the temperature at which cementite loses its magnetism had become known, approximately at least, through a measurement by Wologdine.[†] The material, suspended within a liquid of sufficiently high boiling point, formed a chain between the poles of an electromagnet, and the temperature to which the liquid could be raised before the chain broke down was observed to be 180°C.

Although this method is obviously only approximate, Wologdine appears to take the view that it is sufficient. He remarks that the loss of magnetic properties is progressive, and does not permit a more rigorous determination.

In the following year Maurain[‡] found certain peculiarities, at temperatures below 240°C. in the magnetisation curves of two steels containing about 0·2 and 0·9 per cent of carbon respectively. Discussing these peculiarities, he suggests that they are due to the thermomagnetic properties of cementite.

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[*] "Steel Works Analysis," 1898, pp. 326-330; also Chem. Soc. "Trans.," 1894, p. 788; "Journ." Iron and Steel Inst., 1910, Vol. I., p. 174.

[†] "Comptes Rendus," 1909, Vol. CXLVIII., p. 777.

[‡] "Comptes Rendus," Vol. CL., 1910, pp. 777–780; also "Ann. de Chim. et de Phys." [8], Vol. XX., 1910, pp. 353–389.

To account for the want of exact correspondence between his curves and the result given by Wologdine, he puts forward certain considerations to show why an exact correspondence is not always to be expected. These need not be given here, since we hope to show that the main cause of the discrepancy is the inexactness of Wologdine's result.

It is in fact misleading to regard 180°C. as a sufficient approximation to the temperature at which cementite loses its ferromagnetism. The actual temperature is much nearer 240°C.

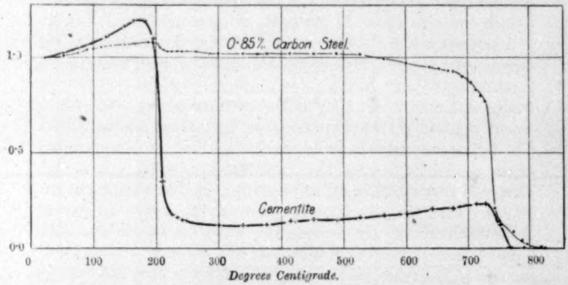


Fig. 1.

We may describe first some experiments which serve to indicate the extent to which we succeeded in extracting free cementite from the steel.

One of several sets of results is shown in Fig. 1. The thick-line curve refers to the cementite and the thin-line curve to a sample of the steel. The ordinates are magnetometer deflections at different temperatures, the deflection at the room-temperature being taken as the unit in each case.

The cementite was contained in a fused silica tube of about 0·5 cm. diameter and occupied about 20 cms. of its length. It was partially protected from the influence of the air by a plug of asbestos. A platinum rhodium thermojunction imbedded in the cementite was used to measure the temperature. The tube, placed along the axis of a magnetising solenoid kept cool by water circulation, was heated electrically. During the rise of temperature a steady current, sufficient to produce a field of about 100 units, traversed the solenoid. The silica tube lay at right angles to the meridian and pointed towards a magnetometer needle. The ordinates represent the deflections due to the cementite alone, other effects upon the needle being counterbalanced in the usual way.

The steel (a short tube) was examined in the same way as the cementite, the same apparatus being used to determine its temperature.

A glance at the curves shows a very pronounced drop (about 90 per cent.) in the magnetism of the cementite between 180°C. and 250°C. There is also a corresponding but very much smaller drop (about 5 per cent.) in the magnetism of the steel (which contains about 13 per cent. of cementite).

A comparison of the curves suggests that the isolation of the cementite was not quite complete, and that it still contained a small quantity of admixed steel. One or two attempts were made, without success, to obtain a specimen of cementite which would not exhibit any appreciable magnetism above 250°C. The failure may have been due to the fact that the susceptibility of the cementite beyond the transition temperature, although relatively very small, is still appreciable. It is much more likely, however, that the effects observed were due mainly to partial decomposition of the cementite after its isolation. We noticed that successive heatings of the cementite occasioned successive increases in the magnetisability above 250°. The cementite of Fig. 1 had been heated to 800° five times before the measurements to which the curve corresponds were made.

We now attempted to fix the transition temperature of the cementite more accurately and to find whether the change was reversible. The procedure was as before except that a copper constantan thermocouple was used to measure the temperature.

The lower half of Fig. 2 is a specimen of the results obtained during heating and cooling between 20°C. and 300°C. In this case the deflection was about 50 per cent. less at 250° than at 200° and the magnetic change was seen to take practically the same course during the rise and fall of temperature. It will be seen that the intensity of magnetisation was distinctly greater at 230° than at 240°. The arrangement was not sufficiently sensitive to decide whether there was any appreciable difference between the readings at 240° and 250°. A shorter and wider tube was used to contain the cementite in this experiment. It was not noticed until the work had been completed that it would have been better to have used the tube already described.

The upper half of Fig. 2, arranged for comparison, shows the behaviour of the steel over the corresponding range in the same apparatus. The scale of ordinates, arbitrary as before, is arranged so that the drop between 200° and 250° (in this case roughly 10 per cent.) is about the same in actual length as the larger (50 per cent.) drop in the cementite.

The upper and lower curves taken together serve to show how the magnetic properties of the cementite make themselves evident in the steel.

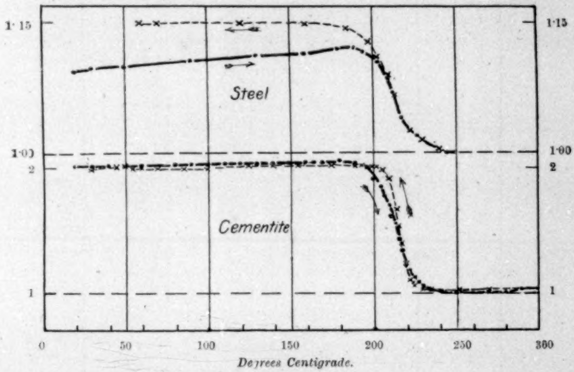


Fig. 2.

The differences below 200° between the rising and falling temperature readings on the upper curve are due to hysteresis, and arise from the method of experiment. A small ring was constructed from a portion of one of the rods, and the permeability of the material in different fields was measured by the ballistic method.

The results for fields of 14 and 3·3 C.G.S. units respectively are shown in Fig. 3. Rising temperature observations are marked by dots and falling temperature observations by crosses. The ordinates in this figure give the approximate intensities of magnetisation in C.G.S. units, the scales for the stronger and weaker fields being shown on the left and right respectively.

The dotted curve in the figure represents the behaviour of the steel after quenching from about 1000°C., in a field

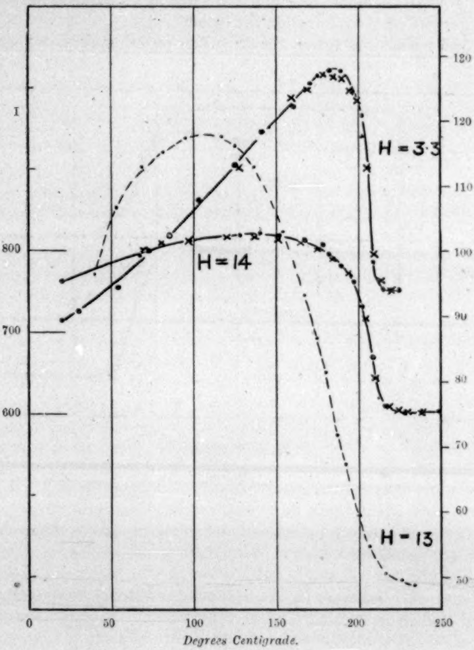


FIG. 3.

H = 13 C.G.S., during heating from 40°C. to 235°C. It is one of a series of results which we hope to describe at a later date. The scale of **I** for this curve is the same as that for the annealed steel in the field **H** = 3·3.

In conclusion we may indicate a modification of Wologdine's method, which has some obvious advantages and may be useful in other cases. It affords a very simple way of confirming the cementite curve of Fig. 2.

A small quantity of the material is dropped into a test tube containing melted paraffin wax. This tube, preferably surrounded by a thin copper sheath open at the sides, is placed between the conical pole pieces of an electromagnet, the bottom of the tube being a few centimetres below the pole ends. The temperature of the tube is raised to about 300°C. by a small Bunsen burner placed in a recess at the bottom of the sheath, and observations are made while, after lowering or removing the burner, the material cools. Successive experiments are made with different exciting currents in the electromagnet coils.

When the current is above a certain value the cementite rises from the bottom of the tube into the space between the pole pieces, immediately after the circuit is completed, although the temperature is still 300°C. The magnetic susceptibility of the (partially decomposed) particles is sufficient to enable the field to raise them against gravity.

Lowering the current step by step, a range of field strengths is reached over which the cementite does not rise until a temperature, quite sharply marked in each case, is attained. This temperature lies between about 240°C. and 200°C. for the range of fields in question—the particles rising sooner the higher the field.

In any field weaker than that which causes it to rise at 200°C., the cementite remains at the bottom of the tube even when the temperature has fallen below 100°C.

These results are clearly in agreement with Fig. 2, and show how the beginning and the end of the interval, over which the susceptibility of the cementite varies rapidly, can be fixed quite easily. The same arrangement can be used to show the presence of cementite in "cast-iron" filings.

ABSTRACT.

The temperature at which cementite (carbide of iron) loses its ferromagnetism is determined sufficiently accurately for purposes of thermomagnetic analysis and examples are given to show the possibility of using the thermomagnetic properties of cementite to determine whether that substance is present in any iron-carbon alloy.

DISCUSSION.

Prof. S. P. **Thompson** has communicated the following remarks upon this Paper: The authors do not say how they in fact extracted the cementite from the steel experimented upon, neither do they give any analysis of its composition. The steel used is stated to contain 0·85 per cent. of carbon. It therefore contained little free cementite, since a steel of that composition is nearly a pure pearlite with a slight excess of ferrite. It would be interesting to know whether similar results are obtained with cementite derived from a steel containing a higher percentage of carbon, and therefore consisting of cementite and pearlite.

Prof. J. O. **Arnold** remarked that he had read the valuable and interesting Paper by Dr. Smith and Messrs. White and Barker with much pleasure, since the physical results obtained agreed in a remarkable manner with hundreds of thermo-metallurgical observations made in the Sheffield University laboratories. In Fig. 1 the sudden drop in the magnetism of the nearly saturated steel was coincident with the thermal transformation of pearlite to hardenite, which commenced about 728°C. and was complete at 732°C. In other words, at Osmond's combined range A_c (1-2-3). In connection with the chemical-physics of the case it was well to remember that the cementite experimented upon by the authors was "B" cementite precipitated in the mass on cooling at 695°C. "A" cementite or the cementite of supersaturation commenced to precipitate at about 900°C., and though the chemical formula of both varieties was Fe_3C, containing 6·6 per cent. carbon, the physical properties were different. Arnold and Read had shown by electrolysis that the "A" carbide was electronegative to the "B" or low temperature cementite dealt with by the authors.

The **Secretary**: In reply to Prof. Thompson's first remark, it should be said that full details of the method of preparing cementite which we employed are given in the book to which we refer. A specimen chemical analysis is also referred to. For convenience in working we used considerable quantities of the material, and there is no doubt that it was not quite pure. Fig. 1, while showing the effect of these impurities, is sufficient to establish the particular property of cementite about which we have written.

The question of the chemical individuality of cementite is frequently raised. It is tersely summarised by Prof. Arnold on p. 268 of Vol. I. of the "Journal" of the Iron and Steel Institute for 1911. The present experiments indicate a new thermomagnetic proof of the thesis which Prof. Arnold there maintains. Already a fair number of materials containing widely different percentages of carbon (and including cast-irons) have been examined thermomagnetically. In every case, in which a sufficient number of observations has been made, peculiarities have been observed between 150°C. and 300°C. Hopkinson makes no mention of such peculiarities in his well-known Papers; but if his curves be examined it will be seen that he made scarcely any measurements in the region in which the presence of cementite is disclosed.

Prof. Thompson raises the question of possible differences between the cementite existing as patches in hyper-eutectoid steels and the lamellar cementite of pearlite, and suggests extraction of the "structurally free" cementite from a high carbon steel. It has to be remembered that much of the carbon of high carbon steels is apt to separate as graphite during annealing, and that, even if this difficulty were overcome, the task of separating two kinds of cementite between which there is no very sharp line of demarcation would still remain. A steel containing twice as much carbon as that we used would still contain almost as much of our cementite as of the other. A question similar to Prof. Thompson's has been considered from the chemical point of view by Saniter amongst others. So far as he could decide, free—*i.e.*, isolated—cementite has the same chemical composition whether relatively coarse-grained or fine. Without committing ourselves, we should be inclined to think that, except perhaps when the carbide is very finely divided, no considerable variation of the present property with size is likely. The question turns upon the relative importance of the surface properties. Prof. Arnold's remarks upon the electrochemical difference between the two kinds of cementite are very interesting in this connection. Surface energy sometimes plays an important part in electrolytic phenomena, and it would produce a difference between the two cementites of the kind which Profs. Arnold and Read have observed. Unless the surface energy is an important modifying influence it seems to us that two structurally different kinds of cementite cannot exist in equilibrium with ferrite except at one particular temperature. Otherwise (as may be the case) the usual interpretation of the phase rule must apparently fail when applied to iron and iron carbide.

Prof. Arnold's expression of interest in the steel curve of Fig. 1 makes us regret that, to avoid confusion, we omitted from that figure the magnetometer deflections observed during cooling. The magnetism of the steel increased as rapidly between 700 deg. and 690 deg. during cooling as it fell near 730 deg. during heating. In contrast with these phenomena, there was no appreciable difference between the temperatures at which the magnetism of the cementite was lost and regained. The eutectoid change is accompanied by changes in distribution of the carbide and iron in the material. The cementite change, on the other hand, is apparently of the same type as that which occurs in iron itself.

The cooling curve of the cementite of Fig. 1 (also omitted to avoid confusion) is instructive with regard to the impurities which the material contained. Part of the magnetism reappeared as soon as the temperature fell below about 770 deg. There was a similar small accession of magnetism at about 690 deg. The main portion of the magnetisability returned between 300 deg. and 200 deg. in the same way as it had disappeared.

There are many questions of interest arising out of the thermomagnetic properties of cementite. The present Paper deals only with one of them. We hope to return to others.

XI. *Note on a Negative Result Connected with Radio-activity.*

By J. H. VINCENT, *M.A.*, *D.Sc.*, *A.R.C.Sc.*, *and* A. BURSILL, *A.M.I.E.E.*

RECEIVED NOVEMBER 10, 1911. READ JANUARY 26, 1912.

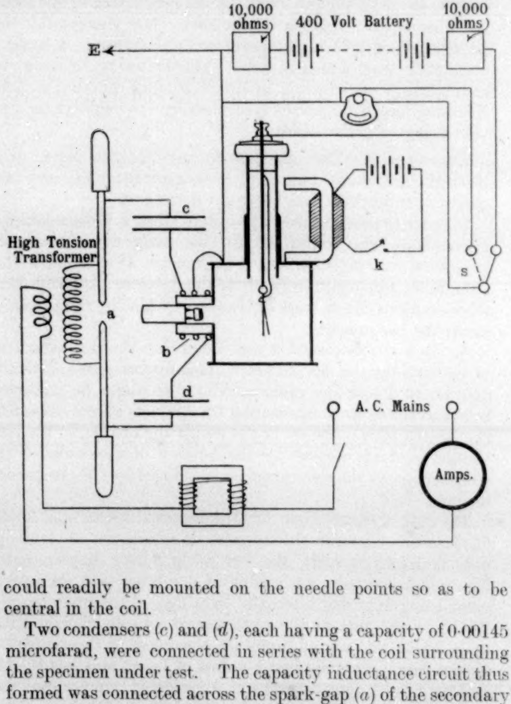
THE modern corpuscular theory of atomic constitution would lead us to admit the possibility of the production of instability in the atom by a strong magnetic field. It is also possible that an alternating field might prove more effective as a cause of instability than a steady field. The following research was consequently undertaken in order to find whether a high-frequency magnetic field causes substances to exhibit a measurable degree of radio-activity.

Experimental.—The substances experimented upon were bismuth, antimony, and iron, both para-magnetic and dia-magnetic substances thus being put to the test.

In order to measure the radio-active effect a Wilson gold-leaf electroscope was employed. In the early experiments the substance was placed about a centimetre away from a thin aluminium window in the walls of the electroscope, but in later work spurious effects were diminished by placing the specimen inside the instrument.

As the work proceeded it was found that the real difficulties of the investigation lay, not in making instruments sufficiently delicate to detect any radio-activity that might be produced, but in avoiding or compensating for spurious effects caused by the electrostatic influence of the oscillating charge, or the convection currents caused by the heating of the walls of the electroscope by the electric currents induced by the alternating magnetic field.

The form of electroscope, and the electrical connections finally adopted are shown in the diagram in which the electroscope only is drawn to scale, the rest being purely diagrammatic. The gold-leaf system was contained in a brass case, the instrument being kept dry internally by a dish of calcium chloride. Fitted to the side of the electroscope was an ebonite tube, upon which was wound a coil (*b*) consisting of three turns of copper wire. This coil had a radius of 1 cm., and was thickly insulated with gutta-percha. A soldered connection was made from the middle turn of the coil (*b*) to the case of the electroscope, and the case was carefully connected to earth, these precautions being taken to prevent electrostatic effects as far as possible. For the same reason, the ebonite tube upon which the coil was wound was painted internally with a thin coat of aluminium paint in order to render its surface slightly conducting. An ebonite plug was turned to fit the tube, and provided with two needle points. The specimens under test were powdered, and cemented to small pieces of cork which



could readily be mounted on the needle points so as to be central in the coil.

Two condensers (*c*) and (*d*), each having a capacity of 0·00145 microfarad, were connected in series with the coil surrounding the specimen under test. The capacity inductance circuit thus formed was connected across the spark-gap (*a*) of the secondary circuit of a high-tension transformer. This transformer had its primary coil connected to the 200-volt alternating current mains, a choking coil being placed in the circuit to prevent the current being excessive. Under these conditions the transformer was capable of developing 100,000 volts in the secondary circuit.

It will be seen that a spark passing across the spark-gap will give rise to electrical oscillations in the capacity inductance circuit, a current thus passing through the coil (*b*) having a high instantaneous value. In this way the specimen under test was subjected to a powerful magnetic field, alternating in direction with a high frequency.

The gold-leaf electroscope could be charged to any potential up to 400 volts, by means of a secondary battery consisting of 200 cells. The P.D. between the gold-leaf and the case of the electroscope when the instrument was being charged, was indicated on a multicellular electrostatic voltmeter, and was maintained constant during each set of experiments. Having charged the leaf by depressing the key (*k*), the brass wire above the sulphur bead in the electroscope could be disconnected from the battery, and earthed by means of the two-way switch (*s*).

Results.—In the last experiments performed the electroscope leaf was charged to —136·5 volts, and allowed to leak for two days. The mean rate of leak was 0·017 microscope divisions per minute. The actual curve of leakage was plotted by taking readings at freqeunt intervals, the initial rate of leak being greater than this mean rate. A small crystal of uranium nitrate (about 0·1 gramme) introduced into the electroscope was found to discharge the gold-leaf almost immediately, the rate of leakage being too high to measure accurately.

Upon switching on the alternating current, with no specimen in the coil, the apparent rate of discharge was very much increased at first, but soon settled down to a value approaching the natural rate of leak. This initial motion of the gold-leaf was found to be due to the convection currents set up in the electroscope by the heating of its walls by induced electric currents. It was found that the convection effects ceased a few minutes after the current was switched off, and the leaf returned to the reading on the natural leakage curve.

The method adopted in looking for radio-active effects was to switch off the current, wait for the apparatus to reach a steady temperature and for convection currents to cease, and then take the electroscope reading.

Taking readings in this manner, the results indicated in all cases that the rates of leakage remained unaltered by the presence of bismuth, antimony and iron in the magnetic field. In conclusion, no evidence was obtained that radio-activity could be produced by the action of a magnetic field.

ABSTRACT.

Specimens of iron, antimony and bismuth were subjected to a high-frequency alternating magnetic field. The air in the neighbourhood was tested for any ionisation that might have been thus produced. The results were negative.

L.C.C. Paddington Technical Institute.

XII. *On the Vibration Galvanometer and its Application to Inductance Bridges.*

By S. BUTTERWORTH, *M.Sc.* (*Manchester*), *Assistant Lecturer in Physics, School of Technology, Manchester.*

(COMMUNICATED BY PROF. E. RUTHERFORD, F.R.S.)

RECEIVED DECEMBER 19, 1911. READ JANUARY 26, 1912.

1. *Introduction.*

OF the many methods which have been proposed for the measurement of inductance, the most satisfactory are those in which a bridge is balanced for both steady and variable currents.

In these methods the detector usually employed is a telephone, but recently instruments have been devised which, besides being more sensitive than the telephone, are independent of the acuteness of the hearing of the operator. These instruments depend for their sensibility upon the principles of resonance and are called vibration galvanometers.

For the purposes of theory they may be divided into two types, according as their moving parts possess only one or an infinite number of degrees of freedom. Wenner * has deveoped a theory of vibration galvanometers of the former type when placed in a circuit containing only pure resistances. His analytical methods, however, become too cumbersome when applied to circuits in which inductance and capacity also act. Since the galvanometers are always used in these more complicated circuits it has been thought desirable to develop a theory which includes the effect of inductance and capacity.

In the following theory it is assumed at first that the galvanometer has only one degree of freedom and the results are afterwards extended to include those possessing an infinite number of degrees of freedom.

2. *Theory of the Vibration Galvanometer of One Degree of Freedom.*

As a typical example of this type of galvanometer we may take Campbell's form of instrument† which is really a D'Arsonval galvanometer having a very low periodic time.

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* Wenner, " Bulletin of the Bureau of Standards," VI., p. 347, 1910.

† Campbell, " Phil. Mag." 6 14, p. 494, 1907.

Its equation of motion is therefore*

$$\alpha\frac{d^2y}{dt^2}+\beta\frac{dy}{dt}+\gamma y=\mathrm{A}i, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where y is the galvanometer deflection at time t, and i is the current at the same instant.

The quantities α, β, γ, A are the intrinsic constants of the instrument. We will call α the constant of inertia, β the constant of damping, γ the constant of restoration, and A the constant of displacement.

If the galvanometer is placed in a circuit of resistance r and of self-induction L, the equation of current will be

$$ri+\mathrm{L}\frac{di}{dt}=e-e_b, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where e is the instantaneous applied electromotive force and e_b is the instantaneous back electromotive force due to the motion of the galvanometer wires. If the constants are properly chosen the right-hand member of equation (1) may be made to represent the force of displacement due to the current. The back electromotive force is then given by

$$e_b=\mathrm{A}\frac{dy}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

When the applied electromotive force is sinusoidal, $e=\mathrm{E}\varepsilon^{jpt}$, whence, after a steady state has been reached,

$$i=\mathrm{I}\varepsilon^{jpt}, \ y=\mathrm{Y}\varepsilon^{jpt}, \ e_b=\mathrm{E}_b\varepsilon^{jpt}, \quad . \quad . \quad . \quad (4)$$

where I, Y and E_b are given by

$$\left.\begin{aligned}(\gamma-\alpha p^2+jp\beta)\mathrm{Y}&=\mathrm{AI}\\ (r+jp\mathrm{L})\mathrm{I}&=\mathrm{E}-\mathrm{E}_b\\ \mathrm{E}_b&=jp\mathrm{AY}\end{aligned}\right\}. \quad . \quad . \quad . \quad . \quad (5)$$

Eliminating E_b and I, $\quad \mathrm{Y}(a+jb)=\mathrm{AE}, \quad . \quad . \quad . \quad . \quad . \quad (6)$

where

$$\left.\begin{aligned}a&=(\gamma-\alpha p^2)r-p^2\beta\mathrm{L}\\ b&=p\{(\gamma-\alpha p^2)\mathrm{L}+\beta r+\mathrm{A}^2\}\end{aligned}\right\}, \quad . \quad . \quad . \quad . \quad (7)$$

or putting $\quad a=\rho\cos\theta, \ b=\rho\sin\theta,$

$$\mathrm{Y}=\frac{\mathrm{AE}}{\rho}\varepsilon^{-j\theta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The amplitude of the vibration is AE/ρ, and its phase lags by an angle θ behind the electromotive force.

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* Gray, "Absolute Measurements in Electricity and Magnetism," Vol. II. p. 393.

If the constants a or γ are adjusted, ρ passes through a minimum when

$$\gamma - ap^2 = -\frac{LA^2p^2}{r^2+p^2L^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and therefore the maximum value of the amplitude attainable by this change is

$$(Y) = \frac{AE}{p\left(\beta\sqrt{r^2+p^2L^2}+\dfrac{rA^2}{\sqrt{r^2+p^2L^2}}\right)} \quad . \quad . \quad . \quad . \quad (10)$$

If now the self-induction or the displacement constant is varied, (Y) reaches a maximum value

$$\frac{E}{2p\sqrt{\beta r}}, \quad \text{when} \quad \frac{A^2}{r^2+p^2L^2} = \frac{\beta}{r}.$$

The current when this absolute maximum has been obtained is given by

$$I = \frac{E}{2r}$$

and it is in phase with the electromotive force.

If capacity is present as well as inductance, $pL - \frac{1}{pC}$ must be substituted throughout for pL.

3. *Vector Diagram for the Vibration Galvanometer.*

The whole of the preceding investigation admits of a very simple geometrical treatment.

Remembering that j is the operator turning a vector through a right angle in the counter-clockwise or positive direction, equations (5) lead to the following construction :

Let $\overrightarrow{OY}$ (Fig. 1) represent the vector Y.

Mark off $\overrightarrow{OA}$ equal to $(\gamma - ap^2)Y/A$.

Draw ABC perpendicular to OA, making $\overrightarrow{AB}$ equal to $p\beta Y/A$ and $\overrightarrow{BC}$ equal to pAY/r. Join $\overrightarrow{OB}$.

Then from the first of equations (5),

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = (\gamma - ap^2)Y/A + jp\beta Y/A = I.$$

Through C draw XX′ perpendicular to OB and cutting OB produced in E ; mark off on it $\overrightarrow{CD}$ equal to $(1/pC - pL)I/r$. Join OD.

Then from the second and third of equations (5),

$$\overrightarrow{\mathrm{OD}}=\overrightarrow{\mathrm{OB}}+\overrightarrow{\mathrm{BC}}+\overrightarrow{\mathrm{CD}}=\mathrm{I}+(jp\mathrm{AY}/r)-j(1/p\mathrm{C}-p\mathrm{L})\mathrm{I}/r=\mathrm{E}/r.$$

The negative sign is used in the third member of this equation because the direction of $\overrightarrow{\mathrm{CD}}$ is obtained from the direction of $\overrightarrow{\mathrm{OB}}$ by a clockwise or negative rotation.

In order to obtain the conditions of maximum sensibility from this diagram we will suppose the reactance $(p\mathrm{L}-1/p\mathrm{C})$ and the constant of restoration (γ) capable of adjustment, and find the conditions that a given vibration is produced by the least electromotive force.

A variation of the reactance will merely alter the position of the point D on the line XX′, while a variation of the restoration constant will alter the length of OA. AB and BC will remain constant throughout.

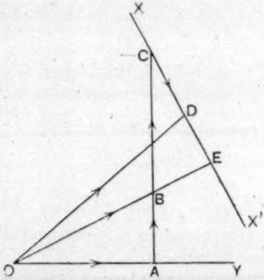

FIG. 1.—VECTOR DIAGRAM FOR VIBRATION GALVANOMETER.

OD, which is the line representing the electromotive force, will be least when the point D coincides with the point E.

Also OE will be least for variations of OA when OB=BE $=\sqrt{\mathrm{AB}\,.\,\mathrm{BC}}$, whence at maximum sensibility.

$$(\mathrm{E})/r=(\mathrm{I})=p\sqrt{\beta/r}(\mathrm{Y})\quad .\quad .\quad .\quad .\quad .\qquad (1\text{A})$$

where (I), (E) and (Y) are the *magnitudes* of the vectors I, E and Y.

Again, from the relations CE/EB=OA/AB, EB=OB,

$$(p\mathrm{L}-1/p\mathrm{C})/r=(\alpha p^2-\gamma)/p\beta,\quad .\quad .\quad .\quad .\qquad (2\text{A})$$

and from the relation $OB^2 = OA^2 + AB^2$,

$$A^2(I)^2 = (Y)^2\{(ap^2 - \gamma)^2 + p^2\beta^2\}.$$

Therefore from (1A) and (2A),

$$A^2p^2.\beta/r = (ap^2 - \gamma)^2 + p^2\beta^2 = \frac{p^2\beta^2}{r^2}\{(pL - 1/pC)^2 + r^2\},$$

whence $$A^2/\{(pL - 1/pC)^2 + r^2\} = \beta/r. \quad . \quad . \quad . \quad . \qquad (3A)$$

This is the condition for maximum sensibility.

Equations (4A) and (3A) agree with the final results of the last section.

4. *The Vibration Galvanometer on Inductance Bridges.*

In practice the vibration galvanometer is nearly always used with some form of inductance bridge, perhaps the most important being Anderson's bridge in which an inductance is balanced against a capacity.

In order to find the best conditions of working we require the theory of the unbalanced bridge.

Anderson's bridge is given in Fig. 2, where *b* is the source of alternating current, *g* is the vibration galvanometer, P, Q, S, U, are non-inductive resistances, R is an inductive resistance and T is a capacity.

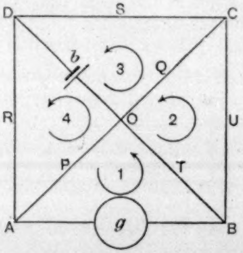

FIG. 2.

If we take different dispositions of inductances and capacities among the various arms of Fig. 2, inductance balances other than Anderson's may be obtained. It will be shown later that this bridge includes also a modified Rimington's bridge, a modified Carey-Foster bridge, and a bridge for measuring frequency.

The method adopted in the following theory is to treat the bridge as if it were composed of resistances only. Then, in order to apply the results to particular bridges, we substitute for the resistances expressions of the form $R+jpL$ in the inductive arms, and of the form $S-j/pC$ in the arms where a capacity is in series with a resistance. In other words the resistances become the resistance operators of Heaviside.

Using Maxwell's method of meshes, the current (i) through the galvanometer in Fig. 2 is given by

$$Di=E[T(QR-SP)-P\{U(S+Q)+SQ\}],$$

where E is the electromotive force in the arm DO, and

$$D=bgA_1+bA_2+gA_3+A_4,$$

in which A_1, A_2, A_3, A_4, have the following values :—

$$\left.\begin{aligned} A_1&=(P+Q+R+S)(T+U)+Q(P+R+S) \\ A_2&=U\{T(P+Q+R+S)+P(Q+R+S)\} \\ &\qquad\qquad +(R+S)\{T(P+Q)+PQ\}, \\ A_3&=(P+R)\{(T+U)(Q+S)+QS\}, \\ A_4&=U\{T(P+R)+PR\}(Q+S)+T\{PR(Q+S) \\ &\qquad\qquad +QS(P+R)\}+PQRS \end{aligned}\right\} \quad . \quad . \quad (1\text{B})$$

If $$T(QR-SP)=P\{U(S+Q)+SQ\} \quad . \quad . \quad . \quad . \quad (2\text{B})$$

the current through the galvanometer is zero and the bridge is balanced. By eliminating U from equations (1B) by means of equation (2B) we get

$$P(Q+S)A_1=B_1G_1,\quad P(Q+S)A\ =B_2G_1,$$
$$P(Q+S)A_2=B_1G_2,\quad P(Q+S)A_4=B_2G_2,$$

where

$$\left.\begin{aligned} B_1&=T(P+Q+R+S)+PQ, & G_1&=Q(P+R), \\ B_2&=T(P+R)(Q+S), & G_2&=T(QR-SP)+PRQ \end{aligned}\right\} \quad (3\text{B})$$

Hence, when there is a balance

$$DP(Q+S)=(G_1g+G_2)(B_1b+B_2).$$

If balance is disturbed by making U become $U(1-x)$, we get

$$Di=E\{T(QR-SP)-PSQ\}x.$$

If x is small, D will be practically unchanged.

Therefore $$i=\frac{EP(Q+S)\{T(QR-SP)-PSQ\}x}{(G_1g+G_2)(B_1b+B_2)} \quad . \quad . \quad . \quad . \quad (4\text{B})$$

Hence for small disturbances of balance the galvanometer behaves as if in a circuit whose resistance operator is $g+G_2/G_1$, and the battery as if in a circuit whose resistance operator is $b+B_2/B_1$.

In determining the best arrangements, we must remember that $g, b, G_2/G_1, B_2/B_1$ are complex. Denote the real part of G_2/G_1 by G_r and the imaginary part by G_i. Similarly denote the real and imaginary parts of B_2/B_1 by B_r and B_i.

Conditions for Best Source.

By placing a suitable inductance or capacity in the battery arm, we can reduce the imaginary part of $b+B_2/B_1$ to zero without disturbing any other term. Then if b_r denote the pure resistance of the battery arm, $b+B_2/B_1$ becomes b_r+B_r.

The source will either be an alternator or an induction coil. In either case, provided the form of the winding is fixed, alterations of b_r will also alter E. In fact, the law connecting b_r and E is the same as the law connecting galvanometer resistance and deflection in an ordinary direct-current galvanometer.

The best value of b is therefore $b_r=B_r$, and when this is the case $E/(b_r+B_r)$ becomes $E_0/\sqrt{B_r}$, where E_0 is a constant depending on the form of winding but not on the diameter of the wire.

Conditions for Best Galvanometer.

The galvanometer behaves as if in a circuit whose resistance operator is $g+G_2/G_1$. The resistance of this equivalent circuit is g_r+G_r where g_r is the resistance of the galvanometer.

The galvanometer can be brought to its maximum sensibility by the methods indicated in sections (2) and (3). When this has been done, the amplitude of vibration (Y), is given by

$$(Y)=\sqrt{\frac{g_r+G_r}{\beta}}\cdot\frac{i}{p},$$

and the imaginary part of $g+G_2/G_1$ is zero.

Hence the galvanometer resistance must be small.

When the battery and galvanometer arms have been properly adjusted, equation (4B) becomes

$$(Y)=\frac{E_0 x}{\sqrt{\beta}}\cdot\frac{P(S+Q)\{T(QR-SP)-PSQ\}}{p\,.\,\sqrt{B_r}\,.\,G_r\,.\,(B_1)\,.\,(G_1)},\quad . \quad . \qquad (5\text{B})$$

provided that the galvanometer resistance is negligible.

Here (B_1) and (G_1) are the moduli of B_1 and G_1.

Best Resistances in Anderson's Bridge.

Put $G_1=a_1+ja_2$, $G_2=b_1+jb_2$; $B_1=c_1+jc_2$, $B_2=d_1+jd_2$.

Then
$$G_r=\frac{a_1b_1+a_2b_2}{a_1^2+a_2^2},\quad B_r=\frac{c_1d_1+c_2d_2}{c_1^2+c_2^2},$$
$$(G_1)^2=a_1^2+a_2^2,\quad (B_1)^2=c_1^2+c_2^2.$$

Therefore $(G_1)(B_1)\sqrt{B_r\,.\,G_r}=\sqrt{(a_1b_1+a_2b_2)(c_1d_1+c_2d_2)}$.

To obtain the values of a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2 in Anderson's bridge, make the substitutions $R=R+jpL$, $T=-j/pC$, $S=QR/P$ in equations (3B). We get

$$a_1=Q(P+R),\qquad a_2=pLQ,$$
$$b_1=\left(\frac{L}{C}+PR\right)Q,\quad b_2=pLQP,$$
$$c_1=\frac{L}{C}+PQ,\qquad c_2=-\frac{(P+Q)(P+R)}{pCP}$$
$$d_1=\frac{L}{C}\frac{Q(P+R)}{P}\qquad d_2=-\frac{Q(P+R)^2}{pCP}.$$

Therefore

$$a_1b_1+a_2b_2=Q^2\left\{\left(\frac{L}{C}+PR\right)(P+R)+p^2L^2P\right\},$$
$$c_1d_1+c_2d_2=\frac{LQ(P+R)}{CP}\left\{\frac{L}{C}+PQ+\frac{(P+Q)(P+R)^2}{p^2LCP}\right\}.$$

Also

$$P(S+Q)\{T(QR-SP)-PSQ\}\equiv Q^2(P+R)\left(\frac{L}{C}-QR\right).$$

If these substitutions are made in equation (5B) it will be seen that the sensibility is increased by diminishing the frequency $(p/2\pi)$ of the alternating current, the limiting value of (Y) for zero frequency being

$$(Y)=\frac{E_0x}{\sqrt{\beta}}\cdot\frac{CP\sqrt{Q}\left(\frac{L}{C}-QR\right)}{(P+R)\sqrt{(P+Q)\left(\frac{L}{C}+PR\right)}}.$$

We require the best values of P, Q and C for a given L and R. Put in the above equation

$$P=lR,\ Q=lR/m,\ C=L/nlR^2,$$

we get
$$(Y)=\frac{E_0x}{\sqrt{\beta}}\frac{L}{R}\frac{\sqrt{l}}{1+l}\cdot\frac{mn-1}{mn\sqrt{(m+1)(n+1)}}.$$

As regards l this expression is a maximum when $l=1$. Also it is symmetrical with respect to m and n.

Hence the best values of m and n are equal and are given by the maximum of

$$\frac{m^2-1}{m^2(m+1)}=\frac{m-1}{m^2},$$

which occurs when $m=2=n$.

Therefore the best conditions of working Anderson's bridge after the galvanometer and battery have been adjusted are

$$p \text{ small, } P=R, \ Q=\tfrac{1}{2}R, \ \frac{L}{C}=2R^2.$$

5. *Anderson's Bridge and its Modifications.*

(*a*) If in the bridge of Fig. 2 we make T a capacity and R an inductive resistance we obtain Anderson's bridge. In order to obtain the conditions of balance we substitute $R+jpL$ for R and $-\frac{j}{pC}$ for T in the equation of balance (2B). Then from the real and imaginary parts of this equation we get as the two conditions for balance

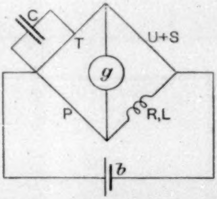

FIG. 3.—RIMINGTON'S BRIDGE.

$$\left.\begin{array}{l} QR=SP \\ \frac{L}{C}Q=P\{U(S+Q)+SQ\} \end{array}\right\} \quad . \quad . \quad . \quad . \quad (1c)$$

(*b*) If we make Q a capacity and R an inductive resistance we obtain a modified Rimington's bridge as may be seen by comparing the diagram for Rimington's bridge (Fig. 3) with Fig. 4, which is the diagram for this modification. The advantage of the modification is that the balance is an absolute one, while in Rimington's bridge the balance is only for integral current. The conditions for balance are

$$\left.\begin{aligned} RT&=P(U+S) \\ \frac{L}{C}T&=PS(U+T) \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (2c)$$

The first adjustment is made with steady currents and the second is made by adjusting S and U, keeping their sum constant. For rapid working a slide wire should be used for U+S.

Of course, the above are only two out of many possible balances, but most of the other modifications are worthless because of the difficulty in bringing about the double adjustment. We may, however, by a simple extension bring about modifications in which mutual induction is present.

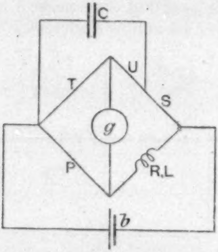

FIG. 4.—MODIFIED RIMINGTON'S BRIDGE.

This extension depends upon the following theorem :—

Let BC, CA (Fig. 5) be two arms of a network, and let them act upon each other inductively.

Let V_A, V_B, V be the potentials at A, B, C respectively, and let the current round the mesh of which BC is a boundary be i_1, the current round the mesh of which CA is a boundary be i_2.

Then, if r_1, r_2 are the resistance operators for BC, CA, and m is the resistance operator introduced by mutual induction,

$$\left.\begin{aligned} V-V&=r_1i_1-mi_2 \\ V_C-V_A&=r_2i_2-mi_1 \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (3c)$$

Now suppose the mutual induction removed and let a new arm BA be introduced. By a suitable adjustment of the resistance operators of the arms BC, CA, AB, we may so arrange that the condition of the remainder of the network is unaltered. In order that this may be so, $V_B - V_C$, $V_C - V_A$, i_1 and i_2 must be unaltered.

To get the required condition, let X, Y, Z be the new resistance operators for the arms BC, CA, AB, and let i_3 be the current in the new mesh ABC.

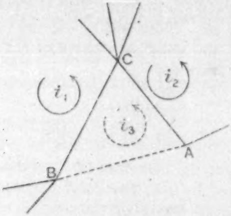

FIG. 5.

Then

$$\left.\begin{aligned} V_B - V_C &= X(i_1 - i_3) \\ V_C - V_A &= Y(i_2 - i_3) \\ (X+Y+Z)i_3 &= Xi_1 + Yi_2 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4c)$$

Eliminating i_3

$$\left.\begin{aligned} V_B - V_C &= X\left(1 - \frac{X}{X+Y+Z}\right)i_1 - \frac{XY}{X+Y+Z}i_2 \\ V_C - V_A &= Y\left(1 - \frac{Y}{X+Y+Z}\right)i_2 - \frac{XY}{X+Y+Z}i_1 \end{aligned}\right\} \quad . \quad . \quad (5c)$$

If (5c) is identical with (3c),

$$X\left(1 - \frac{X}{X+Y+Z}\right) = r_1, \quad Y\left(1 - \frac{Y}{X+Y+Z}\right) = r_2, \quad \frac{XY}{X+Y+Z} = m$$

or

$$X = \frac{r_1 r_2 - m^2}{r_2 - m}, \quad Y = \frac{r_1 r_2 - m^2}{r_1 - m}, \quad Z = \frac{r_1 r_2 - m^2}{m} \quad . \quad . \quad . \quad (6c)$$

By means of this theorem we obtain the following modifications of the bridge of Fig. 2.

(*c*) Remove U and let T and Q act inductively. In order that the general theory of the bridge may still hold, we must put

$$T = \frac{r_1 r_2 - m^2}{r_2 - m}, \quad Q = \frac{r_1 r_2 - m^2}{r_1 - m}, \quad U = \frac{r_1 r_2 - m^2}{m},$$

where r_1, r_2 are the new resistance operators for the arms T, Q, and m is the resistance operator corresponding to mutual induction.

Making these substitutions in the equation for balance we get, after some reduction,

$$(R+P)m-P(r_2+s)=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7c)$$

Now put a capacity C in series with the resistance R so that R becomes $R-j/pC$. Also put $r_2=Q+jpL$, $m=jpM$.

From the real and imaginary parts of equation (7c) the two conditions for balance are

$$\left.\begin{array}{l}\frac{M}{C}=P(Q+S) \\ (R+P)M=PL\end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8c)$$

This bridge is a modification of Carey-Foster's bridge. It was first suggested by Heydweiller,* and applied to the vibration galvanometer by Campbell (*loc. cit.*).

(*d*) Remove T and let Q and U act inductively. The substitutions necessary are

$$T=\frac{r_1r_2-m^2}{m}, \quad U=\frac{r_1r_2-m^2}{r_2-m}, \quad Q=\frac{r_1r_2-m^2}{r_1-m}.$$

The equation for balance becomes

$$(R+P)m-Rr_2+PS=0. \quad . \quad . \quad . \quad . \quad . \quad (9c)$$

Now let R be an inductive resistance so that R becomes $R+jpN$. Also put $r_2=Q+jpL$, $m=jpM$.

The real and imaginary parts of equation (9c) give

$$\left.\begin{array}{r}M(P+R)=LR+NQ \\ p^2N(M-L)=PS-QR\end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10c)$$

This bridge is a good one for measuring frequency since the adjustment for frequency can be made by altering S alone.

Also the range of frequencies measurable is theoretically infinite.

6. *Experimental Results.*

In the following set of experiments all the four modifications of the bridge were used. They were undertaken in order to calibrate a Duddell vibration galvanometer for frequency. This instrument is of the string type (see next section), and the tuning is brought about by adjusting the tension of the vibrating strings.

A pointer attached to the tension screw moves over an arbitrary scale. It was this scale that was calibrated for frequency.

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* Heydweiller, "Ann. der Physik.," Vol. LIII., p. 499, 1894.

The source of electromotive force was a small induction coil the primary current of which was excited by a couple of storage cells.

The frequency of the " make " and " break " of the coil was varied in order to produce the required changes of frequency of the secondary current.

Methods (*a*), (*b*) and (*c*) were employed in order to determine the constants of the coils used in the frequency calibration by method (*d*).

The coils are denoted by A, B and C.

The self-induction of coil B was measured by method (*a*), that of C by method (*b*), and the mutual induction between coils A and B by method (*c*).

The actual observations are only given for the new methods (*b*) and (*d*) as methods (*a*) and (*c*) are well known.

METHOD (*b*).—*Self-induction of Coil* C.

C=2 mfds. P=R=165 ohms.

U+S=T (ohms).........	8,000	7,000	6,000	5,000	4,000	3,000	2,060
S (ohms)..................	1,110	1,120	1,140	1,160	1,220	1,320	2,060
L (henrys)	0·682	0·680	0·681	0·677	0·682	0·680	0·680

Mean self-induction=0·6803 henrys.

METHOD (*d*).—*Calibration of Galvanometer for Frequency.*

Coil A in U arm. Coil B in Q arm. Coil C in R arm.

L=0·00316 henry. N=0·680 henry. M=0·0136 henry.

Q=2·77 ohms. R=165 ohms.

P (calculated from first balancing condition)=12·10 ohms.

The second condition of balance is

$$p^2=1,707\,S-64,000, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

where S is in ohms.

D.	S.	*p*.	Frequency $=\frac{p}{2\pi}$.	Frequency (calculated).
2·0	1,305	1,471	234	236
3·5	2,310	1,970	313	314
5·8	3,330	2,370	377	377
8·3	4,340	2,710	432	430
11·0	5,340	3,007	478	476
13·0	6,120	3,224	513	506
19·0	7,640	3,685	586	581

Column D gives the tension scale reading of the galvanometer when tuned. Column S gives the resistance in ohms out of the S arm to secure balance. *p* is calculated from equation (*a*).

It was found that the frequencies were related to the scale readings by the experimental law

$$f=AD^m,$$

where A=199, m=0·364.

The frequencies in the last column were calculated from this law.

7. *Theory of the String Type of Vibration Galvanometer.*

In its simplest possible form the string type of galvanometer consists of a wire stretched between two fixed points and placed in a transverse magnetic field of strength H. An alternating current whose value is i passes along the wire and sets it in vibration.

The recording part may be represented by a mass M which we will suppose placed on the wire midway between the fixed points.

Let m be the mass of unit length of the wire, $2l$ the length and T the tension.

Take the equilibrium position of the mass M as origin and the undisturbed position of the wire as the x axis. Then, if y is the displacement of a point at distance x from M, we have for small displacements

$$m\frac{\partial^2 y}{\partial t^2}+\rho\frac{\partial y}{\partial t}=\mathrm{T}\frac{\partial^2 y}{\partial x^2}+\mathrm{H}i, \quad . \quad . \quad . \quad (1\mathrm{D})$$

where ρ is the damping constant of the wire.

The end conditions are $x=l$, $y=0$, $x=0$, motion of wire $=$motion of M.

The latter condition gives

$$\mathrm{M}\frac{\partial^2 y_0}{\partial t^2}+k\frac{\partial y_0}{\partial t}=2\mathrm{T}\left(\frac{\partial y}{\partial x}\right), \quad . \quad . \quad . \quad . \quad . \quad (2\mathrm{D})$$

where y_0 is the value of y when $x=0$, and $\left(\frac{\partial y}{\partial x}\right)_0$ is the value of $\frac{\partial y}{\partial x}$, when $x=0$, k is the damping constant of the mass M.

If $i=\mathrm{I}\varepsilon^{jpt}$, the solution of (1D) after a steady state has been reached is $y=\mathrm{Y}\varepsilon^{jpt}$,

where $$\mathrm{Y}=\mathrm{A}_1\cosh qx+\mathrm{A}_2\sinh qx+\frac{\mathrm{H}}{\mathrm{T}q^2}\mathrm{I}, \quad . \quad . \quad (3\mathrm{D})$$

provided that $$\mathrm{T}q^2=-mp^2+j\rho p. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4\mathrm{D}$$

The constants A_1 and A_2 are determined from the end conditions, viz.—

$$x=0,\ \mathrm{Y}=\mathrm{Y}_0\ ;\ x=l,\ \mathrm{Y}=0.$$

These give $$\mathrm{Y}_0=\mathrm{A}_1+\frac{\mathrm{H}}{\mathrm{T}q^2}\mathrm{I}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5\mathrm{D})$$

$$0=\mathrm{A}_1\cosh ql+\mathrm{A}_2\sinh ql+\frac{\mathrm{H}}{\mathrm{T}q^2}\mathrm{I}. \quad . \quad (6\mathrm{D})$$

Further, from equation (2D)

$$(-Mp^2+jkp)Y_0=2TqA_2. \quad . \quad . \quad . \quad . \quad (7\text{D})$$

Eliminating A_1 and A_2 between (5D), (6D) and (7D),

$$(-Mp^2+jkp+2Tq\coth ql)Y_0=\frac{2HI}{q}\,\frac{\cosh ql-1}{\sinh ql} \quad . \quad . \quad (8\text{D})$$

By comparing this equation with the first of equations (5) we see that it may replace that equation provided that we put

the real part of the coefficient of Y_0 for $P(\gamma-ap^2)$, . . (A)

the imaginary part of the coefficient of Y_0 for $P\beta p$, . . (B)

$$\frac{2H}{q}\,\frac{\cosh ql-1}{\sinh ql} \text{ for } PA, \quad . \quad . \quad . \quad . \quad . \quad (\text{C})$$

where P is a multiplying factor to be determined presently

The back electromotive force due to the motion of the galvanometer is given by

$$e_b=2H\int_0^l\frac{\partial y}{\partial t}dx=2Hjp\varepsilon^{jpt}\int_0^l Ydx=2HjpZ\varepsilon^{jpt}(\text{say}),$$

where $Z=\int_0^l Ydx$.

Hence $$qZ=A_1\sinh ql+A_2(\cosh ql-1)+\frac{HIl}{Tq}. \quad . \quad . \quad (9\text{D})$$

Eliminating A_1, A_2 and I between (5D), (6D), (7D) and (9D)

$$q\frac{Z}{Y_0}\sinh ql=\cosh ql-1-\frac{\sinh ql}{2Tq}(-Mp^2+jkp+2Tq\coth ql)\left(2-\frac{ql\sinh ql}{\cosh ql-1}\right) \quad . \quad . \quad . \quad . \quad (10\text{D})$$

In section 2 the back electromotive force was given by $jpAY\varepsilon^{jpt}$.

Hence we must replace A by $\frac{2HZ}{Y_0}$.

This determines the multiplying factor P.

In order to reduce the equations, put q into its complex form $a+jb$.

From equation (4D), a and b are given by

$$T(a^2-b^2)=-mp^2, \quad T(a^2+b^2)=mp^2\sqrt{1+\frac{\rho^2}{m^2p^2}}.$$

If the damping is small, we have, neglecting squares of small quantities

$$a=\frac{\rho}{2\sqrt{m\mathrm{T}}},\ b=p\sqrt{\frac{m}{\mathrm{T}}}\quad .\ .\ .\ .\ .\ .\quad (11\mathrm{D})$$

With the same approximation relation (C) becomes

$$\mathrm{PA}=\frac{2\mathrm{H}}{b}\cdot\frac{1-\cos bl}{\sin bl}.\quad .\ .\ .\ .\ .\ .\quad (12\mathrm{D})$$

Equation (10D) becomes

$$b\sin bl\,.\,\frac{\mathrm{Z}}{\mathrm{Y}_0}=1-\cos bl+\frac{\sin bl}{2\mathrm{T}b}(-\mathrm{M}p^2+2\mathrm{T}b\cot\ bl)$$

$$\left(2-\frac{bl\sin bl}{1-\cos\ bl}\right).\quad .\quad (13\mathrm{D})$$

Also $2\mathrm{T}q\coth ql=2\mathrm{T}b\cot bl+2j\mathrm{T}_{a}\left(\frac{bl-\cos bl\sin bl}{\sin^2 bl}\right)$,

whence the real part of the coefficient of Y_0 in equation (8D) is

$$2\mathrm{T}b\cot bl-\mathrm{M}p^2\equiv\mathrm{P}(\gamma-\alpha p^2),\quad .\ .\ .\quad (14\mathrm{D})$$

and the imaginary part is

$$kp+2\mathrm{T}a\left(\frac{bl-\cos bl\sin bl}{\sin^2 bl}\right)\equiv\mathrm{P}\beta p.\quad .\ .\quad (15\mathrm{D})$$

If the damping is small, resonance is practically attained when the real part of the coefficient of Y_0 above is zero.

Hence we get as the resonance condition

$$\mathrm{M}p^2=2\mathrm{T}b\cot bl.\quad .\ .\ .\ .\ .\ .\quad (16\mathrm{D})$$

Therefore at resonance equation (13D) becomes

$$\mathrm{Z}=\frac{\mathrm{Y}_0(1-\cos bl)}{b\sin bl},$$

and since

$$\frac{2\mathrm{HZ}}{\mathrm{Y}_0}=\mathrm{A},$$

$$\mathrm{A}=\frac{2\mathrm{H}(1-\cos bl)}{b\sin bl}=\mathrm{PA}\ (\text{by } 12\mathrm{D}).\quad .\ .\ .\quad (17\mathrm{D})$$

Therefore P becomes unity at resonance.

Finally, from equations (11D) and (15D),

$$\beta=k+\rho l\left(\frac{bl-\sin bl\,.\,\cos bl}{bl\sin^2 bl}\right).\quad .\ .\ .\ .\quad (18\mathrm{D})$$

In considering the behaviour of a vibration galvanometer of the string type, the equations we need are (16D), (11D), (17D) and (18D).

Putting ϕ for bl these may be written

$$\cot\phi/\phi = \mathrm{M}/2ml, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \tag{19D}$$

$$p = \frac{\phi}{l}\sqrt{\frac{\mathrm{T}}{m}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \tag{20D}$$

$$\mathrm{A} = \frac{2\mathrm{H}l(1-\cos\phi)}{\phi\sin\phi}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \tag{21D}$$

$$\beta = k + \frac{\rho l(2\phi - \sin 2\phi)}{\phi(1-\cos 2\phi)}. \quad . \quad . \quad . \quad . \quad . \tag{22D}$$

To determine the period we first solve (19D) for ϕ and then determine p from (20D). Equation (19D) has an infinite number of roots. With each root there will be associated a certain resonance period and the appropriate damping and displacement constants will be found from equations (21D) and (22D).

Since, when the galvanometer is working under the best conditions the amplitude will depend only on β, that quantity will determine the value of any particular harmonic in producing vibration.

The table below gives the values of $\frac{2\phi - \sin 2\phi}{\phi(1-\cos 2\phi)}$ for various values of $\mathrm{M}/2ml$.

Fundamental.			First overtone.		
ϕ	$\frac{\mathrm{M}}{2ml}$.	$\frac{2\phi-\sin 2\phi}{\phi(1-\cos 2\phi)}$.	ϕ.	$\frac{\mathrm{M}}{2ml}$.	$\frac{2\phi-\sin 2\phi}{\phi(1-\cos 2\phi)}$.
0°	Infinity	0·667	180°	Infinity	Infinity
10°	32·5	0·670	190°	1·71	31·5
20°	7·89	0·678	200°	0·786	7·76
30°	3·31	0·694	210°	0·472	3·53
40°	1·67	0·714	220°	0·310	2·11
50°	0·96	0·742	230°	0·209	1·49
60°	0·55	0·780	240°	0·137	1·20
70°	0·30	0·834	250°	0·083	1·05
80°	0·12	0·904	260°	0·039	0·99
90°	0·00	1·000	270°	0·000	1·00

This table shows that as the mass of the recording part increases the damping of the harmonics increases rapidly, so that for quite moderate values of M the galvanometer may be treated as having only one degree of freedom. It also indicates that the damping of the fundamental diminishes as the mass of the central part increases, but the assumption is that the damping constant k remains constant. As a matter of fact it would increase with the mass of the central part.

The investigation of this section shows that if a string galvanometer is intended for use in measuring frequency, the mass of the recording part must be moderately large compared with the mass of the vibrating wires. The Duddell galvanometer used in the experiments of the last section is of the string type, having two vibrating wires so arranged that they impart an angular motion to a mirror. The angular vibration is magnified by the usual optical arrangement. The mirror corresponds to the mass M in the theory.

8. *Summary.*

1. The theory of the vibration galvanometer of one degree of freedom when placed in circuits containing inductance and capacity is worked out and it is shown that whatever the damping the best conditions are obtained—

(*a*) When the electromotive force is in phase with the current.

(*b*) When $\left(\frac{\text{displacement constant}}{\text{impedance}}\right)^2 = \frac{\text{damping constant}}{\text{resistance}}$.

2. When the best conditions are attained the amplitude of vibration depends only on the applied electromotive force, the damping constant, and the total resistance of the circuit.

In order to secure a large vibration the resistance of the galvanometer should be as small as possible.

3. The theory of a general inductance bridge is worked out and the best conditions of use with a vibration galvanometer are determined.

The bridge is shown to include at least four important methods, and examples of their use are given.

4. The theory of (1) is extended to include galvanometers possessing an infinite number of degrees of freedom and in which the damping is small.

In conclusion, I wish to thank Prof. Gee for his valuable advice and assistance, and the Principal and Committee of the Manchester School of Technology for the facilities placed at my disposal.

ABSTRACT.

Vibration galvanometers are divided into two types according as their moving parts possess only one or an infinite number of degrees of

freedom. A theory of the former type of galvanometer when used in circuits containing inductance and capacity is completely worked out and the conditions of maximum sensibility are determined.

The same theory is shown to be applicable to the string type of galvanometer provided that the damping is small. The effect of harmonics is also considered in this case.

The results are applied in the case of a general inductance bridge which includes: (*a*) Anderson's bridge, (*b*) a modified Rimington's bridge, (*c*) Heydweiller's modification of the Carey-Foster bridge, and (*d*) a bridge for measuring frequency.

The best conditions for working Anderson's bridge with the vibration galvanometer as dectector are obtained.

Experimental results for methods (*b*) and (*d*) are quoted, the latter method being used to calibrate a Duddell vibration galvanometer for frequency.

DISCUSSION.

Mr. A. CAMPBELL said Mr. Butterworth showed very clearly how the best voltage sensitivity may be obtained with a vibration galvanometer by adding self-inductance. For certain purposes we may neglect the self-inductance and treat the problem in the following way: In a moving-coil galvanometer of high sensitivity the ohmic resistance R may often be quite small compared with the effective resistance R′.

Then we have

$$R' \doteqdot R' - R = 1{\cdot}11 \times 10^{-5} B s N n \sigma,$$

where B = flux-density in air-gap, N = number of turns in moving coil, s = mean area of one turn, and σ = current sensitivity in mm. at 1 meter per microampere. Thus, if we increase the current sensitivity by reducing the damping, we raise the effective resistance in much the same proportion. Again, if the ohmic resiscance R is negligible, the voltage sensitivity in mm. at 1 meter per microvolt

$$\doteqdot 9 \times 10^{-4} / B s N n,$$

and is accordingly independent of the damping, the control couple, and the amount of inertia of the moving coil. With regard to the various bridge methods, as a standard for inductance measurements, a condenser is not nearly so good as a mutual or self-inductance. Well-designed mutual inductances have only a very small temperature coefficient and are more permanent in value than the best standard resistance coils.

Mr. W. DUDDELL said the theory put forward in Mr. Butterworth's Paper would be most useful. He had used the same formula as Mr. Butterworth derives for the frequency of all his vibration galvanometers, and found it was generally within 10 per cent. Vibration galvanometers really ought to be looked upon as alternate-current motors and the same principles can be applied in their design. To be efficient, therefore, they must be capable of developing a strong back E.M.F. in such a way as to oppose the current and be in phase with it.

Dr. A. RUSSELL congratulated the author on having perfected the theory of the vibration galvanometer by taking inductance and capacity into account. The rules he deduced to obtain the best conditions for practical working would be very useful. The vector diagram given for the working of the instrument was instructive and ingenious. When the frequency of the vibrations was very high, he thought that possibly errors might arise owing to the assumption made in the fundamental equation that the damping torque due to the air friction was proportional to the angular velocity.

The AUTHOR replied that the formula given by Mr. Campbell for the voltage sensibility of his type of galvanometer is very simple, and may, of course, be deduced as a particular case from the general theory. Its application, is, however, limited. If for any given circuit it is required to bring a vibration galvanometer to its best sensibility, relation (3A) in the Paper should be satisfied. This may be done by adjusting either the displacement constant or the self-induction of the circuit, preferably the former, as the self-induction is generally fixed by other considerations. In the Campbell galvanometer, the displacement constant may be adjusted by altering the flux-density in the air-gap. This adjustment does not interfere with the other constants of the galvanometer. Mr. Duddell has compared the vibration galvanometer with the alternate-current motor, and says that the general principles of design are the same in both cases. Personally, I think it is just as simple, and more convincing, to deduce the best conditions from first principles than to rely upon an analogy that is not immediately obvious. A remark is necessary concerning the best resistances for Anderson's bridge. These are deduced by assuming a certain fractional change in the resistance U. Hence these resistances are the best for detecting a small change of U at balance. They are not, however, the best resistances for detecting a small change in the self-induction. In this case the best arrangements are

$$P=Q=R,\ U=0,\ \frac{L}{C}=R_2,$$

so that we fall back on Maxwell's method.

XIII. *Sealing-Metals.* *By* P. E. SHAW, *B.A., D.Sc., University College, Nottingham.*

RECEIVED DECEMBER 29, 1911. READ JANUARY 26, 1912.

PRELIMINARY NOTICE.

THE easiest way of attaching quartz fibres for suspension is by shellac, Faraday cement or other wax. But these materials all melt at temperatures below 100°C., far too low for requirement in many experiments, *e.g.*, when a vessel has to be evacuated to a high degree with prolonged heating at 300°. Again, the hard waxes change shape slowly after solidifying, with consequent possible change in zero of the suspension.

To avoid these evils the practice initiated by Boys* and followed by Threlfall and others is (1) to clean the fibre ends carefully, then (2) silver them, then (3) deposit copper electrolytically on the silver, and (4) tin the copper surface and solder it to the place of attachment.

This is a great labour to undertake for each fibre, considering the short life these often have.

Threlfall (*see* "Laboratory Arts," pp. 182, 225) describes how he has used Margot's solder (92%Sn, 8%Zn) for fixing quartz fibres and glass to metals, using an aluminium soldering bit and *no flux.* This suggestion of Threlfall is valuable, as will appear below, yet it seems to have lain dormant.

Recently the writer gave this method a trial on an apparatus in connection with the Newtonian constant of gravitation.

A quartz fibre, 18 μ diameter, 23 in. long, was soldered by the above process to a brass torsion head above, and to an aluminium wire below. The latter was loaded with about 10 gms., which is nearly the breaking load for this fibre. This suspension was mounted with all usual attendant shocks, hung in a vacuum vessel which was heated to well above 100°C. for several days, and has remained loaded, yet unbroken, for four months.

The time taken for soldering the ends was no longer than would be required if shellac were used for the attachment. One has even found that the care prescribed by Threlfall as to cleanliness is unnecessary. The hot bit served with some of the alloy is rubbed repeatedly on the surface till the latter is well cleaned and uniformly tinned.

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* C. V. Boys, "Phil. Mag.," Vol. XXXVII., p. 463, 1894.

This method having proved so valuable in the above instance, the writer commenced some experiments with the object of discovering whether or not there is any special merit in the formula given by Margot, and, if not, to find the range of application of the method.

Of the materials likely to be used in suspension work, soft glass melts at, say, 600°C., aluminium at 660°, copper, brass and iron at higher temperatures; so only those materials having low melting points can be employed as solders.

The following results have been obtained :—

A. *Margot's Solder* (M.P., 190°). This material works well with either an aluminium or a copper bit, though the former is cleanest. The materials to which it can be applied include glass, quartz, aluminium, brass, iron. Any one of these can be joined to any other.

B. *Tin* (M.P., 230°C.) acts apparently just as well as Margot's solder, an aluminium bit being used.

C. *Zinc* (M.P., 420°C.) though less fusible than the foregoing metals, serves perfectly well for all the above materials.

D. Since A, B, C are all excellent soldering agents it is probable that any alloy whatever of zinc and tin will serve. Such proved to be the case in one or two instances tried.

E. *Tinman's Solder* (M.P., 180°). This also acts quite well on all the materials quoted in A, including aluminium, though, as is well known, the use of the usual flux (zinc chloride) renders a junction impossible for the latter material.

F. *Lead* (M.P., 327°C.). This will not act well, possibly because of rapid oxidation at its melting point. It will adhere to glass, but not very tenaciously.

G. *Aluminium* (M.P., 660°C.). Nothing can be done with this metal by using it on a bit in the usual way, since at its high melting point oxidation is very rapid. But complete success is obtained if the aluminium be melted in a test-tube, and the rod to be soldered be then thrust into it. The crust of aluminium thus formed on glass and iron is found to be tenacious and hard. Without trial it is impossible to say whether quartz fibres would be rendered rotten by annealing if suspended by metals having such high melting points as zinc and aluminium. But, if not, these should be useful in apparatus which are to be subjected to high temperatures. Ordinary commercial metal is used in all the above cases.

One supposes that in ordinary soldering chemical action occurs with interpenetration of surface molecules. But in the present process the behaviour of such non-metals as glass and quartz precludes the idea of chemical change. The molecules of the metal in the molten state come into contact intimately with those of the solid, and adhere to them molecularly. Possibly the pores of the solid are partially penetrated by the molten metal.

Since, then, the action is similar in kind to that of the many waxes, notably sealing-wax, the writer suggests the general name *sealing-metals* for the above metals and alloys and their like. The sticking which takes place is not due entirely and primarily to a shrinkage effect in which a ring of the molten liquid contracts on solidifying on the solid and so grips it; for it will be found on examination of the surfaces that the sealing-metal will often break away from itself rather than from the solid on which it is set. Also it is not necessary, though it may be advantageous, to have a ring of sealing-metal at all; a small patch suffices.

There are many applications for sealing-metals other than for suspending fibres or wires; thus it may be possible to seal a glass window on to an opening in an apparatus. An optically worked glass window cannot be fused on by melting it to the surrounding glass, yet it is sometimes necessary to have a seal which will stand a much higher temperature than the melting point of shellac.

ABSTRACT.

The established method of fixing quartz fibres for accurate torsion experiments is due to Prof. C. V. Boys. It involves careful cleaning, silvering, electrolytic deposit of copper, tinning and finally soldering. This considerable trouble can be avoided by the simple means given below, while the resulting joint is in some cases stronger. Prof. Threlfall used Margot's solder (92 per cent. Sn, 8 per cent. Zn) to fasten glass, aluminium or quartz surfaces to any other. The writer finds this material acts perfectly and is very simple, the bit being of aluminium and there being no flux. Further investigation shows that there is no special merit in Margot's formula. In place of Margot's solder the following will be found to act very well: (*a*) Tin, (*b*) zinc, (*c*) various alloys of tin and zinc, (*d*) tinman's solder, (*e*) aluminium. Lead does not stick well, though it might be made to do so if oxidation were prevented. Then there is a variety of materials with melting point ranging from 180 deg. to 660 deg. For all these and like materials which act in the same manner as sealing-wax the writer suggests the term sealing-metals. They have the advantages over any wax in (*a*) high melting point, (*b*) non-emission of vapour when temperature is raised.

There are obvious applications other than for tension fibres where joints to withstand temperature are required.

DISCUSSION.

The **President** stated that the method would be extremely useful to physicists if it could be used for making vacuum-tight joints that would stand a high temperature.

Mr. A. **Campbell** mentioned that metals could be soldered to glass by first platinising the glass.

Mr. W. **Duddell** stated that he had successfully fixed quartz fibres with ordinary solder.

Mr. R. **Appleyard** thought that condensers could be made to keep more constant in capacity if the tinfoil could be adhered to the mica.

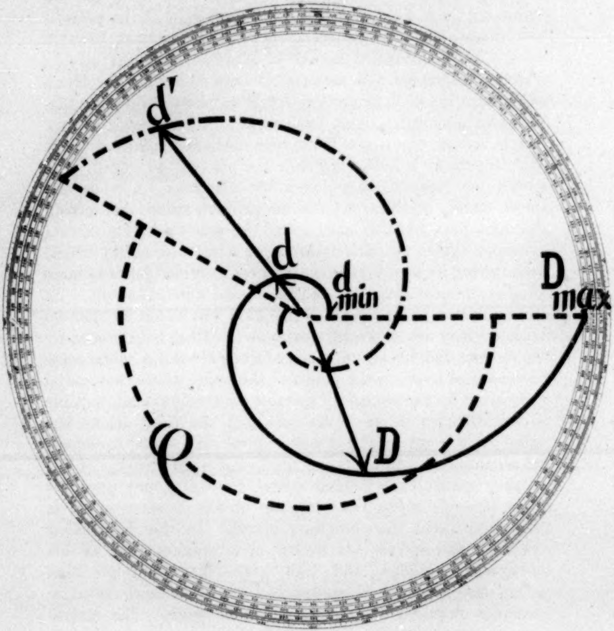
Mr. A. **Campbell** stated that silvered mica condensers had been made, but were not so reliable as the ordinary tinfoil ones. This was probably due to the very sharp edge of the silver. Perhaps platinised mica would give better results.

The **Author**, in reply, stated that metals were very much more porous than glass, so too much must not be expected from joints made with fusible metals. He said it was desirable to find a sealing-metal having a small co-efficient of expansion, for this would have a better chance of adhering strongly after solidification. He thought Mr. Appleyard's suggestion valuable. There should be no difficulty in building up a condenser made on those lines.

XIV. *On a Direct-reading Instrument for Submarine Cable and other Calculations.* *By* R. APPLEYARD, *M.Inst.C.E.*

READ JANUARY 26, 1912.

THE logarithmic spiral has frequently been used for determining by a graphic method the logarithm of the ratio of two quantities. In acoustics, for example, it has been of service as a convenient means of demonstrating the way in which "frequency" is associated with "interval." If an attempt is made to apply the spiral to the solution of practical engineering problems, such as arise in the design of submarine cables, there is difficulty in obtaining sufficient accuracy, especially for readings near the pole of the spiral. The author has removed this defect by introducing a secondary spiral, similar in all respects to the primary spiral, and having the same pole, but displaced round the pole through a certain constant angle. In this instrument a pair of radial scales, each having its zero at the pole, and each divided into the same number of equal divisions, can be rotated about the pole. At all angular positions a scale of this kind will be cut by the two spirals if they are sufficiently extended. It has been proved by the author that for all angular positions of such a radial scale the distance between the pole and the point where that radial scale is cut by the secondary spiral is always the same multiple of the distance between the pole and the point where the radial scale is cut by the primary spiral. In effect, therefore, the secondary spiral magnifies the radial scale readings of the primary spiral to any desired extent, depending only upon the angle through which the template of the primary spiral is rotated to form the secondary spiral. In the instrument exhibited the spirals are drawn in a manner that avoids ambiguous readings, and that gives maximum precision within the range of diameters of conductors and dielectric coverings required for submarine cable work. The instrument is provided with two similarly divided radial scales, one corresponding to d, the diameter of the conductor, and the other corresponding to D, the diameter of the dielectric. The angle between the two scales, corresponding to any pair of values of d and D, is then a measure of $\log \frac{D}{d}$. The scales can be marked to indicate weights of conductor and dielectric, and the circle of degrees to which the spiral is drawn can be marked to indicate $\log \frac{D}{d}$, capacity, dielectric resistance, and other functions of D and d, if required, for any definite dielectric, the specific constants of which are known.



The general equation to the spiral may be written

$$\theta = A \log d + B,$$

where A and B are constants. Hence, when $d_{\text{min.}}$ and $D_{\text{max.}}$ are both on the 0 deg. radius vector, any two points d, D on the spiral are associated with their corresponding angles θ_d θ_D in degrees by the three equations

$$\theta_d = A \log \frac{d}{d_{\text{min.}}}, \qquad \qquad (1)$$

$$\theta_D = A \log \frac{D}{D_{\text{max.}}} + 360,$$

$$A = \frac{360}{\log \frac{D_{\text{max.}}}{d_{\text{min.}}}}.$$

To draw the secondary spiral, the primary spiral is rotated backwards about its pole through an angle, ϕ. This is equivalent to rotating both the radial scales through ϕ. The intercepts are now d_1 and D_1—*i.e.*, the radial scale readings are now greater in the ratio $n = \frac{d_1}{d} = \frac{D_1}{D}$. Equation (1) then becomes

$$\theta_d + \phi = A \log \frac{d_1}{d_{\text{min.}}}. \qquad \qquad (2)$$

Subtracting (1) from (2)

$$\phi = A \log \frac{d_1}{d} = A \log n,$$

which gives ϕ for any required magnification, n.

DISCUSSION.

Prof. S. P. THOMPSON remarked on the useful applications of the instrument, and asked if it would be possible to make it up in a cylindrical form instead of on a plane.

Dr. A. RUSSELL pointed out the large number of calculations in which the logarithm of the ratio of two quantities occur.

The AUTHOR, in reply, thought it would be rather difficult to apply the instrument to a cylinder. In cable calculations he used it for finding the capacity and dielectric resistance per mile. By clamping the radial arms and rotating them, an infinite number of values for the diameter of the conductor and the insulation could be picked out that would give the same capacity and dielectric resistance per mile and therefore the same speed of signalling.

XV. *On a Copper-Zinc Uranium Oxide Cell and the Theory of Contact Electromotive Forces.* *By Prof.* A. Anderson, *M.A.*

Received December 26, 1911. Read January 26, 1912.

The cell, a vertical section of which is shown in Fig. 1, was designed for the purpose of investigating the effects of changes of temperature, and of changes of the pressure and humidity of the air, on the values of the contact difference of potential between pairs of different metals.

The method of measurement used was the deflection method described in the "Proceedings" of the Physical Society, Vol. XXIII., Part V., p. 347.

The construction of the cell is simple. It is a copper vessel furnished with a closely fitting lid. The copper tubes D and E pass respectively through the lid and the vessel itself. A is a copper plate supported by three copper legs, forming a tripod, on which is placed the uranium oxide for ionizing the air inside. A copper rod C, part of which is a smooth double cone, passes through two slightly conical plugs, F, F, of amber (blackened in the figure) in a copper tube. These plugs can be screwed up so as to form an air-tight joint, which becomes more effective as the pressure of the air inside is diminished. On this copper rod is screwed a zinc rod, G, bifurcated so as to support the zinc plate B. This zinc rod, when screwed on, touches the inner amber plug, thus preventing the possibility of any ionized air coming in contact with C. C is put through an opening in the metallic cage enclosing the electrometer and connected through a key with one pair of quadrants, the other

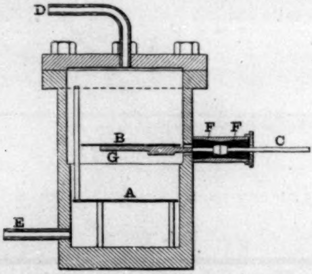


Fig. 1.

pair of quadrants and the copper vessel being earthed. From the deflection of the needle the contact difference of potential can be calculated.

This cell, as has already been stated, was designed for a special purpose, but much simpler forms can, of course, be easily made. Fig. 2 represents a battery of six cells, which will give six times the electromotive force of one.

It is merely a hollow paraffin cylinder, the upper end of which is closed by a copper plate and the lower end by a zinc plate. It is divided into six air-tight compartments by means of double plates, each of which consists of a copper and zinc plate soldered together. In each compartment is placed a small quantity of uranium oxide. The cylinder can be built up of paraffin rings. The current produced by this battery will be exceedingly small owing to the enormously great internal resistance.

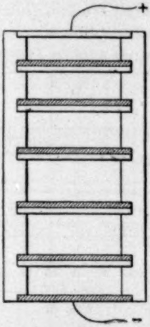


FIG. 2.

A cell such as that represented in Fig. 1 was set up, with plates and inside surface of copper vessel freshly sandpapered. Its E.M.F. at about 15·5°C. was at first 0·73 volt, but gradually diminished. The following table gives the E.M.F. observed after the lapse of the different indicated periods of time :—

20 minutes	0·72
50 minutes	0·71
1 hour 50 minutes	0·7
1 day	0·68
2 days	0·65
3 days	0·63
4 days	0·625

It continued to fall more and more slowly, and, at the end of three months, was 0·34 volt. At the present time, five months after the first measurement, the electromotive force has nearly this value, although the cell, during the last two months, has been heated and cooled many times.

These experiments on the effect of heating and cooling on the contact difference of potential show that the E.M.F. is a linear function of the temperature; and whether the cell is being heated or cooled the E.M.F. has the same value for the same temperature. There is no lag. Fig. 3 gives the result of one such experiment, but all the experiments were consistent, and it was always possible to deduce the temperature of the cell with an error of not more than half a degree from the deflection of the needle, on the assumption that equal increases of deflection corresponded to equal rises of temperature. The electromotive force of the cell is given very accurately by the equation

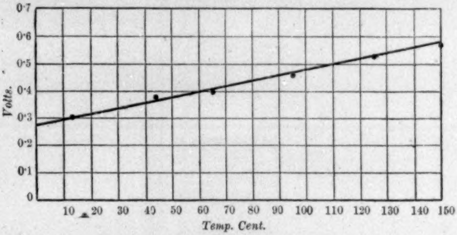


FIG. 3.

$$E = 0{\cdot}27 + 0{\cdot}002t,$$

which gives a rise of one five-hundredth of a volt for an increase in temperature of 1 deg. Centigrade. E, by this formula, would vanish at − 135°C., a temperature much lower than the neutral point of copper and zinc, which is − 67°C.

Helmholtz has shown that, in a reversible chemical cell, there is a quantity of heat absorbed per unit quantity of electricity equal to $T\frac{dE}{dt}$, where T is the absolute temperature. In other words, if the Joule heat be removed as fast as it is produced, the cell will fall in temperature, and in order to keep its temperature constant a quantity of heat equal to $T\frac{dE}{dt}$ per unit quantity of electricity must be given to it. If the cell be not reversible more than this quantity must be given. The equation for the electromotive force of the cell above described may be written

$$E = -0{\cdot}276 + T\frac{dE}{dt},$$

showing that the heat absorbed is more than that which would correspond to the observed electromotive force. The conclusion is irresistible that the cell is a heat cell. And not only that, but if its temperature be kept constant, the heat supplied to it is more than sufficient to account for the Joule heat. Work in some form must therefore be done of amount per unit quantity of electricity equal to that which would produce a back electromotive force of 0·276 volt. The forward electromotive force is, therefore, $0{\cdot}546 + 0{\cdot}002t$ which, of course, vanishes at absolute zero. A cell of this kind working under adiabatic conditions will fall in temperature till its E.M.F. becomes zero—that is, till its temperature becomes −135°C. But it is clear that the heat energy which has disappeared must have been converted into some form of potential energy, possibly at the surface of one or both of the metals. If a current be sent through the cell in the direction opposite to that of the current produced by it, this potential energy will be transformed into heat and heat given out.

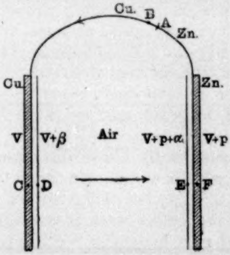


FIG. 4.

To make an attempt to understand what may possibly occur when the cell is working, let us refer to Fig. 4, which represents the state of the cell when there is no ionization of the air—that is, before the uranium oxide is introduced.

Let the potential of the surface of the copper plate be V, that of the surface of the zinc plate $V+p$, that of the layer of air in contact with the copper plate $V+\beta$, and that of the layer of air in contact with the zinc plate $V+p+a$. What is measured by the electrometer is clearly $a-\beta+p$; p is the excess of the potential of A above B, and is supposed to be due to a difference of concentration of the electrons at A and B, the concentration at B being greater than that at A; a and β are, it may be, due to a similar cause. If now the uranium oxide be allowed to ionize the air, there will be a current of electricity, which we may regard for simplicity of statement as the passage in the direction of the arrow-heads of negative electrons round the circuit. When an electron goes from A to B it passes through the bounding surface and is pulled down through a step of potential, p. When it goes from C to D it is forced through the surface and falls up through a step of potential, β. When it goes from E to F, it must get through the surface and at the same time be pulled down through a step of potential, a. If we do not admit the existence of chemical action, the sum of the works at AB, CD and EF will be done at the expense of the heat of the system, and the heat absorbed, corresponding to the passage of a single electron whose charge is ϵ, will be greater than $\varepsilon(a-\beta+p)$ by what may be called the concomitant surface work.

It may be objected to the above that what has been called the concomitant surface work is included in the potential difference values a, β, p; but this is not so, and no potential corresponding to the surface work is measured by an electrometer, no more than a barometer would give any indication of the work done when a liquid molecule passes through the surface of a liquid into the atmosphere above it. It corresponds to the work required in producing evaporation and is possibly positive at CD and negative at EF, the former corresponding to evaporation and the latter to condensation. It may also be objected that the sum of these surface works at AB, CD and FE should vanish. If so, the argument fails, but to this it may be replied that when electrons pass into or out of a metal there may be a change in the state of the surface depending on the metal and involving a storage of potential energy in both cases.

ABSTRACT.

A uranium oxide cell with copper and zinc plates is described, and reference is made to the temperature coefficient of its E.M.F. A difficulty connected with the energetics of the cell is pointed out and a possible explanation put forward tentatively.

XVI. *On Wave-form Sifters for Alternating Currents.* By ALBERT CAMPBELL, *B.A.*

FROM THE NATIONAL PHYSICAL LABORATORY.

RECEIVED JANUARY 16, 1912. READ FEBRUARY 23, 1912.

IN measurements with alternating currents the conditions are often complicated by the presence of harmonics in the waves of current or voltage. Sometimes these harmonics have their origin in the source of supply from which the current is taken, and sometimes they are caused by the presence, in the actual circuits under test, of magnetic or dielectric hysteresis, periodically varying resistance, or other natural or artificial arrangement giving transformation of frequency. When it is desired to suppress all but one of the components of a complex wave form, this is usually done by "tuning" the circuit by arranging capacities and self inductances so as to give resonance at the desired frequency. When the resistance of the circuit is low enough to give good resonance, the desired component may usually be so much increased that the whole of the others are left as only a small proportion of the total wave, and by repetitions of this tuning process a very nearly pure sine wave may be obtained.

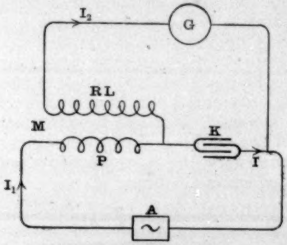


FIG. 1.—CAMPBELL'S WAVE-FORM SIFTER.

In some cases, however, it is useful to be able to suppress entirely a single component (which may be the fundamental or any desired harmonic). This can easily be done by several arrangements. One of the simplest of these is a method which I have already described for the purpose of connecting frequency, capacity and mutual inductance, and measuring any one of them in terms of the other two. It is shown in Fig. 1.

If M is a mutual inductance, K a condenser and A a source of alternating current, then I have already shown that, if

$$\omega_0^2 MK=1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

no current of frequency n_0 will pass through the circuit G, where $\omega_0=2\pi n_0$, M being in henries and K in farads.

Thus if the alternator A gives a current of complex wave form containing a component of frequency n_0, the current through G will contain no component of this frequency.

The combination accordingly forms a very simple waveform sifter, and if M is adjustable (or K) it can be set so as to suppress entirely any desired harmonic (or the fundamental).

For exact suppression, the condenser K should be free from absorption and leakage, and the mutual inductance from distributed capacity; for most purposes these conditions can be nearly enough satisfied. The working circuit G may possess inductance and capacity, but should not contain any iron-cored magnets, synchronous commutators, or other things which of themselves introduce harmonics.

The following investigation shows how the components (of the original current) of frequencies other than n_0 are affected by the sifter. In Fig. 1 let I, I_1 and I_2 be the effective values of the current components of a frequency n, as shown, and let I_1 be of fixed amount. If R and L be the resistance and self inductance of the working circuit containing G (across the condenser terminals), the n_0-component having been suppressed as in equation (1), then

$$I_2^2=I_1^2\frac{\left(\frac{1}{K\omega^2}-M\right)^2\omega^2}{R^2+(L\omega-\frac{1}{K\omega})^2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

which gives I_2 for any particular case.

To get the maximum value of I_2, the working circuit may be tuned by altering L until $LK\omega^2=1$, in which case

$$I_2=I_1(L \sim M)\omega/R.$$

Now $$L/M=\omega_0^2/\omega^2=n_0^2/n^2,$$

and therefore $$I_2=I_1\frac{M}{R}\left[\left(\frac{n_0}{n}\right)^2 \sim 1\right]\omega \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This equation (3) gives the greatest possible value of current component of frequency n allowed through by the sifter when it suppresses entirely the component of frequency n_0, for a given primary current n-component.

When both the primary and the secondary circuits are tuned to frequency n, let the source give a constant voltage having an n-component of effective value E; let P=resistance in the primary circuit (beyond K) and let $b=\frac{\omega_0}{\omega}$.

Then it can be shown that

$$E^2/I_2^2=\frac{P^2R^2}{M^2\omega^2(b^2-1)^2}+M^2\omega^2(b^2-1)^2$$

or

$$I_2=E\,.\,\frac{M\omega(b^2\sim 1)}{\sqrt{P^2R^2+M^2\omega^4(b^2-1)^4}}\,.\quad .\quad .\quad .\quad .\qquad (4)$$

One or two examples of the application of this sifter may be mentioned; others may occur to the reader. In using a telephone as detector in an inductance bridge, when the balance depends to any extent on frequency, perfect silence often cannot be obtained, for when the fundamental disappears one or more harmonics may still be giving sound. By inserting the sifter between the telephone and the bridge, the most prominent harmonic can be suppressed and the point of balance for the fundamental much more readily obtained. By employing two or more sifters in consecutive circuits further harmonics can be suppressed if desired.

Again, the current given by a microphone hummer at 1,000 ∼ per second contains a considerable harmonic of 2,000 ∼ per second. By using a sifter an almost pure wave form at either of these frequencies can be produced and utilized in any desired circuit. The sifters might also be useful in multiple-tone telegraphy.

In the following table are given examples of approximate values of K and M suitable for sifting out various frequencies by the method of Fig. 1.

TABLE.

n ∼ per sec.	MK millihenries and mfd.	M millihenries.	K mfd.
10	256,000	6,400	40
50	10,000	1,000	10
100	2,560	128	20
500	100	10	10
1,000	25·6	25·6	1
5,000	1·0	1·0	1
10,000	0·256	0·256	1

From the values in the table it will be seen that the simple sifter of Fig. 1 is not very suitable for frequencies as low as 50 or 100 ∿ per second, as the value of MK is then inconveniently large. For these lower frequencies a sifter can be arranged as in Fig. 2.

If the resistances are P, Q, R and S, and self inductances L and N, such that SP=QR and LS=NR, then no current of frequency $n_0(=\omega_0/2\pi)$ will pass through circuit G when the mutual inductance M is set so that

$$MK\omega_0^2=\frac{S}{R+S}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

By choosing the ratio R/S large, quite low frequency components can be suppressed when using reasonably small values of K and M (*e.g.*, R/S=250, n_0=100, M=10 millihenries and K=1 mfd.). Such a bridge also forms an excellent means of measuring frequency when M consists of a direct-reading inductometer; the frequency is inversely proportional to $\sqrt{M}$.

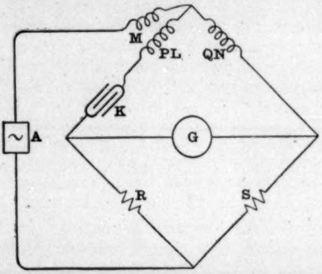


FIG. 2.—CAMPBELL'S BRIDGE WAVE-FORM SIFTER.

I would here point out that almost any null method, such as Hughes's, for example, in which the balance depends on the frequency, may be used as a wave-form sifter in a similar manner to those I have described above. With proper precautions by successfully suppressing each harmonic by means of a sifter, it would appear possible to analyze any given voltage wave form.

In conclusion I may again emphasise the difference between two classes, A and B, of wave-form sifters. Systems of class A *partially* suppress all but one frequency component;

those of class B (such as I have described above) *totally* suppress one single component. In class A capacity and *self* inductance are usually employed; while in class B capacity and *mutual* inductance give the simplest arrangement, but other combinations of self inductance without capacity and with or without mutual inductance. (For instance, in Fig. 2 M and K might be omitted, and a shunt put across the arm Q.) These latter arrangements are independent of absorption in condensers. This absorption, if present, vitiates the method of Fig. 1, unless compensated for by a condenser across the secondary of the mutual inductance. The bridge sifter of Fig. 2, however, is not affected by absorption in the condenser. Sometimes a combination of both systems of sifting is advantageous; formulas (3) and (4) refer to such cases.

An interesting arrangement is given by a slight modification of my null method of testing current transformers ("Phil. Mag.," p 507, 1910, and "Proc." Phys. Soc., p. 214, Vol. XXII.). It is only necessary to replace the transformer by a mutual inductance without iron. How far it is possible to introduce iron cores into the inductances used in sifters of the class B is worth investigating. (*See* M. Joly on "Frequency Transformers," "La Lumière Électrique," p. 195 (14), May 20, 1911.) The question of the efficiencies obtainable also requires consideration.

XVII. *On the Numbers and Ranges of the α-particles emitted by the Emanation and Active Deposit of Thorium.* *By* T. Barratt, *B.Sc., A.R.C.S., East London College, University of London.*

Received January 22, 1912.

1. *Introduction.*

Experiments by Bronson* and by Geiger and Marsden† have shown that the total number of α-particles from thorium emanation and thorium active deposit in equilibrium is about three times the number from the active deposit alone, or the number from thorium emanation is twice as many as from the active deposit in equilibrium with it. These experiments all refer to the *whole* active deposit; no results have been published up to the present which give a relation between the number of α-particles from all the products and those from a *single* product of the series. If ThC_1 and ThC_2 (following the notation proposed by Rutherford and Geiger, "Phil. Mag., XXII., p. 621, 1911) were assumed to be in direct line of descent, the experiments referred to above would indicate that thorium emanation gives four α-particles for each single α-particle from ThC_1 or ThC_2. The experiments of Geiger and Marsden,‡ however, indicated that "thorium emanation" really consisted of the emanation proper together with a short life product now called thorium A of half period 0·145 second.§ This would still require ThEm and ThA each to emit two α-particles for one from ThC_1 or ThC_2. Various considerations led Rutherford and Geiger to suppose that ThC_2 was not directly derived from ThC_1, and Marsden and the author‖ finally proved this assumption correct by showing that the number of α-particles from ThC_2 is greater than the number from ThC_1 in equilibrium with it, the numbers being in the ratio of 65 : 35. If, therefore, ThEm and ThA each give the same number of particles as ThC_1 and ThC_2 together, a scheme of disintegration similar to that given by Marsden and Barratt[*] may be assumed :—

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* "Phil. Mag.," 16, 291, 1908.

† "Phys. Zeit., 11., Jan. 1, 1910.

‡ *Loc. cit.*

§ Moseley and Fajans, "Phil. Mag.," XXII., p. 629, 1911.

‖ "Proc. Phys. Soc.," pp. 50-61, Dec. 15, 1911.

$$\begin{tikzcd}
 & & & \mathrm{ThC}_1 \arrow[rrd, dash, "?"] & & \\
\mathrm{ThEm} \arrow[r] & \mathrm{ThA} \arrow[r] & \mathrm{ThB} \arrow[ru] \arrow[rd] & \alpha(4{\cdot}8\text{ cm.}) & & \mathrm{ThD},\dagger \\
\alpha & \alpha & \beta & \mathrm{ThC'}_1 \arrow[r] & \mathrm{ThC}_2 \arrow[ru, dash] & \beta \\
 & & & & \alpha(8{\cdot}6\text{ cm.}) &
\end{tikzcd}$$

in other words ThC_1 and ThC_2 are on different branches arising from a common root ThB (formerly ThA). The results referred to above, however, are not in themselves sufficient to establish the whole of the conclusions. For example, if "thorium emanation" consists of *three* products, each emitting the same number of α-particles as ThC_2 (say, 65 for each, and 35 for ThC_1), we shall have

$$\frac{\text{No. of particles from emanation and active deposit}}{\text{No. of particles from active deposit alone}}=\frac{295}{100},$$

agreeing very closely with Bronson's, and Geiger and Marsden's figures. On the other hand, some experiments recently published by Miss Leslie[‡] lead to the conclusion that "thorium emanation" gives four α-particles. The present experiments therefore aimed at an exact determination of—

1. The relative number of α-particles from each of the four products ThEm, ThA, ThC_1 and ThC_2.

2. The "ranges" of the α-particles from the same four products.

2. *Arrangement of Apparatus.*

A mixture of air and thorium emanation was drawn through a shallow box, AB, 1 mm. in depth, the back surface being of glass and the front of copper. A circular aperture, M, in the front surface, of about 1 cm. diameter, was covered with mica of thickness equivalent to 1·05 cm. of air. (In some of the experiments thicknesses of mica equivalent to 0·6, 1·5 and 2·5 cm. of air respectively were employed without any appreciable difference in the results.) Diaphragms of various known areas could be placed over M, so that the number of particles at a greater distance could be obtained approximately equal to those at shorter ranges. Emanation from 2 mg. of mesothorium[||] at T was intimately mixed with air entering at C,

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[*] "Proc. Phys. Soc.," pp. 50-61, Dec, 15, 1911.

[†] Mr. Marsden has since obtained results which prove that ThD comes from ThC_1 only. He has also proved the existence of the product $\mathrm{ThC'}_1$.

[‡] "Le Radium, VIII., p. 365, 1911.

Very kindly lent by Mr. F. H. Glew, who also supplied the zinc sulphide screens used, and passed through the apparatus at a rate which could be varied within wide limits by adjusting the rate of flow of water through the Bunsen filter pump connected to the rubber tube P. A microscope, sliding along a millimeter scale with vernier, had a zinc sulphide screen accurately in focus attached to it, and determinations were made of "*n*" the number of scintillations per second observed at various distances "*d*" of the zinc sulphide screen from the mica. The value of "d^2n" was then taken as proportional to the number of a-particles at the given distance *d*.

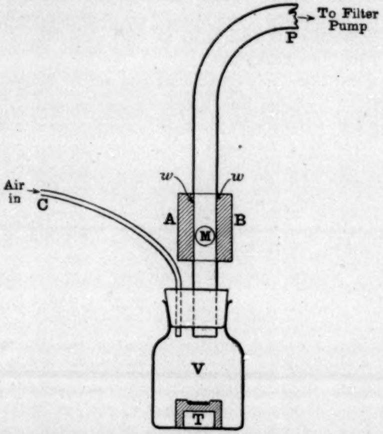

FIG. 1.

A special form of apparatus, employed in experiments where an electric field was to be applied (*see* section IV.) consisted of a shallow box about 3 mm. in depth, similar to AB in Fig. 1, except that the back surface was of copper, had a brass terminal attached to it, and was separated from the front by small side blocks of ebonite, around which, on the parts of ebonite facing the box, two copper wires, *ww*, were led. The negative pole of a battery of about 300 volts was connected to these wires (which were out of the direct line from the aperture M to the microscope), the positive pole being joined to the brass terminal on the back surface of the box. It was found as in Geiger's experiments* on actinium "emanation" that a considerable fraction of the α-particles (in accordance with Rutherford's observations that the atoms of active deposits are positively charged) was attracted to the negatively charged wires when thorium emanation was drawn through the apparatus.

3. *Experiments with the Active Deposit alone.*

The meso-thorium was left in position for periods varying from 24 to 72 hours to allow the active deposit to attain approximate equilibrium; it was then removed, and determinations made as indicated above. The results obtained were in excellent accord with those obtained by Marsden and the author,† indicating that

1. The number of α-particles from ThC_2 is 1·8 times the number from ThC_1.
2. The "ranges" of the α-particles from ThC_1 and ThC_2 are respectively 4·8 cm. and 8·6 cm., the "ranges" having been corrected for temperature and pressure so as to reduce all results to the equivalent values at pressure 76 cm. and temperature 0°C.

4. *Experiments with Thorium Emanation alone.*

The mixture of air and emanation was drawn through the apparatus, any active deposit therein having been first allowed to decay. Countings were carried out at different distances as before, corrections being made for the growth of ThC_1 and ThC_2 during the time taken (usually two or three hours) for the whole experiment. This was sometimes done by causing the current of air to flow in the reverse direction through AB, and counting the number of scintillations due to the active deposit at various stages; on other occasions one counting only—at the end of the experiment—was taken, and the active deposit allowed for from a knowledge of the rate of rise of ThC_1 and ThC_2. One of the curves thus obtained is given in Fig. 2, and indicates that the maximum range of ThEm (+ThA) is 5·4 cm., which agrees well with the range usually given (5·5 cm.) for ThEm (Hahn, "Phil. Mag." 1906).

It is important to observe in this curve that the "slope" commences (at B) at an equivalent distance of about 4 cm.

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* "Phil. Mag.," XXII., p. 201, 1911.

† *Loc. cit.*

from the mica, while in the case of the active deposit alone it commences about 3·8 cm. away. In the latter case the "range" of the product examined is 4·8 cm., so that the curve in question indicates the presence of α-particles of range 5·0 cm.

Further, at the point C (distance 4·4 cm.) a steeper gradient is exhibited, showing that another product is producing an effect. From D to E the slope is less, intimating that the second product only is responsible. There is no evidence of a third or fourth product.

In experiments with the second form of apparatus described in section 2, countings were made at different distances (1) without the electric field, (2) with the electric field. The results of one of these experiments are given below (Table I).

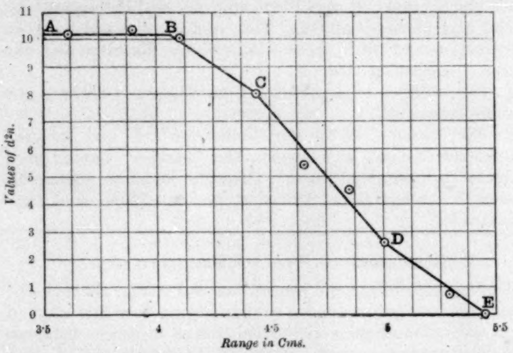

Fig. 2.

TABLE I.

(1) Range (cm.)......	3·5	4·0	4·5	4·7	4·9	5·0	5·2	5·5
(2) d^2n without field	13·26	11·86	7·32	5·55	2·96	1·79	1·18	0
(3) d^2n with field ...	8·67	7·43	3·52	2·62	1·06	0·478	...	...
(4) ThA deviated ...	4·59	4·43	3·80	2·93	1·90	1·312	...	...
(5) Total ThA	6·63	6·41	5·48	4·24	2·74	1·90	1·18	0
(6) Total ThEm ...	6·63	5·45	1·84	1·31	0·22	0	0	0

The second and third "rows" were obtained by experiment. The numbers in the fourth row, being the difference between the values of "d^2n" with and without the electric field, represent the number of α-particles deviated from the central aperture by the applied field. Assuming that at the "range" 3·5 cm. equal numbers $\left(=\frac{13·26}{2}\right)$ of α-particles from ThEm and ThA were emitted, it is evident that a fraction $\frac{4·59}{6·63}=0·69$ particles from ThA were drawn away to the negatively charged wires. Taking this fraction at all ranges, the numbers given in row 5 were obtained for the relative numbers of scintillations due to α-particles from ThA. By subtraction of this number from the total number of particles at the given range, the equivalent number of α-particles from ThEm was obtained (row 6). Curves could then be drawn giving the relative numbers of particles from ThEm and ThA at various distances. A typical pair of curves (corresponding to the numbers given in Table I.) is shown in Fig. 3.

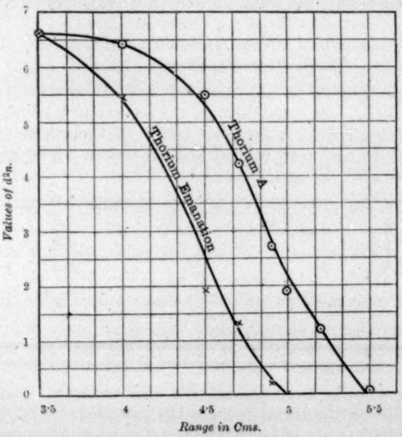

Fig. 3.

The figure indicates that the maximum range of the α-particles from ThEm is 5·0 cm., this being the result foreshadowed in the preliminary experiments. Towards the end of the range the scintillations are very faint and easily missed, but there is no doubt that the curves give the end of the range quite well. In this set of experiments it was found that the maximum fraction of the whole number of particles that could be attracted to the negatively charged wires was only about 0·35 even when a field of 700 volts was applied. This is again strong evidence that the "emanation" gives only two sets of α-particles, for Geiger[*] obtained results of the same order of magnitude in the corresponding case for ActEm and ActA, where there is no question of more than two products. Further evidence on the same point is afforded in the experiments on the laws of probability applied to the emission of α-particles by Marsden and Barratt[†], where excellent accord was obtained between theory and experiment on the assumptions (1) that only two products (ThEm and ThA) were present and (2) that these products gave an equal number of α-particles.

5. *Experiments with Thorium Emanation and Thorium Active Deposit in Equibrium.*

The meso-thorium having been left in position with a constant stream of emanation through the apparatus (the tap was connected to a cistern in the roof), until equilibrium was established, countings were carried out as before. The results agreed well with the assumptions :—

If (*a*) Th Em gives 100 α-particles of range 5·0 cm.
then (*b*) ThA „ 100 „ „ 5·4 „
(*c*) ThC_1 „ 35 „ „ 4·8 „
and (*d*) ThC_2 „ 65 „ „ 8·6 „

A single experiment gave the following results (Table II.) :—

TABLE II.

Range (cm.)	3·5	3·7	4·0	4·3	4·5	4·8	5·0	5·2	5·5	6·0	6·5	8·6
d^2n	6·20	6·05	4·95	4·17	3·76	2·93	1·93	1·80	1·44	1·38	1·35	0

The curve of Fig. 4 co-ordinates d^2n with corrected "range." The part EF of the curve includes the particles of ThC_2 only, and its ordinates are of height 1·35. The part AB measures the number of particles from ThEm, ThA, ThC_1 and ThC_2 and the height of its ordinates is 6·0.

Thus $\dfrac{\text{No. of particles from } ThC_2}{\text{No. of particles from all four products}} = \dfrac{67{\cdot}5}{300}$ which agrees well with the fraction obtained $\left(\dfrac{65}{300}\right)$ on the assumption that when ThEm and ThA each give 100 particles then ThC_1 gives 35 and ThC_2 65. The shape of the curve also supports the conclusion arrived at above, that the "emanation" contains only two products.

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[*] *Loc. cit.*

[†] *Loc. cit.*

When the logarithms of the ranges obtained in the present series of experiments are plotted against the logarithms of the corresponding transformation constants, according to the method of Geiger and Nuttall,* a straight line is obtained which is approximately parallel to those given by them for the radium and actinium families, thus affording a further confirmation of the accuracy of the determinations. The results also agree well with a formula, $\log \lambda = a + bv^n$, recently given by Swinne† where λ is the transformation constant, a and b are constants, v is the initial velocity of the α-particles, and n has the value 1 or 2.

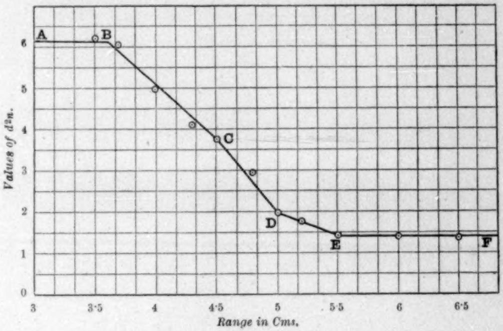

Fig. 4.

Summary.

1. The relative numbers of α-particles emitted by ThEm, ThA, ThC_1 and ThC_2 and their ranges are as follows:—

ThEm,	100	particles of	range	5·0 cm.
ThA,	100	,,	,,	5·4 ,,
ThC_1	35	,,	,,	4·8 ,,
ThC_2	65	,,	,,	8·6 ,,

The experiments described in the present Paper were carried out at East London College and the Wandsworth Technical Institute, and my thanks are due to the Governors of those Institutions and to Prof. C. H. Lees for their kindness in placing the facilities of the laboratories at my disposal.

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* "Phil. Mag.," XXII., p. 613, Oct., 1911.

† "Phys. Zeit.," Jan. 1, 1912, pp. 14-21.

THE PHYSICAL SOCIETY OF LONDON.

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Feb. 12, 1909.	Benoît, R. *Bureau International des Poids et Mesures, Sevres, France.*
Feb. 11, 1910.	Curie, Madame Sklodowska. *Faculté des Sciences de Paris, Rue Cuvier* 12, *Paris.*
Feb. 11, 1910.	Hale, Prof. G. E. *Mt. Wilson Solar Observatory, Pasadena, California, U.S.A.*
Feb. 7, 1890.	Hittorf, Prof. W. *Münster, Westphalia.*
Feb. 8, 1907.	Lippmann, Prof. Gabriel. *La Sorbonne, Paris.*
Feb. 14, 1902.	Lorentz, Prof. H. A. *Hoorgracht* 18, *Leyden.*
Feb. 9, 1906.	Michelson, Prof. A. A. *The University, Chicago, U.S.A.*
Feb. 9, 1912.	Nagaoka, Prof. H. *The University, Tokio, Japan.*
Feb. 11, 1882.	Quincke, Prof. G. *Friedrichsbau, Heidelberg.*
Feb. 13, 1903.	Righi, Prof. Aug. *The University, Bologna.*
Feb. 12, 1892.	Van der Waals, Professor J. D. *The University, Amsterdam.*

Ex-Officio Fellows.

Dec. 20, 1910.	The President of the American Physical Society.
Mar. 16, 1878.	The President of the French Physical Society.
Feb. 14, 1902.	The President of the German Physical Society.

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Mar. 10, 1911.	Abraham, R. M. 11-15 *Rochester Row, Victoria Street,* S.W.
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May 12, 1888.	Addenbrooke, G. L. 35 *Holland Villas Road,* S.W.
Nov. 24, 1911.	Airey, H. Morris. *Armstrong College, Newcastle-on-Tyne.*
Apr. 28, 1911.	Airey, J. R., M.A., B.Sc. *Rugby House, Morley, Yorks.*
Jan. 26, 1906.	Allen, H. Stanley, M.A., D.Sc. 5 *Presburg Road, Malden, Surrey.*
Mar. 10, 1911.	Allen, W. H., M.Inst.C.E. *Bromham House, Bromham, nr. Bedford.*
Dec. 4, 1891.	‖Anderson, Prof. Alex., M.A., LL.D. *Queen's College, Galway.*
Jan. 19, 1878.	Angell, John. 6 *Beacon's Field, Derby Road, Withington, Manchester.*

Date of Election.	
Nov. 9, 1894.	Appleyard, Rollo, M.Inst.C.E. 79 *St. Mary's Mansions, Paddington*, W.
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Jan. 22, 1897.	Ashworth, J. R., D.Sc. 105 *Freehold Street, Rochdale.*
Apr. 26, 1907.	Ayres, Thomas, M.Sc. *The Grove, Wandsworth*, S.W.
May 24, 1907.	Ayrton, Mrs. Hertha. 41 *Norfolk Square*, W.
Nov. 12, 1887.	Bacon, Rear-Admiral R. H. S., R.N. *Wolston Manor, Coventry.*
Mar. 27, 1903.	Baillie, G. H. 82 *Victoria Street*, S.W.
Dec. 16, 1876.	§†‖Baily, Walter, M.A. 4 *Rosslyn Hill, Hampstead*, N.W.
Feb. 8, 1907.	Bairstow, L., A.R.C.Sc. "*Altyre,*" *Holmesdale Road, Hampton Wick.*
Nov. 24, 1911.	Baker, Bevan B., B.Sc. *Frontenac, Donnington Road, Harlesden*, N.W.
Jan. 26, 1912.	Barker, S. G. *Imperial College of Science, South Kensington. Westbourne Villa, York.*
Feb. 25, 1910.	BARKLA, Prof. C. G., D.Sc. (MEMBER OF COUNCIL). *Trewyth, Northwood, Middlesex.*
June 9, 1899.	Barnes, E. *C/o Priest & Ashmore, Newton Works, Eyre Street, nr. Moorhead, Sheffield.*
Feb. 28, 1902.	Barr, J. M. McG. *The Savile Club, Piccadilly, London*, W.
Feb. 25, 1910.	Barratt, T., B.Sc. *Wandsworth Technical Institute, Wandsworth*, S.W.
Feb. 23, 1884.	‖Barrell, Prof. F. R., M.A., B.Sc. *The University, Bristol.*
	§**‖Barrett, Prof. Sir W. F., F.R.S. 6 *De Vesci Terrace, Kingstown, Co. Dublin.*
Mar. 11, 1892.	Barton, Prof. Edwin Henry, D.Sc., F.R.S.E., A.M.I.E.E. *University College, Nottingham.*
Jan. 21, 1898.	Bartrum, C. O., B.Sc. 32 *Willoughby Road, Hampstead*, N.W.
	§*‖Baynes, R. E., M.A. *Christ Church, and* 2 *Norham Gardens, Oxford.*
June 14, 1895.	Beale, S. *Banbury Municipal School, Banbury.*
June 22, 1906.	Beckwith, Emmerson, B.Sc. 10 *Albyn Terrace, Middlesbrough.*
May 22, 1886.	Bell, Chichester A., M.B. "*Torbrex,*" *Headington Hill, Oxford.*
May 14, 1909.	Bell, W. H., A.M.Inst.C.E. *Faraday House, Southampton Row*, W.C.
Nov. 27, 1903.	Bennett, Wm., B.Sc. *School House, Brighton College, Brighton.*
Nov. 25, 1910.	Berry, G. H. 3 *Station Road, Winchmore Hill*, N.
Mar. 12, 1909.	BEVAN, Prof. P. V., M.A. (MEMBER OF COUNCIL). *Royal Holloway College; and Hillside, Egham.*
Apr. 26, 1907.	Blackie, Alfred, Junr. *C/o Mr. Gibbs-Smith, Cromer House, Hampton Road, Teddington.*
Mar. 12, 1909.	Blaikie, L., M.A. *Civil Service Commission, Burlington Gardens*, W.
May 28, 1881.	‖Blaikley, D. J. 13 *Elsworthy Terrace, Primrose Hill*, N.W.

Date of Election.	
Nov. 28, 1885.	§†Blakesley, T. H., M.A., M.Inst.C.E. 7 *Cromwell Crescent, Earl's Court, Kensington*, W.
Mar. 8, 1907.	Boardman, E., B.Sc., F.C.S. 619 *St. Helen's Road, Bolton.*
Mar. 8, 1895.	Borns, H., Ph.D. 5 *Sutton Court Road, Chiswick*, W.
May 13, 1876.	§‖Bosanquet, R. H. M., M.A., F.R.S. *Castillo Zamora, Realijo Alto, Tenerife, Canary Isles*
Mar. 8, 1901.	Bose, Prof. Jagadis Chunder, M.A., D.Sc. *Presidency College, Calcutta.*
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	*‖Bourne, C. W., M.A. *Frating Rectory, nr. Colchester.*
Feb. 12, 1892.	‖Bower, William Richard. *Technical College, Huddersfield.*
June 12, 1880.	§‖Boys, C. Vernon, F.R.S. 66 *Victoria Street*, S.W.
Feb. 22, 1907.	§Bragg, Prof. W. H., M.A., F.R.S. *The University, Leeds.*
Mar. 10, 1905.	Brinkworth, J. H., B.Sc. 31 *Albert Mansions, Battersea Park*, S.W.
Feb. 8, 1895.	Briscoe, Albert Edward, B.Sc. *The Hoppet, Little Baddow, Essex.*
Nov. 21, 1874.	‖Brown, Prof. F. D., M.A. *University College, Auckland, New Zealand.*
Mar. 12, 1881.	‖Browne, Colvile, M.R.A.C. *Swanley Junction, Kent.*
Jan. 22, 1909.	Brunskill, H. 55 *Abbot's Road, Southall.*
Dec. 14, 1894.	‖Bryan, G. H., M.A., F.R.S. *Plas Gwyn, Bangor.*
Feb. 25, 1882.	‖Buchanan, J., D.Sc. *Gordon's College, Aberdeen.*
May 10, 1895.	‖Buckmaster, C. A., M.A. 16 *Heathfield Road, Mill Hill, Park, Acton*, W.
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Mar. 25, 1904.	Burns, G. J., B.Sc. 209 *Shooters Hill Road, Blackheath*, S.E.
Jan. 25, 1907.	Burrow, S. G., B.Sc. *Baroda College, Baroda, India.*
June 14, 1907.	Burton, Dr. C. V. *Boar's Hill, Oxford.*
Mar. 22, 1907.	Butcher, W. D., M.R.C.S. *Holyroad, Ealing*, W.
Dec. 9, 1898.	‖CALLENDAR, Prof. H. L., M.A., LL.D., F.R.S. (Past President). *Imperial College of Science, South Kensington*, S.W.; *and* 49 *Grange Road, Ealing*, W.
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Feb. 28, 1885.	Chadwick, O., M.Inst.C.E. 16 *West Halkin Street, Belgrave Square*, S.W.
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Apr. 8, 1910.	Charlesworth, John, M.Sc. 15A *Loraine Mansions, Widdenham Road, Holloway*, N.
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Dec. 11, 1896.	CHREE, C., M.A., Sc.D., LL.D., F.R.S. (Past President). 75 *Church Road, Richmond, Surrey.*
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Date of Election.	
Mar. 8, 1912.	Churton, Thos. Harding, M.I.E.E. *Hillary Place, Leeds.*
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Nov. 22, 1895.	Clark, G. M., M.A.
Mar. 24. 1911.	Clark, Hugh, B.Sc. *Durban Tech. Inst., Russell Road, Durban, Natal.*
June 9, 1911.	Clarke, F. C. 20 *Cowley Road, Ilford, Essex.*
Dec. 10, 1881.	‖Clayden, A. W., M.A. 5 *The Crescent, Mount Radford, Exeter.*
Nov. 27, 1875.	‖CLIFTON, Prof. R. B., M.A., F.R.S. (PAST PRESIDENT). *The Museum, Oxford.*
Nov. 24, 1899.	Clough, W. T. "*Chelladene,*" *St. Margarets Road, Wanstead Park, Essex.*
Mar. 23, 1889.	‖Cockburn, A., F.C.S. *Longfield, Broom Road, Hampton Wick.*
Mar. 29, 1903.	Codd, M.A. 51A *Berners Street, Oxford Street,* W.
Mar. 11, 1876.	§‖Coffin, Dr. W. H. *National Liberal Club, Whitehall Pl.,* S.W.
Mar. 12, 1909.	Coker, Prof. E. G., M.A., D.Sc. *City & Guilds' Technical College, Finsbury,* E.C.
May 16, 1890.	‖Coleman, Prof. W. M., A.B. 4710 *Davenport Street,* N.W., *Amer. Univ. Park, Washington,* D.C , U.S.A.
Nov. 24, 1911.	Cooper, Richard. 19 *Birdhurst Road, Croydon.*
Feb. 22, 1895.	COOPER, W. R., M.A., B.Sc. (SECRETARY). 113 *Tulse Hill,* S.W.
	Crawford and Balcarres, Earl of, F.R.S., F.R.A.S. 2 *Cavendish Square,* W.
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	*‖Crawshaw, Edward. 25 *Tollington Park,* N.
	*‖Crisp, Sir Frank, B.A., LL.B., J.P. 5 *Lansdowne Road,* W. ; *and Friar Park, Henley-on-Thames.*
Apr. 18, 1890.	§‖Croft, William Bleaden, M.A. *The College, Winchester.*
Mar. 23, 1889.	§‖Crompton, Lt.-Col. R. E. B., C.B. *Thriplands, Kensington Court,* W.
May 12, 1888.	§**‖Crookes, Sir W., F.R.S. 7 *Kensington Park Gardens,* W.
Mar. 13, 1886.	‖Cunynghame, Sir H. H., K.C.B. *Home Office, Whitehall,* S.W.
	‖Curtis-Hayward, A. C., M.A. *Maxwell House, Arundel Street, Strand, W.C.*

May 12, 1911.	Dakin, A., M.A., B.Sc. 16 *Florence Park, Redland, Bristol.*
Mar. 8, 1912.	Darling, Chas. R. 186 *Eglinton Road, Plumstead,* S.E.
Mar. 26, 1898.	Darwin, Horace, M.A., F.R.S. *The Orchard, Cambridge.*
May 12, 1877.	‖Darwin, Major L., R.E., F.R.S. 12 *Egerton Place,* S.W.
Dec. 4, 1891.	‖Davey, Henry, M.Inst. C.E. *Conaways, Ewell, Surrey.*
Feb. 26, 1904.	Davidge, Prof. H. T., B.Sc., M.I.E.E. *Ordnance College, Woolwich,* S.E.
Mar. 11, 1898.	Davidson, Sir J. M., M.B., C.M. 76 *Portland Place,* W.
Mar. 26, 1909.	Davies, B. *Wilford, Chase Green Avenue, Enfield.*
Dec. 9, 1904.	Davies, Henry, B.Sc. *Municipal Technical Institute, Portsmouth.*
Feb. 13, 1875.	‖Day, R. Evan. *Culver, Plaistow Lane, Bromley, Kent.*
May 26, 1877.	Debus, Dr. H., F.R.S. 4 *Schlangenweg, Cassel, Germany.*
June 24, 1876.	Dewar, Sir James, F.R.S. *Cambridge ; and Royal Institution, Albemarle Street,* W.
Nov. 23, 1900.	Dickinson, R. H. *Technical School, Glossop, nr. Manchester.*

Date of Election.	
Feb. 25, 1882.	‖Dockrell, Capt. H. J., R.N. *Sydney House, Shefford, R.S.O., Beds.*
Jan. 26, 1912.	Donaldson, Harold. *National Physical Laboratory, Bushy House, Teddington.*
Oct. 30, 1896.	§Donnan, F. G., M.A., Ph.D. *Muspratt Laboratory, The University, Liverpool.*
Mar. 11, 1898.	Drysdale, C. V., D.Sc. 49 *Rotherwick Road, Hampstead Garden Suburb*, N.W.
Mar. 2, 1900.	†DUDDELL, W., F.R.S. (TREASURER). 47 *Hans Place*, S.W.
	*‖Durham, Henry. 13 *Coleherne Road, South Kensington*, S.W.
Feb. 14, 1902.	Dutton, F., B.Sc. "*Rivington*," *Woodwarde Road, Dulwich*, S.E.
May 26, 1877.	‖Dyer, Sir W. T. Thiselton, K.C.M.G., M.A., B.Sc., F.R.S. *The Ferns, Witcombe, Gloucester.*
Mar. 12, 1909.	Eagle, A., B.Sc. 63 *Hereford Road, Bayswater*, W.
Nov. 28, 1902.	§Eccles, W. H., D.Sc. 37 *Chelsea Gardens, Chelsea*, S.W.
Feb. 24, 1905.	Edgcumbe, K. *Heath Cottage, Letchmore, Watford.*
Feb. 12, 1892.	‖Edser, Edwin. 3 *Hillyfields Crescent, Brockley*, S.E.
Feb. 24, 1911.	Egerton, A. C. G. *Chilton House, Thame, Oxon.*
June 26, 1903.	Eggar, W. D., M.A. *Eton College, Windsor.*
Dec. 11, 1886.	†Elder, H. M., M.A. 41 *Netherhall Gardens, Hampstead*, N.W.
June 26, 1891.	Enright, James, B.Sc. 202 *Holland Road*, W.
Apr. 11, 1902.	Erskine-Murray, J. R., D.Sc., F.R.S.E. *Oswaldestre House, Norfolk Street*, W.C.
Nov. 27, 1875.	Esson, W., M.A., F.R.S. *Merton College, Oxford.*
May 27, 1904.	Etchells, E. F., A.M.I.M.E. *Building Acts' Office, L.C.C., Spring Gardens*, S.W.
Mar. 8, 1895.	Eumorfopoulos, N., B.Sc. 24 *Pembridge Gardens*, W.
Feb. 21, 1890.	‖Evershed, Sydney. *Acton-Lane Works, Chiswick*, W.
Apr. 22, 1898.	†Ewing, Sir J. A., F.R.S. *Froghole, Edenbridge, Kent.*
June 9, 1911.	Farr, C. C. *Canterbury College, Christchurch, New Zealand.*
	§†**‖Festing, Maj.-Gen. E. R., R.E., F.R.S. 56 *Queen's Gate Terrace*, S.W.
Feb. 8, 1879.	‖Finch, G. B., M.A. *Howes Close, Cambridge.*
Nov. 28, 1902.	Finn, M., M.Sc. *Sea View, Ambleside Drive, Southend-on-Sea.*
Dec. 14, 1894.	‖Fitzpatrick, Rev. T. C., M.A. *The Lodge, Queen's College, Cambridge.*
	§†*Fleming, Prof. J. A., M.A., D.Sc., F.R.S. *University College, Gower Street*, W.C.
Mar. 25, 1882.	§†‖Fletcher, L., M.A., F.R.S. *British Museum* (*Natural History*), *Cromwell Road*, S.W.
Apr. 14, 1883.	§‖Forbes, Prof. G., M.A., F.R.S. 11 *Little College Street, Westminster.*
Nov. 25, 1898.	Forsyth, R. W. *Imperial College of Science, London*, S.W.
	**‖FOSTER, Prof. G. Carey, F.R.S. (PAST PRESIDENT). *Ladywalk, Rickmansworth, Herts.*
June 14, 1879.	‖Fox-Pitt, St. George Lane. *Travellers' Club, Pall Mall*, S.W.
Jan. 21, 1898.	‖Francis, William. *Red Lion Court, Fleet Street*, E.C.

Date of Election.	
Dec. 13, 1895.	‖Fridlander, E. D. *Hertford Place, Coventry.*
Nov. 25, 1892.	‖Fry, Francis James, J.P. *Messrs. Fry & Sons, Bristol.*
Jan. 22, 1909.	Fuller, F. P., B.Sc. 114 *Chesterton Road, Kensington.*
Feb. 22, 1895.	Gardiner, J. H. 59 *Wroughton Road, Balham*, S.W.
Mar. 13, 1908.	Garrett, A. E., B.Sc. 127 *Lothair Road, Finsbury Park*, N.
Feb. 22, 1907.	Garrett, C. A. B., B.Sc. 5 *Azalea Road, Blackburn.*
Nov. 1, 1889.	Garrett, T. A., M.A. *Sunnymead, Reigate Road, Reigate.*
May 13, 1892.	‖Gerrans, H. T. 20 *St. John's Street, Oxford.*
Jan. 24, 1902.	Gheury, M. E. J. 40 *Westmount Road, Eltham Park, London*, S.E.
Nov. 13, 1896.	Gifford, J. W. *Oaklands, Chard, Somerset.*
Dec. 13, 1879.	‖GLAZEBROOK, R. T., C.B., D.Sc., F.R.S. (Past President). *Bushy House, Teddington, Middlesex.*
Nov. 28, 1907.	Gordon, Rev. J. M. *St. John's Vicarage, Redhill.*
Mar. 23, 1889.	‖Gorham, J. M., M.Inst. C.E. *Drove House, Singleton, Sussex.*
Feb. 28, 1902.	Grace, G. M. 33 *Weltje Road, Hammersmith*, W.
Feb. 13, 1875.	‖Gray, R. K. *Telegraph Works Company, Silvertown*, S.E.; *and* 106 *Cannon Street*, E.C.
Feb. 23, 1912.	Green, Harry H. 12 *St. John's Road, East Ham*, E.
Mar. 11, 1910.	Griffith, I. O., M.A. 49 *Banbury Road, Oxford.*
Feb. 22, 1896.	§Griffiths, A., D.Sc. *Birkbeck College, Bream's Buildings, Chancery Lane*, E.C.
Feb. 23, 1884.	‖Grundy, J. 8 *Grosvenor Gardens, Cricklewood*, N.W.
Mar. 2, 1900.	Gundry, P. G. Ph.D., B.Sc. *Transvaal University College, Box* 43, *Pretoria, S. Africa.*
Mar. 12, 1909.	Hadfield, Sir R. A., F.R.S. 22 *Carlton House Terrace*, S.W.
Mar. 13, 1908.	Hall, A. E. 6 *Andover Street, Swindon, Wilts.*
	*‖Harding, Alfred B. *Iona, Ravensbourne Park, Catford*, S.E.
	Hare, A. T. *The Orangery, St. Margaret's, Middlesex.*
Feb. 14, 1880.	§Harker, J. A., D.Sc., F.R.S. *Cape House, Kingston Road, Teddington, Middlesex.*
Mar. 12, 1909.	Harlow, F. J., B.Sc. *Beau Séjour*, 28 *Warwick Road, Earl's Court*, W.
Jan. 22, 1909.	Harris, T., B.Sc. 2 *Apsley Villas, Acton, London*, W.
Mar. 2, 1900.	Harrison, Prof. E. P. *The Observatory, Alipore, Calcutta*
Apr. 28, 1893.	‖Harrison, Fred., M.A. *School House, Newcastle, Staffordshire.*
Jan. 24, 1896.	Harvey, H. S. S. 270 *G.P.O., Brisbane, Queensland.*
Jan. 13, 1893.	‖Haselfoot, C. E., M.A. *Hertford College, Oxford.*
Apr. 26, 1896.	Hay, Prof. A., D.Sc. *Imp. Inst. Science, Bangalore, Mysore, India.*
May 6, 1882.	‖Heaton, W. H., M.A. *University College, Nottingham.*
Nov. 26, 1909.	Hemmy, Prof. A. S. *Lahore. India.*
	§**Henrici, Prof. O., Ph.D., F.R.S. 34 *Clarendon Road, Notting Hill*, W.
May 9, 1885.	‖Herroun, E. F. *Ravenshaw, Croydon Road, Reigate, Surrey.*
Mar. 16, 1878.	Hicks, Prof. W. M., M.A., F.R.S. *Leamhurst, Ivy Park Road, Sheffield.*
Dec. 11, 1875.	‖Higgins, Clement, M.A. *Winterdyne, Bournemouth, West.*

Date of Election.	
Mar. 8, 1912.	Higgins, W. F., B.Sc. 15 *Nicoll Road, Harlesden*, N.W.
Mar. 24, 1911.	Hirst, Hugo. 71 *Queen Victoria Street, London*, E.C.
Dec. 4, 1891.	Hoffert, Hermann, H., D.Sc., H.M.I. *Lime Grove, Brooklands, Sale, Manchester.*
Feb. 7, 1890.	‖Holland, Charles E., B.A. 4 *Redington Road, Hampstead*, N.W.
Nov. 22, 1901.	‖Hopkinson, Prof. B., F.R.S. 10 *Adams Road, Cambridge.*
Apr. 22, 1882.	‖Hopkinson, Edward, M.A., D.Sc. *Ferns, Alderley Edge, Cheshire.*
Apr. 26, 1907.	Hopwood, F. Lloyd, B.Sc. 5 *Birley Road, Totteridge Lane*, N.
Jan. 24, 1884.	Howard, Alfred, A.R.S.M., A.R.C.Sc. 186 *Stanstead Road, Forest Hill*, S.E.
May 13, 1876.	‖Howard, David. *Devon House, Buckhurst Hill, Essex.*
Jan. 24, 1908.	Howarth, J. G. *Danes Dyke, Chart Road, Reigate.*
Feb. 25, 1910.	Howe, G. W. O., M.Sc. 14 *Bernard Gardens, Wimbledon Park*, S.W.
Feb. 8, 1907.	Hunter, J. de G., M.A. *Survey of India, Dehra Dun, United Provinces, India.*
	*†‖Huntington, Prof. A. K. *King's College, Strand*, W.C.

May 26, 1877.	‖Jack, Prof. W., M.A., LL.D. 10 *The University, Glasgow.*
Mar. 25, 1882.	‖Jackson, M. J., M.A., D.Sc. *Applegarth, Aldergrove, British Columbia.*
Mar. 8, 1873.	‖Jacob, F. *Messrs. Sicmens Bros. & Co., Woolwich*, S.E.
Feb. 8, 1907.	Jakeman, Christopher. "*Claremont*," *Cromwell Road, Teddington.*
May 22, 1889.	‖Johnson, W. Claude. *Broadstone Farm, Coleman's Hatch, Sussex.*
Apr. 8, 1910.	Jolley, A. C. 81 *Welford Road, Norbury, London*, S.W.
Feb. 17, 1886.	‖Jones, D. E., B.Sc. *Eryl-Daf, Radyr, Cardiff.*
Feb. 26, 1904.	Jones, Prof. E. Taylor, D.Sc. *University College, Bangor.*
Nov. 27, 1880.	‖Jones, H. Chapman, F.I.C. *Imperial College of Science, South Kensington*, S.W.
Feb. 24, 1911.	Jones, Jonathan, B.Sc. *The County School, Aberystwith.*
Nov. 27, 1908.	Jones, Prof. R. Ll. *The Observatory, Madras, India.*
Apr. 10, 1880.	‖Judd, J. W., LL.D., F.R.S. 30 *Cumberland Road, Kew.*
Dec. 15, 1877.	‖Judson, J. E., B.A. *Charterhouse, Godalming.*

Mar. 10, 1911.	Kaye, G. W. C., D.Sc. *Culver, St. James Avenue, Hampton Hill, Teddington.*
Nov. 24, 1911.	Kempson, E. W. E. 2 *Whitehall Gardens, Rugby.*
Nov. 12, 1909.	Kennett, W. H., B.A. *Wellington College, Berks.*
	*‖Kesselmeyer, C.A. *Rose Villa, Vale Road, Bowdon, Cheshire.*
	*‖Kesselmeyer, W. J. *Edelweiss Villa*, 13 *Albert Road, Hale, nr. Altrincham, Cheshire.*
Feb. 23, 1889.	Kilgour, Hamilton, *Holly Hall, Harrison Street, East Orange, N.J., U.S.A.*
Dec. 5, 1874.	‖Kinch, Edward. *Royal Agricultural College, Cirencester.*
Feb. 24, 1905.	Kingsbury, J. E. *Burton Dassett, Herne Hill*, S.E.

Date of Election.	
Apr. 14, 1905.	Kirkman, J. C., B.Sc. 42 *Beechwood Road, Uplands, Swansea.*
Mar. 2, 1878.	‖Kirkman, J. Parke, M.A. *Glanyrafon, Bedford.*
Mar. 13, 1896.	Korda, Désiré. 15 *Rue Ambroise-Thomas, Paris.*
	*‖Lambert, Prof. C. J., M.A. *Buckhurst Lodge, Westerham Hill, Kent.*
Jan. 25, 1879.	‖Lankester, Prof. Sir E. Ray, K.C.B., M.A., F.R.S. 29 *Thurloe Place,* S.W.
June 9, 1911.	Larard, C. E. 106 *Cranley Gardens, Muswell Hill,* N.
June 22, 1894.	‖Larmor, Prof. Sir Joseph, M.A., D.Sc., F.R.S. *St. John's College, Cambridge.*
May 27, 1876.	‖Law, Channell. *The Museum, Torquay.*
Jan. 25, 1907.	Lean, Clement, B.Sc., A.M.I.Mech.E. *Thanet House,* 231 *Strand,* W.C.
Feb. 8, 1807.	LEES, Prof. C. H., D.Sc., F.R.S. (VICE-PRESIDENT). *Greenacres, Woodside Road, Woodford Green, Essex.*
Nov. 22, 1895.	§‖Lehfeldt, R. A., D.Sc. *Transvaal Technical Institute, Johannesburg, S. Africa.*
May 11, 1889.	Lewes, Prof. V. B. *Royal Naval College, Greenwich,* S.E.
Feb. 28, 1896.	Lewis, Prof. D. Morgan, M.A., F.R.S. *University College of Wales, Aberystwyth.*
Feb. 14, 1880.	‖Lewis, T. C., M.A. *West Home, West Road, Cambridge.*
Jan. 17, 1882.	‖Lewis, Prof. W. J., M.A. *Mineralogical Museum, Cambridge.*
June 24, 1904.	Liebert, W. *Checkfire Door Co., Camel Road, Silvertown,* E.
Mar. 10, 1911.	Lightley, E., B.Sc. *Fernlea, Salisbury Road, Harrow.*
June 14, 1895.	Littlewood, T. H., M.A. *Bancroft's School, Woodford Wells, Essex.*
Feb. 22, 1879.	‖Liveing, Prof. G. D., M.A., F.R.S. *Newnham, Cambridge.*
Mar. 25, 1876.	‖Liversidge, Prof. A., M.A., F.R.S. *Hornton Cottage, Hornton Street, Kensington,* W.
Feb. 3, 1877.	Lockyer, Sir J. Norman, K.C.B., F.R.S. *Solar Physics Observatory, South Kensington,* S.W.
Feb. 13, 1875.	‖LODGE, Principal Sir O. J., D.Sc., F.R.S. (PAST PRESIDENT) *Mariemont, Edgbaston, and The University, Birmingham.*
Dec. 11, 1903.	Lough, Victor. *Glendale, High Road, Loughton, Essex.*
Feb. 23, 1884.	‖Love, E. F. J., M.A. *The University, Melbourne, Australia.*
Nov. 27, 1896.	Lowater, Miss Fanny, B.Sc., Ph.D. *Rockford College, Rockford, Illinois,* U.S.A.
Mar. 27, 1903.	Lownds, Louis, B.Sc., Ph.D. *South-West London Polytechnic, Manresa Road, Chelsea,* S.W.
Mar. 6, 1891.	Lucas, William, M.A. 25 *Glasslyn Road, Crouch End,* N.
	§†*‖Lupton, Sydney, M.A. 102 *Park Street, Grosvenor Square,* W.
Jan. 23, 1903.	Lyle, Prof. T. R., Sc.D. *The University of Melbourne, Victoria, Australia.*
Jan. 25, 1879.	Macdonell, Prof. A., B.A. 28 *Avenue Gardens, Mill Hill Park,* W.
June 14, 1901.	McEwen, J. M. 17 *Wellington Road, Stockport.*
Dec. 14, 1878.	M'Gregor, Prof. J. Gordon, M.A., D.Sc., F.R.S. 24 *Dalrymple Crescent, Edinburgh.*

Date of Election.	
May 11, 1878.	‖Magnus, Sir Philip, B.Sc., M.P. 16 *Gloucester Terrace, Hyde Park*, W.
Oct. 30, 1896.	‖Mair, D. B., M.A. *Civil Service Commission, Burlington Gardens*, W.
Feb. 22, 1901.	Makower, Walter, D.Sc. *Maylands, Brook Road, Fallowfield, Manchester.*
Nov. 18, 1876.	‖Malcolm, Col. E. D., R.E., C.B. *Poltalloch, Loch Gilphead, Argyll.*
Mar. 7, 1890.	‖Mance, Sir Henry, LL.D. *Old Woodbury, Sandy, Beds.*
Mar. 11, 1910.	Marsden, E., B.Sc. *Physics Laboratory, The University, Manchester.*
Mar. 8, 1907.	Martyn, G. H., B.Sc. 38 *Hogarth Hill, Garden Suburb, Hampstead*, N.W.
Feb. 24, 1911.	Mason, C. C., M.A. *Beverley, Cavendish Avenue, Cambridge.*
Mar. 23, 1889.	‖Massey, W. H., M.Inst.C.E. *Twyford, R.S.O., Berks.*
May 14, 1887.	§‖MATHER, Prof. T., F.R.S. (VICE-PRESIDENT). 37 *Wyatt Park Road, Streatham Hill*, S.W.
June 22, 1900.	Mayall, R. H. D., M.A. *Sidney Sussex College, Cambridge.*
	**‖Meldola, Prof. R., F.R.S. 6 *Brunswick Square*, W.C.
Mar. 6, 1891.	‖Miers, Principal Sir H. A., D.Sc., F.R.S. 23 *Wetherby Gardens*, S.W.
Feb. 13, 1886.	‖Mills, A. E., M.A., M.Inst.E.E. *Halcyon, North Finchley*, N.
	§**‖Mills, Prof. E. J., F.R.S. 64 *Twyford Avenue, West Acton*, W.
Mar. 13, 1880.	§Minchin, Prof. G. M., M.A., F.R.S. 149 *Banbury Road, Oxford.*
Feb. 23, 1906.	Modi, E. M., D.Sc., F.C.S., LL.D., Litt.D., F.R.S. (Edin.). *Meher Buildings, Tardeo, Bombay, India.*
Dec. 4, 1891.	‖Mond, Robert Ludwig, B.A., F.G.S. 20 *Avenue Road*, N.W.
Jan. 26, 1912.	Moore, Harry. 21 *Waldegrave Road, Hanger Lane, Ealing*, W.
Apr. 13, 1889.	Mordey, W. M. 82 *Victoria Street*, S.W.
May 12, 1877.	‖Morley, H. F., M.A., B.Sc. 5 *Lyndhurst Rd., Hampstead*, N.W.
Feb. 24, 1911.	Moss, E. W. 20 *Huddleston Road, Tufnell Park*, N.
Mar. 22, 1895.	‖Morris, Dr. D. K. *Messrs. Morris & Lister, Ltd., Carlton Works, Coventry.*
May 11, 1906.	Morrow, John, M.Sc., D.Eng. *Armstrong College, Newcastle-on-Tyne.*
Mar. 22, 1895.	‖Morton, Prof. W. B., M.A. *Queen's College, and Glencarse, Notting Hill, Belfast.*
Mar. 8, 1895.	‖Mostyn, S. G., M.A. *Health Office, Houndgate, Darlington.*
June 28, 1879.	Moulton, Sir John Fletcher, M.A., F.R.S. 57 *Onslow Square*, S.W.
Mar. 23, 1889.	‖Mudford, F. J. *Arrow Electrical Works, Jackson Road, Holloway*, N.
	**‖Muirhead, Alexander, D.Sc., F.R.S. *The Lodge, Shortlands, Kent.*
	§*‖Müller, Dr. Hugo, F.R.S. 13 *Park Square East*, N.W.
Nov. 24, 1911.	Murdoch, W. H. F., B.Sc., M.I.E.E. 14 *Howitt Road, Hampstead*, N.W.
Mar. 12, 1909.	Narbeth, B. M. *Technical Institute, Durban, Natal, S. Africa.*

Date of Election.	
Dec. 11, 1886.	Natanson, Ladislas. *Professor, Cracow University;* 3 *Studencka str., Cracow* (*Austria*).
Mar. 24, 1911.	Nelson, George, A.R.C.S., B.Sc. *The Technical College, Sunderland.*
Nov. 26, 1909.	Nettleton, H. R., B.Sc. *Birkbeck College, Bream's Buildings, Chancery Lane*, E.C.
	‖Neville, Francis Henry, M.A., F.R.S. *La Verna, Norton Ways, Letchworth, Herts.*
Apr. 8, 1910.	Neville, B. M. *Silverleigh*, 42 *Constantine Road, Hampstead*, N.W.
Feb. 25, 1888.	‖Newall, Prof. H. F., M.A., F.R.S. *Madingly Rise, Cambridge.*
Nov. 12, 1909.	Nicholson, J. W., M.A., D.Sc. *Trinity College, Cambridge.*
Nov. 11, 1910.	Nicol, James. 43 *Beversbrook Road, Tufnell Park*, N.
Nov. 12, 1881.	Niven, Sir W. D., M.A., F.R.S. *Eastburn, St. John's Road, Sidcup.*
Oct. 25, 1895.	Noble, Sir Andrew, K.C.B., F.R.S. *Jesmond Dene House, Newcastle-on-Tyne.*
Nov. 25, 1898.	Noble, Wilson. *Barkham Manor, Wokingham.*
Apr. 26, 1895.	‖Notcutt, S. A., LL.M., B.A., B.Sc. *Constitution Hill, Ipswich.*
Apr. 10, 1875.	‖Odling, Prof. W., M.A., F.R.S. 15 *Norham Gardens, Oxford.*
Dec. 9, 1904.	Owen, D., B.Sc. *Berkswell*, 11 *Manor Villas, Squires Lane, Church End, Finchley.*
July 8, 1910.	O'MEARA, Major W. A. J., C.M.G. (MEMBER OF COUNCIL). 12 *Longridge Road, Earl's Court*, S.W.
Nov. 13, 1908.	Palmer, B. J., B.Sc. 12 *Chancellor Road, Southend-on-Sea.*
May 27, 1904.	Parker, F. H., B.Sc. 72 *Greenvale Road, Eltham, Kent.*
Feb. 8, 1907.	Paterson, C. C., A.M.Inst.C.E. *National Physical Laboratory, Teddington.*
	**‖PERRY, Prof. J., D.Sc., F.R.S. (PAST PRESIDENT). *Imperial College of Science, South Kensington*, S.W.
May 25, 1878.	‖Phelips, W. R. *Montacute House, Ilminster.*
Feb. 24, 1899.	Phillips, C. E. S. *Castle House, Shooter's Hill, Kent.*
Apr. 22, 1904.	Phillips, W. C. S., B.Sc. *Municipal Technical School, Suffolk Street, Birmingham.*
Mar. 13, 1886.	‖Pickering, Prof. S. U., M.A., F.R.S. *Harpenden, Herts;* *and* 60 *Palace Court*, W.
Jan. 16, 1891.	‖Pidgeon, Winter Randall, M.A. *Lynsted Lodge, St. Edmund's Terrace, Regent's Park*, N.W.
Apr. 24, 1903.	Polley, L. J. N. *Rutlich School, Merton*, S.W.
May 12, 1899.	‖Pollock, Prof. J. A., B.Sc. *University of Sydney, N.S.W.*
Jan. 16, 1891.	§Porter, Alfred William, B.Sc., F.R.S. *University College, Gower Street*, W.C.
Nov. 11, 1892.	‖Porter, Dr. T. C., M.A. *Upton Park, Slough.*
Dec. 13, 1879.	‖POYNTING, Prof. J. H., M.A., Sc.D., F.R.S., J.P. (PAST PRESIDENT). 10 *Ampton Road, Edgbaston, Birmingham.*
Mar. 16, 1878.	‖Prance, Miss Edith. *The Ferns, Frognal Lane, Hampstead*, N.W.
Nov. 4, 1876.	‖Preece, Sir W. H., K.C.B., F.R.S. *Gothic Lodge, Wimbledon.*

Date of Election.	
Mar. 25, 1876.	‖Preston, Dr. S. Tolver. *Papenkamp, Klein Flottbek bei Altona, Germany.*
May 8, 1886.	§‖Price, W. A., M.A. 135 *Sandyford Road, Newcastle-on-Tyne.*
June 27, 1885.	§‖Ramsay, Sir W., K.C.B., Ph.D., F.R.S. *University College, Gower Street*, W.C.; *and* 19 *Chester Terrace*, W.
Dec. 9, 1892.	Rawson, Col. Herbert E., C.B., R.E. *Home Close, Heronsgate, Herts.*
Dec. 15, 1877.	§†‖Rayleigh, The Right Hon. Lord, O.M., M.A., D.C.L., F.R.S. *Terling Place, Witham, Essex.*
Feb. 8, 1907.	‖Rayner, E. H. *Tiviot Dale, Gloucester Road, Teddington.*
Mar. 10, 1911.	Read, R. S., B.A., B.Sc. 6 *Archibald Road, Exeter.*
	**‖REINOLD, Prof. A. W., C.B., M.A., F.R.S. (PAST PRESIDENT). 9 *Vanbrugh Park Road, Blackheath*, S.E.
Feb. 13, 1903.	Rheinberg, J. 23 *The Avenue, Brondesbury Park*, N.W.
May 8, 1903.	Richardson, Prof. O. W., M.A., D.Sc. *Princeton, N.J., U.S.A.*
June 12, 1886.	‖Rideal, S., D.Sc. 28 *Victoria Street, Westminster*, S.W.
Nov. 9, 1894.	Rintoul, D., M.A. 26 *College Road, Clifton, Bristol.*
Mar. 11, 1910.	Robinson, James, M.Sc., Ph.D. *Seghill, Northumberland.*
May 11, 1889.	Robinson, Prof. W., M.E., M.Inst.C.E. *University College, Nottingham.*
May 8, 1880.	‖Roscoe, Prof. Sir Henry E., Ph.D., F.R.S. 10 *Bramham Gardens, Whetherby Road*, S.W.
Jan. 24, 1885.	‖Rose-Innes, J., M.A., B.Sc. *King's College, Cambridge.*
Feb. 8, 1907.	Rosenhain, Dr. W. "*Warrawee*," *Combe Lane, Kingston Hill.*
	*‖RÜCKER, Sir Arthur W., M.A., F.R.S. (PAST PRESIDENT). *Everington, near Newbury, Berks.*
Apr. 26, 1907.	Russ, Sidney, D.Sc. *Cancer Research Laboratories, Middlesex Hospital*, W.
Jan. 26, 1906.	RUSSELL, Alexander, M.A., D.Sc. (VICE-PRESIDENT). 10 *Richmond Bridge Mansions, East Twickenham, Middlesex.*
Jan. 19, 1878.	‖Russell, J. W., M.A. 28 *Staverton Road, Oxford.*
Mar. 22, 1907.	§Rutherford, Prof. E., D.Sc., F.R.S. *The University, Manchester.*
May 26, 1877.	‖Sale, Lt.-Col. M. T., R.E., C.M.G. *Ugia Bank, Abbey Wood, Kent.*
Jan. 15, 1876.	‖Salomons, Sir D. L., Bart. *Broom Hill, Tunbridge Wells.*
Jan. 30, 1875.	‖Salter, M. J. *Greyrick House, Mickleton, Gloucestershire.*
May 13, 1898.	Sampson, J. *University, Liverpool.*
June 27, 1885.	‖Sanderson, F. W., B.A. *The School House, Oundle.*
Dec. 9, 1882.	‖Saunders, Rev. S. T. H., M.A. 39 *Buckland Crescent*, N.W.
Nov. 26, 1909.	Schofield, H. 30 *Fielding Road, The Avenue, Bedford Park*, W.
Feb. 28, 1902.	Schofield, J. *Oakwood, Big Valley, Meltham Road (Berry Brow), Huddersfield.*
	†§**‖SCHUSTER, Prof. A., Ph.D., D.Sc., LL.D. F.R.S. (PRESIDENT). *Kent House, Victoria Park, Manchester.*

Date of Election.	
Apr. 26, 1907.	Scoble, Walter A., A.R.C.S., B.Sc. *Finsbury Technical College, Leonard Street*, E.C.
Feb. 21, 1883.	Scott, Prof. A. W. *St. David's College, Lampeter, South Wales.*
Feb. 24, 1911.	Sears, J. E., Junr. *National Physical Laboratory, Teddington, Middlesex.*
Dec. 19, 1894.	‖Searle, G. F. C., M.A., D.Sc., F.R.S. *Wyncote, Hills Road, Cambridge.*
Mar. 17, 1877.	‖Seaton, W. S. *Montague House, Richmond, Surrey.*
May 9, 1885.	‖Selby, Prof. A. L., M.A. 29 *Palace Road, Llandaff, Cardiff.*
June 14, 1901.	Selby, F. J., M.A. *Ravensbourne, Gloucester Road, Teddington, Middlesex.*
Feb. 24, 1899.	Shand, Prof. John. *University of Otago, Dunedin, New Zealand.*
Mar. 8, 1895.	Shaw, P. E., B.A., D.Sc. *University College, Nottingham.*
Jan. 26, 1906.	Shaw, Walter, B.Sc. 236 *Hampton Road, Ilford.*
Feb. 8, 1879.	§‖Shaw, W. N., M.A., Sc.D., F.R.S. 10 *Moreton Gardens, South Kensington*, S.W.
Mar. 7, 1890.	‖Shorter, Lewis R., B.Sc. 55 *Campden Hill Road, Kensington*, W.
Oct. 25, 1895.	‖Shurlock, F. W., B.A., B.Sc. 71 *Wilson Street, Derby.*
Apr. 13, 1889.	‖Siemens, A. *Caxton House, Westminster.*
Jan. 22, 1881.	‖Simpson, G. P., B.Sc. *Board of Trade, Exhibitions Branch, Queen Anne's Chambers, Westminster*, S.W.
Dec. 10, 1881.	‖Sircom, Prof. the Rev. Sebastian, M.A. *The College, Wimbledon*, S.W.
Dec. 14, 1894.	†‖SKINNER, S., M.A. (MEMBER OF COUNCIL). *South-west London Polytechnic, Manresa Road, Chelsea*, S.W.
Dec. 13, 1895.	‖Slater, R. C., M.A. *Charterhouse, Godalming, Surrey.*
Nov. 23, 1878.	‖Sleeman, Rev. P. R., F.R.A.S. 65 *Pembroke Road, Clifton, Bristol.*
Mar. 22, 1901.	Smart, E., B.A., B.Sc. *Tillyloss, Tullylamb Terrace, Perth.*
Mar. 10, 1911.	Smart, W. A. M., B.Sc. *Caer Glow, Erskine Hill, Golders Green*, N.W.
June 10, 1904.	Smith, E., B.Sc. 14 *Woodriffe Road, Leytonstone*, E.
Feb. 7, 1890.	‖Smith, Edward Wythe. 19 *Newburgh Road, Acton.*
Feb. 8, 1907.	SMITH, F. E., A.R.C.S. (MEMBER OF COUNCIL). *Redcot, St. James' Avenue, Hampton Hill, Middlesex.*
Jan. 24, 1908.	Smith, R. Stuart, B.A. 58 *Burton Road, Melton Mowbray.*
Nov. 26, 1897.	SMITH, S. W. J., M.A., D.Sc. (SECRETARY AND LIBRARIAN). *Imperial College of Science & Technology, S. Kensington*; 12 *Fordhook Avenue, Ealing Common*, W.
Mar. 12, 1909.	Smith, Thomas, B.A. *National Physical Laboratory, Teddington.*
Apr. 10, 1880.	‖Smith, W. O. *Century Works, Lewisham*, S.E.
June 26, 1896.	‖Sodeau, W. H., D.Sc. *Torpedofabrik, Fiume, Hungary.*
June 8, 1878.	‖Solly, R. H., M.A., F.G.S. *Care of* A. Hutchinson, *Pembroke College, Cambridge.*
May 12, 1883.	Soward, Alfred W., C.B., F.C.S., F.S.S., A.Inst.E.E. 28 *Therapia Road, Honor Oak*, S.E.
Feb. 24, 1899.	Sowter, R. J. 137 *St. Julians Farm Road, W. Norwood*, S.E.

Date of Election.	
Mar. 8, 1895.	‖Stansfield, H., D.Sc. *The University, Manchester.*
Apr. 23, 1898.	Starling, S. G., B.Sc. *Municipal Technical Institute, Romford Road, West Ham.*
June 11, 1881.	‖Stead, J. E., F.R.S. *Redcar.*
June 28, 1895.	‖Stewart, J. J., M.A., B.Sc. 2 *Stow Park Crescent, Newport, Monmouthshire.*
	*Stirling, Hon. Sir James, M.A., F.R.S. *Finchcocks, Goudhurst, Kent.*
Dec. 15, 1877.	‖Stocker, Prof. W. N., M.A. *Brasenose College, Oxford.*
Mar. 13, 1880.	§‖Stroh, Augustus. 98 *Haverstock Hill*, N.W.
June 9, 1893.	‖Stroud, Prof. Henry, M.A., D.Sc. *Armstrong College, Newcastle-on-Tyne.*
Feb. 26, 1887.	§‖Stroud, Prof. W., D.Sc. *Caxton Street, Anniesland, Glasgow.*
Mar. 12, 1909.	‖STRUTT, Prof. the Hon. R. J., F.R.S. (MEMBER OF COUNCIL) 54 *Onslow Square*, S.W.
Dec. 10, 1887.	§‖SUMPNER, W. E., D.Sc. (MEMBER OF COUNCIL). *Municipal Technical School, Suffolk Street, Birmingham.*
Dec. 6, 1889.	‖Swan, Sir J. W., F.R.S. *Overhill, Warlingham, Surrey.*
Nov. 23, 1906.	Swann, W. F. G., D.Sc., A.R.C.S. 435 *Glossop Road, Broomhill, Sheffield.*
May 22, 1886.	§†‖Swinburne, J., F.R.S. 82 *Victoria Street*, S.W.
Feb. 8, 1895.	§Swinton, A. A. Campbell, M.Inst.C.E. 66 *Victoria Street*, S.W.
May 27, 1876.	Taylor, Herbert A. 4 *Great Winchester Street*, E.C.
Feb. 13, 1903.	Templeton, Prof. W. S., M.A., B.Sc. 27 *St. Hilary Terrace, Stoke, Devonport.*
May 28, 1897.	Thomas-Davies, G. *Bank Chambers, Regent Street, Clifton, Bristol.*
Dec. 11, 1875.	‖THOMPSON, Prof. S. P., D.Sc., F.R.S. (PAST PRESIDENT). *Finsbury Technical College, Leonard Street*, E.C.
Mar. 22, 1895.	‖Thomson, Prof. Sir J. J., O.M., M.A., D.Sc., LL.D., Ph.D., F.R.S. *Holmleigh, West Road, Cambridge.*
Mar. 22, 1907.	Thomson, W., M.A. 47 *Nassau Road, Barnes*, S.W.
Nov. 28, 1885.	‖Thorpe, Prof. Sir Edward, Ph.D., F.R.S. *Athenæum Club*, S.W.
Mar. 9, 1889.	§‖Threlfall, R., M.A., F.R.S. *Oakhurst, Church Road, Edgbaston, Birmingham.*
Jan. 26, 1900.	Tomkins, J. A. *Municipal Technical College, Bradford.*
	§*‖Tomlinson, Herbert, B.A., F.R.S. *Gilling, Upper Station Road, Bexhill.*
June 8, 1900.	Tomlinson, H. J. 56 *Harold Road, Upper Norwood.*
June 26, 1903.	Townsend, Prof. J. S., M.A., F.R.S. *The Electrical Laboratory, Oxford.*
Dec. 8, 1888.	‖Trotter, Alexander Pelham, B.A. 8 *Richmond Terrace, Whitehall*, S.W.
Mar. 8, 1895.	§Trouton, Prof. F. T., M.A., Sc.D., F.R.S. *University College, Gower Street*, W.C.
Feb. 13, 1903.	Tucker, W. S., B.Sc. 50 *Empress Avenue, Ilford.*
Feb. 22, 1901.	Tutton, A. E. H., M.A., D.Sc. (Oxon), F.R.S. *Duart, Yelverton, S. Devon.*
Nov. 25, 1904.	TWYMAN, F., A.I.E.E. (MEMBER OF COUNCIL). *Messrs. A. Hilger*, 75A *Camden Road*, N.W.

Date of Election.	
Mar. 8, 1912.	Tyndall, A. Mannering, D.Sc., M.Sc. 68 *Trelawney Road, Bristol.*
	§**‖Unwin, Prof. W. C., B.Sc., F.R.S. *Palace-Gate Mansions,* 29 *Palace Gate, Kensington,* W.
Nov. 22, 1895.	‖Vincent, J. H., D.Sc., M.A. 20 *Mostyn Avenue, Wembley, Middlesex.*
Apr. 14, 1905.	Vinter, P. J., M.A. 5 *Acacia Road, Acton,* W.
Feb. 13, 1902.	Wagstaff, C. J. L. 62 *Westbere Road, Cricklewood,* N.W.
Feb. 28, 1900.	Walker, G. W., M.A. *The Observatory, Eskdalemuir, Langholm, N.B.*
Dec. 1, 1883.	§†‖Walker, J., M.A. 30 *Norham Gardens, Oxford.*
Mar. 7, 1890.	‖Waller, Augustus D., M.D., LL.D., F.R.S. 32 *Grove End Road,* N.W.
Nov. 24, 1888.	‖Walmsley, R. M., D.Sc. *Northampton Institute,* E.C.
Dec. 11, 1891.	§†‖Watson, William, D.Sc., F.R.S. 7 *Upper Cheyne Row,* S.W.
Mar. 8, 1895.	Watt, J. *Royal Naval College, Osborne, I.W.*
Dec. 11, 1886.	‖Wertheimer, Prof. Julius, B.A., D.Sc. *Merchant Venturers' Technical College, Bristol.*
Mar. 8, 1912.	West, Gilbert D., B.Sc. 51 *Chesilton Road,* S.W.
Dec. 14, 1894.	§‖Whetham, W. C. D., M.A., F.R.S. *Trinity College, Cambridge.*
Dec. 9, 1989.	WHIPPLE, R. S. (MEMBER OF COUNCIL). *Wexcombe, Luard Road, Cambridge.*
Nov. 28, 1902.	Whitaker, S. T. *Emanuel School, New Wandsworth,* S.W.
May 10, 1895.	§‖White, Prof. S. A. F., M.A. *King's College,* W.C.
June 8, 1906.	White, W. H. 2 *Bushy Park Villas, Hampton Wick.*
Feb. 24, 1899.	Whitehead, C. S., M.A. 3 *St. Catherine's Terrace, Hove, Sussex.*
Dec. 15, 1894.	§‖Wilberforce, Prof. L. R., M.A. *The University, Liverpool.*
Mar. 10, 1905.	Wilkinson, Ambrose, B.Sc. *University College, Nottingham.*
Mar. 13, 1908.	Williams, W. R., B.Sc. 3 *Elm Grove, Aberdare, S. Wales.*
Jan. 22, 1909.	Willings, W. B.Sc., 67, *Florence Road, Brighton.*
Jan. 27, 1907.	WILLOWS, R. S., M.A., D.Sc. (MEMBER OF COUNCIL). 128 *Rodenhurst Road, Clapham Park,* S.W.
June 12, 1903.	§Wilson, Prof. H. A., M.A., D.Sc., F.R.S. *McGill University, Montreal.*
Jan. 26, 1912.	Wilson, W., Ph.D. 33 *Cloudesdale Road, Upper Tooting,* S.W.
Jan. 22, 1887.	Womack, Frederick, B.Sc., M.B. *Bedford College, Baker Street,* W.
Dec. 15, 1877.	‖Wood, Sir H. T. ,B.A. *Society of Arts, Adelphi,* W.C.
	§*‖Wormell, Dr. R. *Roydon, nr. Ware.*
Jan. 24, 1884.	Worthington, Prof. A. M., M.A., F.R.S. 1 *The Paragon, Blackheath,* S.E.
May 14, 1909.	Wright, Arthur, M.I.E.E. 3 *Addison Road,* W.
Mar. 27, 1886.	Wright, A. R. *H.M. Patent Office, Southampton Buildings, Chancery Lane,* W.C.
Dec. 13, 1880.	‖Wrightson, Sir T., C.E. *Neasham Hall, Darlington.*
Oct. 28, 1892.	Wyatt, G. H., B.Sc. *Emanuel School, New Wandsworth,* S.W.

Date of Election.	
May 12, 1905.	Yorke, J. P. *L.C.C. School of Marine Engineering, High Street, Poplar*, E.
Feb. 27, 1886.	§‖Young, Prof. Sydney, D.Sc., F.R.S. *University Chemical Laboratory, Trinity College, Dublin ; and 12 Raglan Road, Dublin.*

Students.

Jan. 27, 1911.	Coursey, P. R. "*Gwynash*," 3 *Coniston Road, Muswell Hill*, N.
June 25, 1909.	Frecheville, G. 21 *Rutland Court, Knightsbridge*, S.W.
Jan. 26, 1912.	Guild, John. 72 *Castle Street, Battersea*, S.W.
Feb. 23, 1912.	Hobbs, F. C. 18 *Kildare Terrace, Bayswater*, W.
Jan. 26, 1912.	Kebby, C. H. 75 *Sterndale Road, West Kensington Park*, W.
Jan, 26, 1912.	Norman, H. A. 38 *Waldemar Avenue, Fulham*, S.W.
Nov. 10, 1911.	Richmond, V. C. 121 *The Avenue, Highams Park, Chingford.*
Dec. 20, 1910.	Thetford, C. F. 77 *Spencer Street, Catton, Norwich.*
Feb. 26, 1909.	Wall, W. G. P. 21 *Norwood Grove, Liverpool.*
Mar. 26, 1909.	Waters, J. W., B.Sc. 15 *Bloomfield Road, Kingston-on-Thames, Surrey.*
June 30, 1911.	Welch, A. P. *Enfield Villa, Putney Road, Enfield Wash.*
Dec. 20, 1910.	White, William. 4, *Rosenau Crescent, Battersea, S.W.*
Jan. 22, 1909.	Wood, D. O., B.Sc. *Vernon Lodge, Wellesley Road, Gunnersbury.*

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XVIII. *A Critical Examination of the Possible Causes of Terrestrial Magnetism.* *By* ARTHUR SCHUSTER, *F.R.S.*

PRESIDENTIAL ADDRESS, FEBRUARY 9, 1912.

1. The periodic and regular disturbances of the magnetic needle have been studied in various directions during recent years, but the origin of the earth's magnetisation has been left to take care of itself, partly, perhaps, because there is little chance, at present, to test any theory that might be proposed; but mainly, I believe, because difficulties cease to trouble us when we have become familiar with them.

I have no theory of terrestrial magnetism to propose, but believe that a useful purpose may be served by discussing various possibilities and restricting the range of choice.

To begin with, it is well to be clear as to what it is that we want to explain. We know that the earth behaves like a magnet with its axis inclined at an angle of about 12 deg. to the geographical axis of the earth. Is this near coincidence between the two axes merely accidental? If it is, our problem is much simplified, because we might then simply draw the inference that the iron inside the earth is in a state in which it can retain any magnetisation which is imposed upon it; but if we consider the coincidence as fundamental, we must face the task of finding a reason for it. In a choice which depends on a question of probabilities, we must give weight to every hint that we receive, such as that furnished by the secular variation, which seems to point clearly in favour of a real connection between the axes of magnetisation and of rotation; for the slow changes of the magnetic elements show that the magnetic axis of the earth is not stationary, but has moved during the last four centuries in such a way as to suggest a fairly regular rotation round the geographical pole. Our information is not sufficient to speak with certainty, but the arguments brought forward by Dr. Henry Wilde in favour of a uniform rotation of one system within the other cannot be lightly set aside. I believe, therefore, that I am in agreement with the great bulk of scientific opinion if I assume that the near approach between the geographical and the magnetic poles is intimately connected with the ultimate cause of terrestria magnetism.

2. The numerical value of the average intensity of the earth's magnetism in C.G.S. units is 0·08. I do not know of any experiments on the maximum magnetic intensity that can be given to a sphere, but it is not likely to be above 100. Accepting that number, a sphere of iron as large as the earth would have a magnetic moment a thousand times stronger than that of the actual earth. A comparatively small fraction of the earth, strongly magnetised, or a large portion of its mass magnetised to the one-thousandth part of full magnetisation, would therefore account for the observed magnetic forces.

This view of the origin of terrestrial magnetism is commonly rejected because the iron in our laboratories loses its power of magnetisation at a temperature which must exist at very moderate depths below the surface, even with the most modest estimate of the internal temperature of the earth.

It may be argued, however, on the other side, that the enormous pressure to which the iron in the earth is exposed may very well alter its molecular condition, and allow it to retain its magnetic properties. To form a rough estimate of the possible effects of pressure, we may suppose, for the sake of argument, that the critical temperature depends mainly on the molecular distance. If α be the coefficient of dilatation at the critical temperature and β the resistance to change of volume, it will require an increase of pressure $\beta^{-1}\delta p$ to balance an increase of temperature $\alpha\delta t$, which gives us $\alpha\beta$ for the ratio $\delta p/\delta t$. The product $\alpha\beta$ has a value of about 45×10^6 at ordinary temperatures, and if the order of magnitude be the same at the critical temperature of iron, a change of pressure of 100 atmospheres should produce a rise of over 2 deg. in the critical temperature of iron. Taking the temperature gradient near the surface of the earth as 0·01°C. per foot, and allowing for the conductivity of iron, which must be at least 16 times greater than that of the average rock, I find that an increase of depth, such that the rise in pressure is 100 atmospheres, would be accompanied by a rise in temperature of only one quarter of a degree. The effect of pressure on the critical temperature need, therefore, only be the eighth part of the estimated amount in order to prevent iron from losing its permeability through the internal heat of the earth. The assumption on which our estimate has been made is, however, quite arbitrary, and experiment only can show to what extent—if at all—pressure can raise the critical temperature of iron.[*]

Such experiments have been in progress under my supervision during the last three years. The pressure furnace in which the iron can be heated and tested magnetically was designed by Prof. Petavel, and the experiments are being carried out by Mr. F. Whaley. The difficulties are very great, and the results have hitherto not been very consistent, mainly, I believe, because small changes in the iron, due more especially to the absorption of gases, seem to have a considerable effect. In the first experiments there seemed, indeed, to be a rise in the critical temperature of about half the estimated amount, but when greater precautions were taken to ensure the absorption of gases, pressure seemed to act in the reverse direction, and in the later experiments this was consistently the case. Now, when we come to apply this result to our problem, we must bear in mind that the iron in the earth may not be chemically pure, and probably does contain large quantities of dissolved gases, and that we may be misled if we experiment only with pure iron and protect its surface against the entry of such gases. At present we must keep an open mind as regards the effects of pressure, but there seems no reason to exclude the possibility that iron inside the earth may be permanently magnetic.

3. Rejecting—prematurely, in my opinion—an iron core of the earth as the cause of terrestrial magnetism, scientific men have turned to electric currents inside the earth. But grave objections can be raised against the existence of such currents. They must either be maintained by permanent E.M.F.s or be a survival of an old state of things, and therefore constantly diminishing in intensity. Only the latter view can be defended, because there is nothing inside the earth except the unequal distribution of temperature to supply the energy necessary to overcome resistance, and thermoelectric force could not act so as to maintain the currents necessary for the purpose.

The idea that electric currents circulate inside the earth as a survival of an old state of things is, no doubt, attractive, and there is indeed some indication that the magnetic moment of the earth has diminished during the last century. The rate at which currents decay in a conducting sphere has been calculated by Prof. H. Lamb, whose results show that if the earth as a whole were to conduct as well as copper, it would take 10 million years to reduce the magnetic intensity in the ratio of $e : 1$. A hundred million years ago the earth would have been 27,000 times stronger in magnetic intensity than it is now, and assuming the conductivity to be that of iron, this number would have to be multiplied by 10^{12}. Considering that the crust of the earth has probably been solid during not less than 500 million years, we can see that these currents must originally have had enormous intensities. But even disregarding the difficulty presented by the magnitude of the currents, we should not find it easy to account for their existence. It might be suggested that swarms of electrons, to which magnetic storms are often ascribed, might have been circulating round the earth with sufficient velocity and in sufficient density to establish the magnetic field of the earth. Such a view will not, however, bear examination, because it seems impossible to account for the energy which at some time or other must have been supplied to set the swarm in motion, and also because electrostatic repulsions would quickly dissipate any swarms which are sufficiently dense to produce appreciable magnetic effects. I have discussed this matter in a recent communication,* and it need not be further entered into here. To sum up: the difficulties of explaining terrestrial magnetism by means of electric currents seem to me to be insuperable.

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[*] It has been pointed out to me by Dr. S. W. J. Smith that the above argument is open to objection, because with iron of different degrees of purity a diminishes near the critical temperature, and in some cases becomes even negative. This suggests that iron contracts when it changes from the magnetisable to the unmagnetisable state, and this, again, would indicate that pressure *lowers* the critical temperature in accordance with the majority of the experiments. I acknowledge the force of the criticism, which shows how much further experiments with pure iron and other magnetisable metals are required.

4. Some reference ought, perhaps, to be made to the view, which has recently been revived, that terrestrial magnetism is induced in a magnetic core of the earth by outside forces which are still acting. This view is untenable, because not more than 1 per cent. of the observed magnetic forces comes to us from outside. A magnetic sphere, placed in a uniform field, would increase the primary field at its poles and diminish it at its equator, and whatever the permeability, the forces at the equator can never be reversed. While at the poles, even an infinite permeability could only double the primary field. Less than one-hundredth part of the earth's magnetism can, therefore, be ascribed to this cause.

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* Proc. Roy. Soc., Vol. **85**, p. 48.

5. All theories which are based on magnetic matter inside the earth share in the weakness that additional assumptions have to be made in order to account for the near coincidence of the magnetic and geographical axes. This difficulty is, no doubt, not felt so strongly in the case of the hypothetical swarm of electrons, which may reasonably be supposed to circulate in the equatorial plane, and it is probably this advantage which has brought the theory into favour.

Whatever view we adopt, we cannot escape introducing some new phenomenon for which there is at present no experimental evidence, and we are justified, therefore, if we go a little further afield, and ask whether there may not be a definite connection between the rotation of the earth and its magnetisation. Without laying claim to any originality for an idea which has probably occurred to many, I am not aware of any published statement of the question prior to a lecture I gave at the Royal Institution in 1891, on eclipse observations. " The form of the corona," I remarked at the end of the lecture, " suggests a further hypothesis, which, extravagant as it may appear at present, may yet prove to be true. Is the sun a magnet ? We know that a body at such a high temperature cannot be magnetisable, but may not a revolving body act like a magnet, and may not the earth's magnetisation be similarly due to its revolution about its axis ? It can be shown that, although a revolving body may act like a magnet sufficiently to account for terrestrial magnetism, our instrumental appliances would yet be quite insufficient to allow us to detect the phenomenon in bodies set into rotation artificially on the surface of the earth, so that there is no *a priori* reason against the hypothesis." In the following year I returned to the subject in my presidential address before Section A of the British Association, putting again the question : " Is every large rotating mass a magnet ? "

Towards the end of the same year, Lord Kelvin discussed magnetic relationships between the sun and the earth in a presidential address to the Royal Society, in which the following passage occurs : " Considering probabilities and possibilities as to the history of the earth from its beginning to the present time, I find it unimaginable but that terrestrial magnetism is due to the greatness and the rotation of the earth."

6. In what way may a body become a magnet merely by virtue of its rotation ? There are at least two ways. A rotating molecule may be a magnetic doublet, or a moving mole-

cule may behave as if it carried an electric charge. If rotation is the determining cause, we may either suppose that a definite magnetisation is connected with a definite angular velocity or we may imagine that rotation merely sets up a magnetising force which may or may not produce a magnetic field according to the nature of the body. Let us separately examine those alternatives, and begin with the supposition that rotation and magnetisation are connected directly.

Consider two spherical bodies (Fig. 1), and compare the magnetic effects of two similar elements in corresponding positions P and *p*, at corresponding points Q and *q*. The magnetisation being equal, the magnetic moments of P and *p* are proportional to the volumes and therefore to the cubes of the linear dimensions. As the effects at Q and *q* due to elements having equal moments are inversely proportional to the cubes of the distances, equal magnetic intensities of the spheres will cause equal magnetic forces at corresponding points. If, therefore, the magnetic intensity were determined solely by the angular velocity, the magnetic forces at the surface of each sphere would be the same for the same angular velocity, no matter what the size of the sphere might be. Magnetic forces equal to those of the earth could therefore be obtained in our laboratories at the surface of any body that turned round once in 24 hours, and if the effect be proportional to the angular velocities, 10 rotations per second would set up a magnetic field greater than any it is at present in our power to produce. We may therefore dismiss the direct correlation of angular velocity and magnetic force as an explanation of terrestrial magnetism.

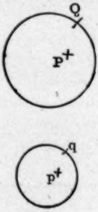

FIG. 1.

7. The alternative hypothesis, that a moving molecule carries a magnetic field with it, as if it were electrically charged, demands a closer examination. The method of dimensions may again be applied to tell us whether appreciable effects might be expected in laboratory experiments if terrestrial magnetism were due to such a cause. The magnetic effect of current elements at P and p would still be proportional to the volumes, but, in distinction from the previous case, they would, at corresponding points, Q and q, vary inversely as the *squares* of the distances. To give equal effects, the linear velocities at corresponding points would have to be inversely proportional to the linear dimensions of the system. It follows that the magnetic effects of rotating spheres at corresponding points would be proportional to ωr^2, if r be the radius of the sphere and ω the angular velocity. The linear velocity at a point of the earth's equator is 46,510. Taking a sphere of 10 cm. circumference, and a rotational frequency of 1,000, an equatorial velocity of 10,000 might possibly be produced, but would probably exceed the limits of safety. Under these experimental conditions, the magnetic forces at the surface would be smaller than the earth's force in the ratio of 3×10^9 to 1. As we cannot detect so minute a magnetic force, there is no prospect of verifying our hypothesis by experiment.

If magnetisation and rotation go together, the sun and the planets would all be magnetic, and in the case of the sun we have some reason to believe that this is so. The intensity can be calculated from that of the earth, if we know the manner in which it depends on the dimensions of the system, and according to our present supposition it should be proportional to the product of the maximum linear velocity and the radius. The linear velocity at the sun's equator and the radius of the sun being respectively 4·3 and 102 times greater than the corresponding values for the earth, the magnetic intensity of the sun should be about 440 times greater than that of the earth. It is likely that our experimental methods will soon be able to allow us to detect the Zeeman effects produced by a field of this strength. So far our hypothesis has not involved us in any difficulties; these begin when we come to examine the effects of the earth's motion through space.

8. Independently of any application to cosmical physics, it is interesting to discuss the magnetic effects of a rotating electrified sphere. The problem is simple enough when the exploring magnet is at rest, but complications arise when its

points of support are fixed to the sphere. It may easily be shown that a sphere charged with a quantity of electricity, E, and rotating with angular velocity ω produces a magnetic potential which at a point of its surface, having a polar distance θ, is $\frac{1}{5}E\omega \cos\theta$ or $\frac{1}{3}E\omega \cos\theta$, according as the charge is distributed uniformly through the volume or over the surface. The potential here considered is that which can be explored by a stationary magnet, but special treatment is required when the forces are referred to a system moving with the sphere. According to our present knowledge, a translatory velocity of the sphere would produce a magnetic field which would cease to be noticeable to an observer moving with the sphere. There must, therefore, be a compensating effect neutralising the primary one when the supports of the exploring magnet are fixed to the sphere. This compensating effect, which is complete when the sphere has only a translatory velocity, must also be taken into account when the sphere rotates. A closer examination of this point, to be given later, shows that, in order to get rid of the effects of translation, the magnetic field due to a rotating body has to be modified in such a manner that it no longer agrees with the observed facts. The theory, though not absolutely killed by this objection, can only meet it if it is altered so materially that it takes a different form altogether. For the present, therefore, we may dismiss it.

9. Let us return to the discussion of the first alternative, but instead of assuming that rotation causes magnetisation independently of the nature of the body which rotates, let us suppose that it only sets up a magnetic force, which may or may not produce an appreciable magnetisation according to the nature of the body. We now get rid of the objection which was fatal to the theory in its original form, on the ground of the excessive magnitude of the magnetic effects whenever a body rotates even with a small angular velocity, because the material now has to be taken into consideration. On the other hand, we lose something, because the burden of terrestrial magnetism would again fall on iron, and the old difficulty depending on the temperature of the interior of the earth would reappear. Further, as the iron of our laboratories does not, when set into rotation, become magnetised to the same degree as the earth, we should be driven to assume a peculiar state inside the earth in which iron or some other metal has properties which we cannot reproduce in our laboratories,

Nevertheless, the magnetising effects of rotation deserve a more thorough examination both theoretically and experimentally, because the difficulty only concerns the magnitude of the observed effect. It is, on the contrary, quite in accordance with our present views that every rotating body should be subject to a magnetising force along the axis of any rotation that may be impressed upon it. If magnetisation be due to a circulation of electrons within the molecules, these should to some extent behave like gyrostatic compasses, setting themselves parallel to the axis of rotation of the body which contains them. Further properties of molecular constitution have, however, to be specified before we can say whether any actual magnetisation results. If the electron is free, the main result, I believe, would be only a magnetic precession round the axis of rotation, and in this we may find a powerful argument, because the same cause which gives us the magnetic force also gives us the secular variation. Eleven years ago, I showed in a Paper read before this Society, that if terrestrial magnetism were due to electric currents in the earth, a magnetic precession agreeing with the secular variation would result, but that the period of rotation of the magnetic axis round the geographical pole would be far too great to serve us in accounting for the secular variation. The length of the period depends on the size of the sphere, or rather on the thickness of the shell within which the electric currents are supposed to be circulating, and I pointed out that, in order to reduce the period to 500 years, the thickness of the shell would have to be reduced to molecular dimensions. Towards the end of the Paper, the following passage occurs: "The fact that a current sheet of molecular dimensions would show a magnetic precession of angular velocity comparable with that of the secular variation suggests the possibility of the phenomena being rather of a molecular than a molar character. If terrestrial magnetism is due to the rotation of the electron round the atom, a precession of the required amount might be produced. But the subject is not capable of theoretical treatment without making some assumption of too speculative a character to be introduced here."

10. To complete, so far as is possible at present, the discussion of the secular variation, I may refer to a Paper in the first number of "Terrestrial Magnetism," in which I showed that a sphere, rotating in a conducting medium at rest, and magnetised along an axis inclined to the axis of rotation, induces

currents in the medium, the magnetic forces of which tend to turn the magnetic axis in accordance with the secular variation.

There is a further effect tending to destroy any component of magnetisation at right angles to the axis of rotation, so that whatever the position of the magnetic axis might have been to begin with, it will tend more and more towards the geographical axis. The latter effect increases with the conductivity of the outer medium, but the couple which tends to displace the magnetic axis vanishes when the conductivity is infinitely great, and in the case of the earth would be practically zero, with a conductivity of 10^{-8} C.G.S. The maximum rotational effect takes place with a conductivity of $2{\cdot}4 \times 10^{-14}$, and this is about the same order of magnitude as the conductivity I obtained for the higher regions of the atmosphere from a discussion of the Diurnal Variation of Terrestrial Magnetism. It is, however, much greater than that of atmospheric air near the soil, which is of the order 10^{-24}. The magnetic force due to any induced currents in space, if appreciable, should, of course, be observable, but they have to be disentangled from the primary forces of the earth. This demands a more accurate survey of the earth as a whole than we possess at present, and we look forward to the magnetic survey of the "Carnegie" for the required data. So far as our present observations go, the calculations of Prof. Ad. Schmidt do not lend support to the existence of appreciable currents induced in the manner contemplated in this paragraph. A partial agreement might be obtained if the medium in which the currents are induced circulated round the earth with a greater angular velocity than that body, but even then the observed difference of phase remains irreconcilable. It must further be understood that my calculation was based on the permanent state to which the system tends when the sphere is kept rotating; it would have to be modified if, owing to the translatory velocity, there were a relative motion between the earth and the matter in space in which the currents are induced.

11. Briefly reviewing the arguments, we may assert that little can be said in favour of the hypothesis which ascribes the origin of terrestrial magnetism to electrical currents inside the earth. It fails to account for the existence and maintenance of these currents, and also for the changes which are indicated by the secular variation. If we base our theories on magnetic iron, we must reconcile the high temperature of the interior of

the earth with the maintenance of the magnetic properties of iron. This difficulty might be removed if we could show that pressure raises the critical temperature, but experiments have not hitherto led to decisive results. The relationship between the magnetic and geographical axes and the secular variation might be explained if the earth were surrounded by a region which conducts electricity and does not take part in the earth's rotation, while it follows the earth in its motion through space, but the observed magnetic effects, so far as they depend on magnetic forces having their centres outside the earth, do not agree with those to be expected from currents induced in a medium at rest by the earth's rotation. At present this evidence is not conclusive on account of the uncertainty of the observations. If neither hypothesis agree with the fact, we must look for some novel and independent cause, such as possible effects of rotation in directing the magnetic axes of molecules. But rotation alone cannot determine magnetic intensity, because, if it did, the effects of rotating bodies would be too great to have escaped detection hitherto. An alternative hypothesis, that a moving molecule behaves magnetically, as if it carried a charge, gets us into difficulties when we take account of the translatory motion of bodies through space. There remains the possibility that rotation sets up a magnetising force which acts differently on different substances. The analogy with the gyrostatic compass presents itself, and the natural manner in which the secular variation would be explained according to this theory should count in its favour.

As regards further experiments and observations, I should place as the most important those intended to bring out—if it exists—a magnetic effect of rapidly rotating bodies. Such experiments are at present in progress in the Physical Laboratories of the University of Manchester. The further investigation of the effects of pressure on the temperature at which iron loses its magnetic properties may—it is hoped—define the conditions under which magnetic iron can exist in the interior of the earth. As regards observation, it would be most helpful if it could be definitely settled whether the sun is a magnet, and if so, what is its magnetic moment. The evidence at present rests entirely on the form of certain rays of the corona, which—assuming that they indicate the path of projecting particles—seem to be deflected as they would be in a magnetic field, but this evidence is not at all decisive.

12. I have referred to the possibility that a neutral molecule

moving through space might carry a magnetic field with it, as if it were charged electrically, and, assuming this to be the case, it was easy to calculate the magnetic forces due to a rotating body. The hypothesis was set aside owing to the absence of any effects which could be referred to the translatory motion of the earth through space. Now, an electrically charged sphere would not act on a magnet carried with the sphere, so long as there is only translation, and it remains to be shown whether the compensating action which takes place in this case might also do so in the other, without materially interfering with the observed effects of rotation. Assuming that no electrodynamic action can exist between an electric charge and a magnetic pole, so long as there is no relative motion the observable forces cannot be changed by giving the whole system a uniform motion in any direction. Let us, therefore, take a sphere charged with a quantity of electricity, E, distributed uniformly throughout its volume, and calculate the force acting on unit pole placed on its surface, the observer moving with the sphere. Endow the whole system with a velocity equal and opposite to that of the point at which the magnetic effect is required. This will reduce the point of observation to rest. If that point lies at an angular distance, θ, from the pole, the velocity required is $a\omega \sin \theta$ parallel to the circle of latitude. The magnetic force due to a uniform translation is connected with the electric force by a simple relation,* which, when applied to the present case, becomes $E\omega \sin \theta/a$, acting along a meridian from south to north, if the product $E\omega$ is positive. If the point of observation is at a distance, r, greater than a, the force is $E\omega \sin \theta/r$, so that, outside the sphere, the magnetic force due to the translation is subject to a potential, $E\omega \cos \theta$. The rotation of the sphere sets up a potential, $E\omega a^2 \cos \theta/5r^2$, giving at the surface a horizontal force towards the south equal to $E\omega \sin \theta/5a$. The total southerly horizontal force is, therefore $-4E\omega \sin \theta/5a$, and the vertical force downwards is $-2E\omega \cos \theta/5a$. If the sphere be charged negatively and rotate in the positive direction, the north end will point southward and downwards, if charged positively it will point northward and upwards, and it could never be made to agree with the lines of force observed on the earth. By applying Gauss's theory to the results of his experiments, an observer placed on an electrically charged and rotating sphere would come to entirely erroneous conclusions. If he were to assume a magnetic potential made up of the sum of two terms, one proceeding by positive and the other by negative powers of r, the magnetic forces observed at the surface would agree with the potential $-\frac{2}{15}\left(\frac{a^2}{r^2}+\frac{5r}{a}\right)\mathrm{E}\omega\cos\theta$, and he would conclude that the part of the potential which vanishes at the centre, and which is commonly ascribed to outer causes, is five times as large as that part which is zero at infinity. But, if the same observer could pursue his observations at such distances from the earth's centre as would allow him to distinguish between r and a, he would find that he would have to correct his results, and write for the potential $\frac{1}{5}\left(\frac{a^2}{r^2}-5\right)\mathrm{E}\omega\cos\theta$, From this he would draw the further erroneous conclusion that the space outside the earth contains magnetic matter having a volume density, $\mathrm{E}\omega\cos\theta/2\pi r^2$.

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* Proc. Roy. Soc., Vol. 85, p. 48.

13. Some of the difficulties encountered in the previous paragraphs would disappear if we were free to assume a separation of electricities in the interior of the earth, an outer shell and an inner core containing volume electrifications of opposite kinds. We might consider, for instance, how, owing to centrifugal force, the free negative electrons within the earth would be driven towards the equator. The resulting magnetic forces in this case would not, however, be of the kind required to explain terrestrial magnetism, because, in order that the earth should be equivalent to a homogeneously magnetised sphere, the volume density must be constant within each concentric shell. The question raises very delicate considerations, which deserve attention. How does gravity affect the negative electron or the positive charge? In the discussions of the paths of electrons projected from the sun or drawn towards the earth by magnetic force, gravity has hitherto been left out of account altogether. Whether the results obtained are affected by the omission it is difficult to say, but the problem must be faced before any real progress can be made in questions concerning the electric economy of the earth. Lorentz's theory of gravitation seems well adapted to serve as basis of discussion, especially if we put it into a somewhat more general form. Assuming that the attraction between a negative and a positive unit of electricity is stronger than the repulsion between two units of the same sign, gravitation might be explained if the excess of attraction were about $0{\cdot}36\times10^{-36}$, the attrac-

tions and repulsion measured separately being unity.* What strikes us most is the extraordinary precision to which different electrons must be adjusted in order that so small a fraction of their electric force should yield a force possessing the constancy of gravitation. Now, if such small quantities are of importance other questions arise. Assuming that molecules are made up of definite positive and negative electric charges, we are at liberty to define a quantity of electricity by the number of elementary charges it contains. But would such a definition be identical with the usual definition of equal forces at equal distances ? In other words, would a certain number of charges when placed at unit distance from an equal number of similar charges repel each other with equal forces, whether the charges were positive or negative ? This would no doubt be the case to a high degree of accuracy, but we may hesitate to assert that equality holds down to the 10^{-36}th part of the whole. In order to leave the question open, assume the repulsion between two negative elementary charges to be $1-\alpha$ and that between two positive charges $1-\beta$, while the attraction between an elementary negative and positive charge is unity. If a molecule contains n negative and n positive charges, a body containing N molecules will exert an attraction expressed by $\alpha n\mathrm{N}$ on a negative and an attraction $\beta n\mathrm{N}$ on a positive charge. The attraction on a neutral molecule, which we should interpret as gravitational attraction, would be $\mathrm{N}n^2(\alpha+\beta)$. In Lorentz's original theory $\alpha=\beta$, and gravitation would be equally divided between positive and negative charges. It is to be noticed that the proportionality between mass and gravitation is here abandoned so far as the elementary constituents of matter are concerned, because negative electrons would be acted on gravitationally far more than their mass would lead us to expect. If we wish to preserve the relationship between mass and gravity, we should have to give to α a considerably greater value than to β. In our ignorance of these matters it may be well to leave the question open, and even to admit the possibility that one of the quantities is negative, which would mean a gravitational repulsion on one of the units of electricity. When α and β are unequal, a large mass containing an equal quantity of elementary charges of both kinds would attract one kind more than the other. It would, in fact, appear to act as a solidly electrified body. At first it might seem that the apparent negative charge of the earth confirms this view, but surrounded as it is by a conducting medium, there can be no electrostatic force outside, which means that a charge would accumulate on the surface neutralising exactly the effects of the inner mass. Intricate questions arise, however, when we ask ourselves whether, owing to a difference in gravitational force, the different layers of the interior of the earth would not, in course of time, assume volume charges which would vary from the centre towards the surface. Magnetic forces might result in this manner, but the whole subject is too obscure at present to allow of any definite theory being formed. In conclusion, we shall do well to remember that though it seems necessary to bring in some new cause to explain the facts of terrestrial magnetism, any theory which bases it directly on rotation will have to explain why the magnetic axis does not coincide accurately with the geographical axis. A want of homogeneity or of symmetry in the internal constitution of the earth might account for the displacement if it were permanent, but the secular variation shows that this is not the case. If we could imagine the earth's axis of rotation to have shifted in course of time, the present state may be partly due to a survival and partly to a progressive readjustment which would tend to bring the two axes into a coincidence through a path which is spiral, owing to a precessional effect.

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* Take two neutral molecules of hydrogen and let the charge of each kind on each of them be e. If c^2e^2 be the repulsion at unit distance between charges of the same kind and $(1+\alpha)c^2e^2$ the attraction between charges of opposite kinds (c is the velocity of light), the residual attraction which we should interpret as gravitation is $2\alpha c^2e^2$. If m be the mass of a molecule of hydrogen, this attraction must be equal to γm^2, where $\gamma=6{\cdot}5\times10^{-8}$. The value of α is therefore $\gamma m^2/2c^2e^2$ or, substituting numerical values $0{\cdot}36\times10^{-28}\times m^2/e^2$. The smallest possible value of e is that appearing in the electrolysis of water, and we therefore get the largest allowable value for α by substituting 10^4 for m/e.

ABSTRACT.

In forming any theory of the cause of terrestrial magnetism, the first question that arises is whether we are to consider the near coincidence of the geographical with the magnetic axis as accidental or significant. The secular variation argues for the second alternative, and scientific opinion has always favoured the view that there is a definite reason for the close approach of the magnetic to the geographical pole.

The view that iron is responsible for the observed magnetic field has generally been put aside, because iron loses its magnetisation at temperatures lower than those which all are agreed must hold at moderate depths below the surface. But the objection raised on this ground disregards the possibility that the critical temperature of iron may be raised by pressure. It may be shown that if that temperature depends on molecular distance, and the ratio of compres-

sibility to thermal dilatation is assumed to be independent of pressure, the rate of increase of temperature with the depth is less than the rate of increase of the critical temperature, so that iron would retain its magnetic properties. Experiments on the effects of pressure on the critical temperature of iron have been in progress during the last four years, but have not hitherto led to a decisive result. For the present we must, therefore, keep our minds open to the possibility that the iron contained in the earth is magnetisable.

The alternative opinion that electric currents circulating inside the earth are the cause of electric currents is next examined. Though this view seems to be favoured by scientific opinion, it has to overcome formidable difficulties before it can be accepted. Are the electric currents permanent, or are they only survivals of a former state of things which is dying out ? If permanent, we must account for the electromotive forces, and we know of no such forces which could act in the manner required. If the electric currents are survivals, their intensity when the earth crust first formed must have been at least 10^{12} times as strong as it is now, and the difficulty of accounting for their original production would be correspondingly increased. In the opinion of the author, the difficulties which stand in the way of basing terrestrial magnetism on electric currents inside the earth are insurmountable.

The question whether the rotation of the earth may be responsible for its magnetic field is next examined, and different possibilities are considered. If the rotation of a sphere independently of the chemical nature of the substance were to produce a magnetic field, it is shown by the theory of dimensions that the field at the surface of the sphere would be proportional to the angular velocity and independent of its size. The effect of rotating bodies set into rotation in our laboratories would give, therefore, magnetic forces equal to those of the earth with one rotation per day, and with angular velocities of the order of one per second the effects would be so large that they could not have escaped detection.

A second supposition, that a neutral molecule in its motion behaves as if it carried a charge, leads to the result that if terrestrial magnetism be due to such a cause, the effects to be expected in any laboratory experiment that we could propose are far too small to be detected. But the theory is finally negatived by the additional conditions which must be imposed to destroy the effects of translation independently of rotation. A distinction must here be made between possible magnetic effects of a rotatory body on a magnet which is at rest and on one the supports of which are fixed to the rotating body. Returning to the effects of rotation, the possibility is considered that rotation instead of directly determining magnetic intensity, determines magnetic force which may or may not cause magnetisation according to the nature of the body. This view deserves attention, because there is some theoretical foundation for it, inasmuch as molecules may be considered to behave in a manner analogous to that of a gyrostatic compass. The theory would also explain in a natural manner the secular variation by a precessional motion of a magnetic molecule. On the other hand, the theory would have to explain why the iron inside the earth becomes—by rotation—more strongly magnetised than the iron in our laboratories. There is, of course, always the possibility of some substance being subject to the effects of rotation in a much higher degree than iron. Such questions can only be settled by experiments which are now in progress. Other theories of the secular variation are considered.

In conclusion, the question is raised whether the negative electron is subject to gravitation, and if so, whether in comparing the weight of a negative electron with that of the unit positive charge, we must consider the proportionality between gravitation and mass to hold. In Lorentz' theory of gravitation this is abandoned. It is pointed out how fundamental questions have a bearing on the problem of terrestrial magnetism.

XIX. *Krypton and the Auroral Spectrum.* By T. W. PAGE, *A.R.C.S., B.Sc.*

RECEIVED NOVEMBER 14, 1911.

IT has been pointed out by Runge (" Astrophysical Journal," Vol. XVIII.) and further by Baly (" Astrophysical Journal," Vol. XIX.) that there exists a notable correspondence between the spectrum of the aurora and that of krypton.

The most important coincidence is that of the green auroral line $\lambda=5570$ (Sykora) with the green krypton line $\lambda=5570{\cdot}5$ (Baly), and it has been suggested that the aurora is due to some electrical discharge in the upper atmosphere in the carrying of which krypton is directly concerned.

Even if this identity were established, the absence of the yellow krypton line $\lambda=5871{\cdot}12$ would demand an explanation. If under any kind of electrical discharge the yellow line were absent from the spectrum of krypton, or if its intensity compared with that of the green line were materially reduced, it would be possible to ascribe the auroral line to krypton with greater confidence.

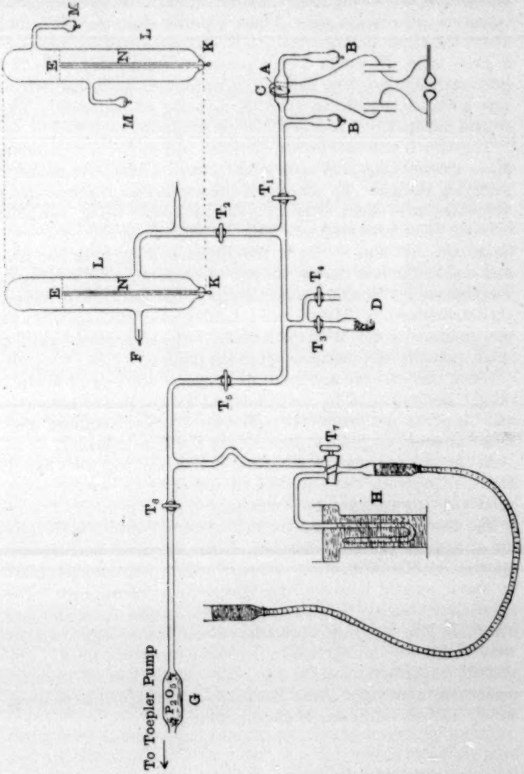
The experiments here described were undertaken at the suggestion of Prof. Hon. R. J. Strutt, and were performed under his direction, with a view to finding if, under any circumstances, a spectrum of krypton could be obtained in which the intensity of the yellow line was less than that of the green line to any marked extent. The first type of discharge was the ring discharge, and the second the canal rays in krypton. The apparatus used is shown in the accompanying diagram.

The gas used in the first instance was obtained from liquid air residues. It was sparked over mercury with excess of oxygen in presence of KOH, and the excess of oxygen was removed by means of phosphorus. The resulting gas was rich in krypton. It was introduced into the apparatus at H, and was confined in the tube A, the last traces of nitrogen being removed by means of a discharge between electrodes (B), consisting of an alloy of potassium and sodium.

The green and yellow lines of mercury were usually present, especially in the light from the wider portions of the tube, but by keeping the tap T_1 closed, and passing the discharge for some time, the lines became less intense and eventually quite faint.

The tube A was surrounded with a coil of wire, C, of about eight turns, which was connected with the outer coatings of two

Leyden jars, the inner coatings of which were connected to the discharging knobs of a large Wimshurst machine. The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spectrum was observed by means of an ordinary single prism spectrometer, and a small direct vision prism spectroscope was also used. In all cases the green and yellow krypton lines appeared equal in intensity. The canal rays were obtained in the tube L. The electrode E consisted of a perforated aluminium disc which fitted the tube tightly. The perforations consisted of parallel slits about 1 millimetre in width, and about the same distance apart. It was supported by means of a glass tube, N, down which passed a connecting wire, the portion near the disc being of aluminium, and that which was brought out of the glass at K being of platinum. The second electrode consisted of a single platinum wire, F.

The tube L was also provided with a pair of K-Na electrodes, M, as shown in the side view of the tube. These were used for purifying the gas. By making F the anode and E the cathode the canal rays were obtained, the pressure being suitably reduced by surrounding the tube D, containing charcoal, with liquid air. At first, owing to the presence of argon in the gas, and also to the fact that at the low pressure required hydrogen was emitted by the aluminium electrode, the spectrum obtained was unsatisfactory. The former difficulty was overcome through the kindness of Sir William Ramsay, who generously gave a small quantity of practically pure krypton.

Using this gas, the spectrum of krypton was clearly visible, though accompanied by the spectra of hydrogen and mercury, and the green and yellow krypton lines were both present, and, as far as could be judged, were about equal in intensity.

At the low pressure required the luminosity was very small, and the spectrum was observed by directing the spectroscope vertically downwards, above the discharge tube.

The result of these experiments does not decide definitely for or against the identification of the krypton and auroral spectra, such identification resting chiefly upon the agreement in wave length between the green lines mentioned. The agreement between the faint lines of the aurora spectrum and others in the krypton spectrum would carry little weight were it not for the agreement between the green lines. The present experiments show that the yellow line of krypton persists in about the same intensity as the green one under widely varied conditions of stimulation.

XX. *A Balance Method for Comparison of Quantities of Radium and Some of its Applications.* By *Prof.* E. RUTHERFORD, *F.R.S., and* J. CHADWICK, *B.Sc., University of Manchester.*

RECEIVED FEBRUARY 21, 1912. READ FEBRUARY 23, 1912.

IN many radio-active measurements it is now of great importance to determine with accuracy the quantities of radium or of radium emanation employed in various investigations. The International Committee appointed by the Congress of Radiology at Brussels, in 1910, have now in preparation an International Radium Standard, which will contain a known weight of pure radium salt. All quantities of radium will ultimately be expressed in terms of this International Standard. It is consequently of great importance to develop reliable methods of comparing quantities of radium accurately in terms of the primary standard. The most suitable method for this purpose is to compare the γ-ray activities of the two specimens. If the radium is enclosed in a sealed tube, the γ-ray activity reaches a practical maximum after two months, and the intensity of the penetrating γ-rays which are emitted serves as a definite measure of the quantity of radium present. The greater part of the γ-rays are emitted not by radium itself but by radium C, and recent investigations by Moseley and Makower * have shown that under ordinary conditions about 11·5 per cent. of the total γ-ray activity of radium is to be ascribed to radium B. The γ-rays from the latter are on the average much less penetrating than those from radium C, and are completely absorbed by a lead screen 2 cm. thick.

In making a comparison, it is essential to know that the radium compound contains only radium and its products and no meso-thorium or radio-thorium. Both the latter substances emit γ-rays of about the same penetrating power as those given out by radium, and without a special examination it is difficult to tell whether the thorium products are absent. Since meso-thorium and radium are always separated together, and are chemically closely allied, it is impossible to isolate pure radium compounds from minerals containing both uranium and thorium. Fortunately the uraninite deposits at Jochimstahl contain only a trace of thorium, so that the radium separated from this ore can be obtained practically free from mesothorium.

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* Moseley and Makower, "Phil. Mag.," January, 1912.

The γ-ray activity of radium preparations has been usually compared by electroscopic methods. The electroscope is either composed of lead plate about 3 mm. thick or is completely surrounded by a lead cover of this thickness. Under such conditions the primary β-rays are completely stopped by the lead and the ionisation in the electroscope is due to the more penetrating γ-rays and to the β-radiation to which they give rise. The rate of movement of the gold leaf of the electroscope between two fixed points of the scale of the observing microscope is taken as proportional to the intensity of the γ-radiation. With a well-constructed electroscope it is possible to compare approximately equal quantities of radium with a probable error of not more than $\frac{1}{2}$ to $\frac{1}{3}$ per cent. If, however, the quantities of radium differ largely in amount—for example, in the ratio of 10 to 1—the ionisation currents in the electroscope for a given distance of the radium from the electroscope differ in this ratio, and the comparison cannot be made with the same accuracy as in the case of nearly equal quantities. The accuracy of the method obviously depends upon the certainty of complete saturation of the ionisation in the electroscope in the two cases. Unless a special investigation be made to determine the variation of current with voltage under the experimental conditions it is difficult to be certain that saturation has been reached. Any method of comparison of the intensity of the γ-rays from two preparations by the direct measurement of the ionisation they produce is subject to the same limitations.

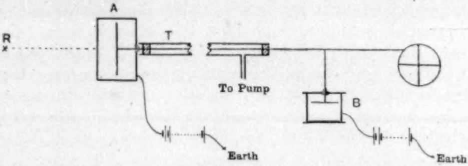

FIG. 1.

In order to avoid this difficulty the writers have developed a balance method for accurately comparing quantities of radium by their γ-ray effects. The general principle of the method is clearly seen from Fig. 1. The γ-rays from the radium preparation at R enter the ionisation chamber A. The saturation ionisation current in A is balanced against an equal but opposite ionisation current due to a standard preparation of uranium oxide in the vessel B. The accuracy of the balance is tested by a Dolezalek electrometer. In practice, the ionisation current due to B is kept fixed, and the standard radium preparation is moved until a balance is obtained. The standard is then removed to a distance, and a balance is again obtained for the second preparation. Since the intensity of the γ-rays from a point source of radium fall off nearly inversely proportional to the square of the distance, the relative γ-ray activity can be deduced from the distances of the radium from the vessel A in the two cases.

The construction of the ionisation chamber is shown in Fig. 2A. It consists of a lead cylinder 2 cm. thick 10 cm. long and 15 cm. internal diameter. One end of the cylinder is closed by a lead plate 2 cm. thick, the other, where the γ-rays enter, by a lead plate whose thickness can be varied between 1 mm. and 2 cm. of lead. The plate ordinarily employed is of thickness 1 cm. The interior electrode consists of a circular aluminium plate 1 mm. thick and 12 cm. diameter. This is insulated from the cylinder by a sulphur stopper, provided with a guard ring in the usual manner.

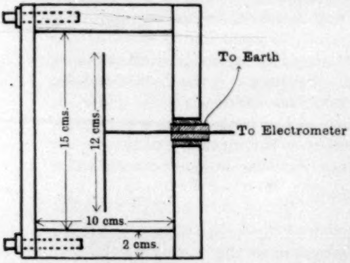

FIG. 2A.

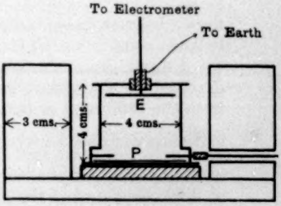

FIG. 2B.

The construction of the uranium vessel B is shown in Fig. 2B. It consists essentially of a brass cylinder 4 cm. high and 4 cm. diameter. On the bottom is placed a metal plate, P, which is covered with a uniform film of uranium oxide prepared by McCoy's method. Above the plate is an iris diaphragm of the type used in microscopes. The aperture of the diaphragm could be varied by means of an attachment between 0·5 cm. to 3 cm. By this means the ionisation in B can be varied over a considerable range. The base of the vessel is screwed on to an ebonite block which is fastened to a lead plate 1 cm. thick. The vessel is protected from external γ-rays by surrounding it by a lead cylinder 6 cm. high and 3 cm. thick. The interior electrode E is a circular copper plate insulated by a sulphur stopper provided with a guard ring. The electrode in A (Fig. 1) is connected with the electrometer by a wire passing through a thick lead tube, T, exhausted by means of a water pump to a pressure of about 2 cm. In this way the ionisation of the air surrounding the wire due to an external source of γ-rays was made very small. A was kept charged to -120 volts and B to $+200$ volts. The electrometer needle was charged through a phosphor bronze suspension to 200 volts.

The radium preparation was mounted on the carriage of a photometer bench, symmetrically in regard to the face of the ionisation chamber, and was moved along the graduated bench until a balance was obtained. Suppose r_1 is the distance of the balance point from the front surface of the air in A, and $r+a$ the distance from the surface of the back plate, a being the depth of the ionisation chamber. The intensity of the γ-rays from the radium at a distance, r, from the source within the ionisation chamber is proportional to $\frac{e^{-\mu_1 d} . e^{-\mu r}}{r^2}$, where μ_1 is the average coefficient of absorption of the γ-rays in the lead plate of thickness d and μ the absorption of the γ-rays by the air. The actual ionisation observed in the chamber A arises in all probability mainly from the β-rays set up by the γ-rays on the front and back plates. Our knowledge of the amounts and distribution of these excited β-rays is at present too inadequate to calculate the intensity of the radiation at each point of the ionisation chamber with certainty. If, however, the distance a is small compared with r it is to be expected that the ionisation in the chamber of depth a should be very nearly proportional to $1/r(r+a)$. It will be seen later that this relation is borne out by experiment. Assuming this formula, it is a simple matter to compare the γ-ray activities of two quantities of radium by determining the distances r_1, r_2 required for a balance. Suppose the standard preparation containing S milligrammes of radium balances at a distance r_1 from the inner surface of the front plate, and the test preparation containing R milligrammes of radium balances at a distance r_2. Since when a balance is obtained, the ionisation in the chamber is equal for both preparations, and the absorption by the lead is the same in both cases—

$$\frac{\mathrm{R}e^{-\mu r_2}}{r_2(r_2+a)}=\frac{\mathrm{S}e^{-\mu r_1}}{r_1(r_1+a)}$$

or

$$\mathrm{R/S}=\frac{r_2}{r_1}\cdot\frac{r_2+a}{r_1+a}e^{-\mu(r_1-r_2)}.$$

For an accurate determination it is necessary to know the absorption of the γ-rays by air. This has been determined as accurately as possible by one of the authors, and an account of the experiments is given in an accompanying paper. If the γ-rays pass through 3 mm. of lead before entering the ionisation chamber the value of μ for air at normal temperature and pressure is given by $\mu=0{\cdot}0000624$ (cm.)$^{-1}$, and $\mu=0{\cdot}000059$ (cm.)$^{-1}$ at standard pressure and 15°C. Suppose, for example, the preparation R balances at a distance 1 metre beyond the standard preparation S at a temperature 15°C., then $e^{-\mu(r_1-r_2)}=e^{0{\cdot}0059}=1{\cdot}0059$. The correction for absorption is thus 0·59 per cent.

Since the absorption is certainly correct within 5 per cent., the maximum error introduced by the correction in the case considered is not greater than 3 parts in 10,000.

If the standard and test preparations are contained in glass tubes of unequal thickness, it is, of course, necessary to correct for the absorption of the γ-rays in glass, and in some cases by the radio-active preparation itself. This latter correction is in general small, and can easily be applied if the weight and nature of the material are known.

It has been found that when the γ-rays have passed through 1 cm. of lead the coefficient of absorption of the γ-rays in ordinary soda glass is given by $0{\cdot}117$ (cm.)$^{-1}$. The correction to be applied for $\frac{1}{10}$ mm. extra thickness of wall is consequently 0·12 per cent., and for an extra thickness of 1 mm. $=1{\cdot}2$ per cent. In properly constructed standards contained in thin-walled glass tubes the correction is in general a fraction of a per cent.

It is, of course, necessary that the ionisation in the smal vessel B, due to the γ-rays from the radium, should be negligibly small in comparison with the ionisation due to the uranium. This can readily be arranged by making the volume of B small compared with that of A. The ratio of volume of

B to A used was 1 to 35. In addition, the γ-rays had to penetrate 5 cm. of lead before entering the ionisation vessel B. The relative effect of the γ-rays on the ionisation of B is clearly greater the larger the quantity of radium under comparison. In the experiments the greatest distance of the balance point from A was about 3 metres, and the distance between A and B was 4 metres. Taking the various factors into consideration, it can be simply calculated that the maximum ionisation due to the γ-rays in B is not more than $\frac{1}{3000}$ of the ionisation due to the uranium, and may consequently be neglected.

Test of Distance Law.—We have seen that the ionisation in the vessel A should vary approximately as $\frac{e^{-\mu r}}{r(r+a)}$, where r and a have been already defined. The accuracy of this relation has been tested experimentally in the following way: Four small glass tubes were each filled with about 20 millicuries of emanation. After radio-active equilibrium was reached, the balance was obtained for each of the four tubes separately and then all together. Correction was made for the decay of the emanation, and consequently of the γ-ray activity during the interval of measurement. Since the time for the five observations was about one hour, the correction for the decay of the emanation was small, and could be made with accuracy. If r_1, r_2, &c., be the distance of balance for the four tubes separately, and r for the four tubes together, the values $\Sigma r_1(r_1+a)e^{\mu r_1}$ for the four tubes should equal $r(r+a)e^{\mu r}$, if the distance law assumed is correct.

The agreement between the values is shown in the following table for a number of separate experiments. In some cases the ionisation in B was varied; in others, the ionisation was kept constant, but the distance of balance from A became smaller owing to decay of the emanation with time.

Approximate distance of single tubes from ionisation chamber.	$\Sigma r_1(r_1+a)e^{\mu r_1}$ for four tubes.	$r(r+a)e^{\mu r}$ for four tubes together.
120 cm.	66,860	66,740
,,	66,090	65,900
,,	65,550	65,400
110 cm.	52,140	52,170
,,	51,210	51,170
70 cm.	22,910	22,860
,,	19,550	19,580
62 cm.	18,490	18,440

At 45 cm. from A the value corresponding to the second column was found to be 0·4 per cent. less than in the third. At this distance, therefore, the deviation from the formula becomes appreciable. Provided the balance is not obtained for distances less than 60 cm., for the particular apparatus employed, the formula is seen to agree with experiment to 1 part in 400.

Another simple method of testing the distance law is to compare the ratio of the γ-ray activity of two radium preparations which differ in considerable ratio—1 to 4, for example—for different values of the ionisation in B. If the distance law is correct the ratio of the γ-ray activities of the two preparations should agree whatever the distance of balance.

Advantages of this Method.—The balance method of comparing γ-ray activities has numerous advantages. In the first place, since the ionisation in the testing vessel is a constant when a balance is obtained, no error due to possible lack of saturation arises. There is no correction to be applied for natural leak provided it does not change during the time of observation. Since comparisons are made when the radium is at considerable distance from the ionisation chamber, the dimensions or shape of the tube containing the radium has little influence, and the correction for distribution is, in most cases, negligible.

In consequence of this the γ-ray activity of radium preparations contained in vessels of different size and shape can be readily compared. If the actual quantity of radium in the tube is to be determined in terms of the standard, correction, of course, must be made for the absorption of the γ-rays in the walls of the containing vessel and in the radio-active material itself.

The method is an ideal one for comparison of nearly equal quantities of radium, such as, for example, will be used in comparing duplicate standards in terms of the primary. Under such conditions, the balance distances are practically the same for the two preparations. Possible errors in the distance law assumed or in the amount of absorption of γ-rays by air do not appreciably affect the accuracy of the comparison. If the two standards to be compared are enclosed in tubes of equal thickness and contain radium of about equal purity, it will be a simple matter to determine the relative value of the standards to 1 part in 1,000 or even closer.

We have seen that the distance law breaks down for a distance smaller than 50 cm. If, however, the apparatus be required to determine small quantities of radium—*e.g.*, $\frac{1}{10}$ milligramme—it will be necessary either to reduce the ionisation in B or to calibrate the apparatus for smaller distances. The latter can be done by observing the alteration of distance of balance for a tube containing emanation as it decays with time. Assuming the period of decay of the emanation (3·85 days) a fairly accurate calibration curve can be obtained.

The investigations of J. A. Gray * have shown that γ-rays are excited by the passage of β-rays through matter. The source of β-rays in his experiments was radium D + radium E. The authors have found by the balance method that the β-rays from radium in falling on matter also give rise to γ-rays, but in amount small compared with the primary γ-rays from the active matter itself. Since the thickness of glass tubes enclosing the radium preparation is in general not sufficient to absorb all the β-rays, the escaping β-rays no doubt produce some γ-rays in the surrounding air. This possible error in the measurements can be corrected for by surrounding the radium tube with a sufficient thickness of a substance of small density, like glass or aluminium, to absorb all the β-rays. Gray has shown that the excitation of γ-rays is small for light substances, and is approximately proportional to the atomic weight of the material.

Determination of Quantities of Emanation.—The amount of emanation in equilibrium with 1 gramme of radium is called 1 curie; from 1 milligramme of radium, 1 millicurie. It is often necessary to determine the amount of emanation stored in a tube by itself. This is simply done by comparing the γ-ray activity of the emanation tube with that due to a standard quantity of radium. Knowing the period of decay of the emanation, the amount of emanation at any time can be deduced.

In comparisons of this kind it is necessary to apply a correction. The γ-rays do not arise from the emanation itself, but from its subsequent products, radium B and radium C. Five or six hours after introduction of the emanation into a tube, a state of changing equilibrium exists between the emanation and its products, and the γ-ray activity then decays at the same rate as the emanation. It is clear, however, from general considerations that the amount of radium B or radium C present at any time must be always greater than corresponds to true equilibrium with the emanation. The theoretical correction can be easily determined. Let λ_1, λ_2, λ_3, λ_4 be the radioactive constants of the emanation, radium A, B and C respectively. If P_0 atoms of emanation are initially present, the amount S of radium C at a time, t, later is given* by

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* Gray, "Proc. Roy. Soc." A. **85**, p. 131, 1911.

$$S = c_1 e^{-\lambda_1 t} + c_2 e^{-\lambda_2 t} + c_3 e^{-\lambda_3 t} + c_4 e^{-\lambda_4 t},$$

where $c_1 = \dfrac{\lambda_1\lambda_2\lambda_3 P_0}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)(\lambda_4-\lambda_1)}$, $c_2 = \dfrac{\lambda_1\lambda_2\lambda_3 P_0}{(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)(\lambda_4-\lambda_2)}$
and so on.

Since the values of λ_2, λ_3, λ_4 for radium A, B and C are large compared with λ_1, after five or six hours the terms in $e^{-\lambda_2 t}$, $e^{-\lambda_3 t}$, $e^{-\lambda_4 t}$ become vanishingly small, and $S = c_1 e^{-\lambda_1 t}$.

At the instant t the amount S_0 of radium C which would be in equilibrium with the amount P of the emanation, kept constant by supply from radium, is given by $S_0 = \lambda_1 P_0 e^{-\lambda_1 t}$.

$$\text{Consequently}\quad S/S_0 = \frac{\lambda_2\lambda_3\lambda_4}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)(\lambda_4-\lambda_1)},$$
$$= 1{\cdot}0089,$$

when the half value periods of emanation radium A, B and C are taken as 3·85 days, 3 minutes, 26·8 minutes, 19·5 minutes respectively.

This shows that the amount of radium C is always 0·89 per cent. *greater* than corresponds to permanent equilibrium with an equal quantity of emanation.

After the γ-radiation has passed through 2 cm. of lead, the γ-rays arise entirely from radium C. If the γ-rays from the emanation tube are compared with the γ-radiation from a radium standard through 2 cm. of lead, the amount of emanation in the tube is 0·89 per cent. *less* than that deduced from direct comparison with the γ-ray effect for the radium standard.

If the γ-rays pass through 3 mm. of lead, Moseley and Makower have shown that about 11·5 per cent. of the γ-radiation arises from radium B and 88·5 per cent. from radium C. The correction in this case is somewhat smaller, but can readily be shown to be about 0·85 per cent.

The amount of radium C present in a vessel after removal of the emanation is often determined by comparison of its γ-ray effect after a definite interval with that due to a radium standard. Correction in this case has to be made for the γ-rays emitted from radium B, unless the γ-rays from the latter are removed by using a lead screen 2 cm. thick. Numbers showing the decay with time of the γ-ray activity of the active deposit of radium have been given for one important case by Moseley and Makower (*loc. cit.*).

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* *See* Bateman, "Proc. Camb. Phil. Soc.," 15, p. 423, 1910.

Experiments with Balance Method.—The decay of the γ-ray activity of a large quantity of emanation was determined over several weeks by the balance method. The decay was found to be exponential with a half value period of 3·854 days. This is in excellent agreement with the results of Mme. Curie and Rutherford, who found values of 3·846 and 3·85 days respectively.

Some experiments were also made to determine experimentally the time required for the γ-ray activity of a tube filled with emanation to attain a maximum. The exact time was noted when emanation was introduced into a small glass tube and sealed off. The amount of γ-ray activity was accurately measured at intervals between 2 and 6 hours later.

The activity rose rapidly at first and was within 1·5 per cent. of the maximum after 3 hours. Four separate determinations were made and the maximum through a lead screen 3 mm. thick was found to occur at 255, 254, 256 and 257 minutes respectively after filling the tube. In this case, about 11·5 per cent. of the γ-radiation was supplied by radium B. Taking this factor into account, it was calculated that the maximum should occur at 255 minutes, an interval agreeing with the mean experimental value.

Using this balance method experiments were made to test whether the emission of γ-rays was affected by placing a tube containing emanation between the pole pieces of a large electromagnet. No detectable effect was observed.

ABSTRACT.

A balance method is described for accurately comparing quantities of radium by their γ-radiation, in which the ionisation due to the γ-rays is balanced against the constant ionisation due to uranium oxide. By observing the distance of the radium preparation from the ionisation chamber when a balance is obtained, the relative γ-ray activities of the two preparations can be determined with an accuracy of at least one part in 400. A method of calibration is given, and the corrections necessary to deduce the quantities of radium present are considered.

Calculations are given showing the correction required to determine a quantity of emanation by comparison of its γ-ray activity with that due to a radium standard.

The balance method was employed to determine the period of transformation of the radium emanation. The half value period was found to be 3·854 days. It was found experimentally that the γ-ray activity due to the radium emanation and its products reaches a maximum 255 minutes after the introduction of the emanation into a sealed tube. Calculations showed that under the experimental conditions the theoretical maximum should be reached after 255 minutes.

It was found that the γ-ray activity of a radium preparation was not appreciably altered by exposure in a strong magnetic field.

University of Manchester, February, 1912.

XXI. *The Absorption of γ-Rays by Gases and Light Substances.*

By J. CHADWICK, *B.Sc.*

RECEIVED FEBRUARY 21, 1912. READ FEBRUARY 23, 1912.

AT the present time all information on the absorption of the γ-rays by gases is indirect, for the absorption coefficients are deduced either by assuming the density law or from the observed absorption coefficients of gases for β-rays. Values obtained in this way are, however, rather indefinite, and vary among themselves by about 50 per cent. For example, the experiments of Russell and Soddy indicate an absorption coefficient for air of about 0·000056 (cm.)$^{-1}$, while Eve,* from the absorption of the β-rays in air, obtains the value 0·000032 (cm.)$^{-1}$. As a fairly accurate knowledge of the absorption coefficient of air was required in the method of comparison of radium standards described in a previous Paper, a special series of experiments was made to determine the absorption of γ-rays by the gases hydrogen, air, and carbon dioxide. The investigation was extended to include the absorption of γ-rays by a few light solids and by some solutions. As is well known, the value of the absorption coefficients for most bodies depends on the thickness of lead traversed by the γ-rays before passing into the ionisation chamber. The absorption coefficients are given for two thicknesses of lead in common use—viz., 3 mm. and 1 cm.

Absorption by Air.—In determining the absorption by air two methods were employed. In the first liquid air was used, and the absorption produced was measured by means of the balance method described in the previous Paper. In most of the experiments the liquid air was contained in a $\frac{1}{2}$ litre flask of about 10 cm. diameter, and in others in a cylinder of 3 cm. diameter. The diameters of the cylinder and flask were determined by finding the absorption of γ-rays produced when full of water. The vessel containing the liquid air was placed immediately in front of the radium tube. The density of the liquid air was determined by taking a sample of the saturated vapour just above the surface of the liquid and analysing it. The density of the liquid air can then be found from the data given by Behn and Kiebitz ("Ann. d. Physik," 1903). The percentage of oxygen in the vapour varied from 20 per cent.

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*Eve, "Phil Mag.," July, 1911.

to 70 per cent., corresponding to a range of density between 0·98 and 1·11. A large number of measurements were made and consistent values were obtained. With a thickness of 3 mm., the values found for μ/D, where μ is the absorption coefficient measured in the usual way and D the density, varied between 0·047 and 0·050, with a mean of 0·0484 $(\text{cm.})^{-1}$. This is probably correct to within 0·5 per cent. With a thickness of lead of 1 cm., the mean value of μ/D was found to be 0·0465 $(\text{cm.})^{-1}$.

In the second method air compressed into a long iron cylinder was used. As it would have been very inconvenient to move the iron cylinder for each reading, the balance method was abandoned and the ordinary rate method was used. The change consisted simply in disconnecting the vessel B. The radium tube was placed at one end of the cylinder and the ionisation chamber was at the other end.

The rate of movement of the electrometer needle was observed when the air in the cylinder was at atmospheric pressure, at 60 atmospheres and at 90 atmospheres. The cylinder used was 120 cm. long, and the iron walls were nearly 3 cm. thick. The pressure gauge used in these experiments was carefully calibrated with a standard gauge made by Schäffer and Budenberg. For a thickness of lead of 3 mm. the value obtained for μ/D was 0·048, and for a thickness of 1 cm. μ/D was 0·046. The error in these experiments is probably within 2 per cent. It will be seen that the above values agree very well with those obtained for liquid air.

Absorption by Carbon Dioxide.—The absorption coefficient of carbon dioxide was determined in the same manner. The pressures used were 23, 33 and 50 atmospheres, and the temperature varied from 13°C. to 17°C. The density of the gas at a temperature of 15°C. and pressure of 50·4 atmospheres was found from a table given by Amagat ("Comptes Rendus," 1892). With a thickness of lead of 3 mm., μ/D is 0·051, and with a thickness of 1 cm. is 0·047.

It will readily be seen that the measurement of the absorption coefficient of carbon dioxide at different pressures would afford a very simple and convenient method of determining the isothermals. All the necessary data can be obtained quickly and accurately, and the only difficulty is to keep the temperature constant.

An attempt was made to determine the absorption coefficient of solid carbon dioxide, but serious difficulties were encountered.

The carbon dioxide snow was put in a large liquid air cylinder and pressed together as tightly as possible, but the density under these conditions was about 0·8, while Behn (" Ann. d. Phys.," 1900) found that the real density of the snow is 1·53. Further, the snow evaporates rapidly from the sides of the cylinder, and thus the thickness of the carbon dioxide is continually decreasing. Such results as were obtained confirmed the values given above.

Absorption in Hydrogen.—As the absorption of the γ-rays by hydrogen is extremely small, a slight modification of the balance method was employed. The ionisation chamber A of the previous Paper was replaced by a cubical lead box, whose sides were 3 mm. thick. The cylinder containing the hydrogen and the radium tube were placed against one face of the box in the same way as before, and a balance was obtained by moving a tube containing a small amount of radium along a scale placed against another face. The cylinder used was an ordinary hydrogen cylinder 150 cm. long, with walls nearly 4 cm. thick. The pressure of the hydrogen was 127 atmospheres. Several measurements were made, and the value of μ/D was found to be 0·047. This is for a thickness of lead of 3 mm. This is probably correct to 5 per cent., allowing for possible error of 2 to 3 per cent. in the measurement of the higher pressures, for the pressure gauge was not standardised above 60 atmospheres. The absorption coefficient for a thickness of lead of 1 cm. was not determined, as the effect to be measured was too small Even with a thickness of 3 mm. the ionisation contributed by the fixed radium tube was only about a third of the total ionisation, and with 1 cm. of lead would only be about a fifth.

The value $\mu/\mathrm{D}=0{\cdot}0484$ found above for air gives $\mu=0{\cdot}0000624$ $(\mathrm{cm.})^{-1}$ for air at 0°C. and 760 mm. pressure, and $\mu=0{\cdot}000059$ $(\mathrm{cm.})^{-1}$ at 15°C. and 760 mm. pressure.

After these measurements were completed, a Paper by Hess appeared (" Phys. Zeit.," November, 1911), in which he finds μ for air to be 0·000045 at 22°C. and 750 mm. pressure. His method consists in measuring the relative ionisation produced by a quantity of radium placed at different distances from a Wulf's electroscope, and assuming that the deviations from the inverse square law were due to absorption of the γ-rays by air. He used distances varying from 10 metres to 90 metres, and obtained results varying by about 20 per cent. He ascribes these differences to variations in the natural leak of the electroscope during the observations. The method is not very suitable for an accurate determination, as the effect to be measured is rather small, and the ratio of the ionisation currents measured is about 100 to 1.

In his Paper on the number of ions produced by the γ-rays of radium C, Eve (" Phil. Mag.," October, 1911) has taken the value of μ for air as 0·000044. Taking the value found in this Paper—viz., $\mu=0{\cdot}000059$—the number of ions produced by the γ-rays is 8×10^{14}, or about 30 per cent. less than Eve's number.

If we assume that the atmosphere is at a uniform temperature of 15°C., and that the pressure at the surface of the earth is 76 cm., then, taking into account the variation of density with height, it can be simply calculated that the γ-rays will pass through 817 metres of air before being reduced to 1 per cent. of their value at the surface. Experiments show, however, that the number of ions per cubic centimetre is the same at a height of 1,000 metres as on the surface. It follows, therefore, that the ions in the air cannot be wholly due to the γ radiation from radioactive material in the earth.

Absorption by Light Substances.—The absorption coefficients of a few solids and some solutions of salts in water were determined by the balance method, and are given in the table below, as they are often useful in radioactive measurements. Two thicknesses of lead were used—viz., 3 mm. and 1 cm.

Thickness of lead = 3 mm.				1 cm.	
Substance.	Density D.	μ in (cm.) $^{-1}$	$100\times\mu/D$.	μ in (cm.) $^{-1}$	$100\times\mu/D$.
Hydrogen.........	...	...	4·7	...	...
Air..................	...	...	4·84	...	4·65
Carbon dioxide...	...	...	5·1	...	4·7
Wood	0·730	0·0380	5·21	0·0320	4·38
Aluminium	2·94	0·133	4·51	0·118	4·01
Iron	7·75	0·350	4·52	0·312	4·03
Lead	11·4	1·029	9·03	0·683	5·99
Water	1·000	0·0558	5·58	0·0472	4·72
$CuSO_4$ solution	1·141	0·0631	5·53	0·0512	4·49
NaCl ,,	1·145	0·0626	5·47	0·0512	4·47
$CaCl_2$,,	1·164	0·0611	5·25	0·0544	4·67
$Pb(NO_3)_2$,,	1·156	0·0671	5·80	0·0662	5·72

The solutions used in the experiments were contained in a rectangular vessel of thin glass 5 cm. across. The absorption coefficient of lead was obtained with a sheet 4 mm. thick. If thicker plates are used, the absorption coefficient is much less—*e.g.*, the absorption coefficient deduced from the absorption by 2 cm. of lead when the thickness of lead over the ionisation chamber is 1 cm. is 0·593.

It will be seen from the table that the value of μ/D is a minimum for substances like aluminium and iron, and the minimum is more pronounced the softer the γ-rays used. This also appears from the absorption experiments of Russell and Soddy.

In conclusion, I have great pleasure in thanking Prof. Rutherford for his help and interest throughout the experiments.

ABSTRACT.

The absorption coefficients of air, carbon dioxide and hydrogen for the γ-rays from radium, have been measured after the rays have traversed 3 mm. and 1 cm. of lead. The gases were contained in an iron cylinder and were under considerable pressure. The values found for the absorption coefficients are, at normal temperature and pressure, 0·000062, 0·000102, and 0·0000042 (cm.)$^{-1}$ for air carbon dioxide, and hydrogen respectively, the thickness of lead being 3 mm. When the thickness of lead is 1 cm. the absorption coefficients of air and carbon dioxide are 0·000059, 0·000092 (cm.)$^{-1}$ respectively.

The absorption coefficient of air has also been determined by using liquid air. The absorption coefficients found in this way are in good agreement with the values found for air under pressure showing that the absorption of γ-rays is not affected by temperature.

The absorption coefficients of a number of light substances have also been found.

DISCUSSION ON THE TWO PRECEDING PAPERS.

Prof. H. L. Callendar fully appreciated the difficulty of accurately comparing quantities of radium by the electroscopic method. He had a year or two ago made some measurements of the heat given out by radium, and found that with only two or three milligrammes the heat could be measured to 1 part in 1000.

Dr. J. H. Vincent commented on the value of the new method for the accurate comparison of quantities of radium. It was also valuable to have further tables of the absorption of γ-rays by different substances.

Dr. S. Russ asked whether Prof. Rutherford attributed the diminution of the coefficient of absorption for γ-rays with increase of distance traversed to the lack of homogeneity of the rays themselves.

Dr. G. W. C. Kaye remarked that the Röntgen Ray Society several years ago considered the question of a Radium Standard. He hoped that when prepared the standard would be placed so that other workers could standardise their radium by it.

Prof. A. W. Bickerton asked whether it was now regarded as finally established that the α-rays are nothing but helium.

Prof. C. H. Lees asked concerning the preparation of the Primary Standard.

The President remarked on the advantages of being able to get accurate quantitative measurements in any science. The early workers on vacuum tube discharges experienced the same difficulty as the early workers on radio-activity in getting accurate quantitative measurements of the effects. He also asked how the thickness of the tube containing the radium was found.

Prof. E. RUTHERFORD, in reply, stated that the thickness of the tube, if not measured before putting the radium into it, could be found approximately by weighing and measuring. Since the absorption it caused in the γ-rays was in general less than 1 per cent., its thickness need only be known approximately. There appears to be no doubt that the γ-rays emitted from radium in equilibrium are complex in character. In addition to the more penetrating γ radiation from radium C, Moseley and Makower have recently shown that radium B emits some γ-rays of a less penetrating type. Recent work had shown that the β-radiation from radioactive substances is exceedingly complex in character. For example, radium B and radium C together give out nearly 30 groups of homogeneous β-rays, some of them with velocities very close to the velocity of light. Since there appeared to be no doubt that γ-rays always accompanied β-rays, although in no very obvious quantitative relation, it might be anticipated that the γ-rays from radium would equally be very complex in character. At the same time, there was some indirect evidence that the γ-rays from radioactive substances consisted of a mixture of the characteristic radiations of the elements from which they arose. It was certainly striking how closely the γ-rays from radium were absorbed according to an exponential law by lead and other heavy substances; but the exponential law only held after the less penetrating rays, which were undoubtedly present, had been absorbed. At present there was no certain evidence that an exponential law of absorption was any guarantee of the homogeneity of the γ radiations. The primary standard of radium consists of about 30 milligrams of a pure radium salt, which has been accurately weighed and contained in a sealed tube. It is thus a weight standard, but duplicates will be compared with it by measuring the intensity of the γ-rays emitted. A very important point has arisen in connection with the preparation of a pure radium standard. If the mineral from which the radium is separated contained thorium, its product, meso-thorium, is isolated with the radium, and cannot be separated from the latter by any known method. The presence of an appreciable quantity of meso-thorium with the radium would render it altogether unsuitable for an accurate standard. Fortunately the Joachimstahl uraninite deposits from which the duplicate and primary standard would be prepared were practically free from thorium, and this difficulty would, in consequence, not arise. There could, he thought, be no doubt now that the α-rays consisted entirely of projected helium atoms.

XXII. *On Wave-form Sifters for Alternating Currents.* By ALBERT CAMPBELL, *B.A.*

THE PAPER IS PRINTED IN THE "PROCEEDINGS" PAGE 107.

CORRIGENDA.

Formula (2). The denominator should be $R^2+\left(L\omega-\frac{1}{K\omega}\right)^2$.

Formula (4) and the one immediately above it. Delete, and read instead $I_2=E\,.\,\frac{M\omega(b^2\sim 1)}{PR+M^2\omega^2(b^2-1)^2}$.

Page 111, line 6. Insert the words "may be used" after "mutual inductance."

ABSTRACT.

If an alternating current of complex wave form be passed through a condenser (K) and the primary coil of a mutual inductance (M) in series with it, any other circuit connected across the condenser and the secondary coil (in series) will receive no component of the current of frequency n if $\omega^2MK=1$, where $\omega=2\pi n$.

This arrangement affords a means of totally suppressing the fundamental or any single harmonic in a complex wave form. This kind of sifting can also be done by means of almost any null method in which the balance depends on frequency. A bridge method suitable for quite low frequencies is also described.

DISCUSSION.

Mr. **DUDDELL** pointed out the value of being able to get rid of particular harmonics, especially of telephonic frequencies. It was very difficult to make a machine to generate a sine wave, and if the wave was not a sine wave the mathematical formulæ did not apply. He thought the chief use of Mr. Campbell's sifter would be to remove the last trace of a troublesome harmonic from a machine that had been designed to give as nearly as possible a pure sine wave. Generally there was only one harmonic that was specially troublesome.

Dr. **RUSSELL** said that the more usual way of getting current or P.D. waves, which were approximately sine shaped, was to use a resonator, which greatly magnified one harmonic. Prof. G. A. Campbell's "wave-form filter" did this. It consisted merely of a condenser and an inductive coil in parallel. Armagnat's oscillograms proved that excellent results could be obtained in practice by simply tuning a condenser and an inductive coil in series. He deprecated the introduction of too many technical terms into electrical theory.

Prof. G.W. O. **HOWE** remarked that a disadvantage of the old-fashioned method of getting pure sine waves by resonance was that the amplitude varied so much with slight changes in the frequency of the generator.

The **PRESIDENT** thought the subject a matter of considerable importance.

Mr. S. **BUTTERWORTH** wrote: Mr. Campbell has described an interesting and useful method of removing harmonics from an alternating current.

With regard to the first form of sifter, it may be important to notice that, *even when the secondary circuit is tuned to the fundamental harmonic, some of the higher harmonics may have their relative values increased in the sifting process.* It may be deduced from equation (2) of Mr. Campbell's Paper that if the tuned frequency is n, and the sifted frequency is bn, all currents of frequencies greater than b^2n will have their values relative to the n component increased by the sifter, provided that the inductance of the secondary circuit is sufficiently small, and that b is greater than unity. Thus, if the 2nd harmonic is removed by the sifter, the 3rd harmonic is diminished in importance, while the 4th and succeeding harmonics become more pronounced.

Mr. B. S. COHEN wrote: Mr. A. Campbell's method of eliminating any particular harmonic is interesting and is doubtless very effective, provided the frequency which it is desired to eliminate is known. The author refers to this device as a sifter. This, however, is rather a misnomer. The wave filter devised by G. A. Campbell (*see* "Phil. Mag.," March, 1903), which eliminates all waves above a certain frequency, can more truly aspire to this title. The latter device, which does not appear to have received the attention in this country it deserves, has been employed by the writer in connection with a small alternator for some years with very satisfactory results. Some disadvantages of Mr. A. Campbell's method which do not apply to the G. A. Campbell wave filter are: The impossibility of including any apparatus containing iron in the desired circuit G without upsetting the tuning. This at once renders this device useless for telephonic measurement purposes. Then, again, the cutting down of the available output when several of these tuning circuits are used together must be considerable, and, unfortunately, the output usually available from small alternators, telephone hummers, &c., is limited. The G. A. Campbell wave filter also, of course, occasions losses, but, it would appear, not to the extent of this device. Lastly, when used with an alternator, if the speed, and consequently the frequency, varied, the device would not act efficiently, whilst with the G. A. Campbell wave filter the same variations need not make any difference in the efficiency.

The AUTHOR makes the following reply: With regard to the introduction of new technical terms, in sciences like botany or zoology, the moment a new plant or animal is discovered it is essential that it should be named immediately. Similarly, in experimental physics, it is highly desirable that when a novel instrument (or method) appears it should receive an appropriate and *distinctive* name. The name should also, as far as possible, be self-explanatory (for example: pressure gauge, wavemeter, synchronoscope). As physicists in general are not usually interested in the science of language, there is at present rather a scarcity than an over-abundance of good clear technical terms in our language. The clearness and beauty of French scientific language is due in great measure to the readiness with which new and logical nomenclature has been adopted. With regard to Mr. Cohen's remarks, further experiments will be required to show how far the presence of iron cores will interfere with the methods. Strong harmonics are not introduced by the presence of iron unless the magnetisation is tolerably high, and this is often not the case in measurements connected with telephonic work. The various methods have not yet been examined from the point of view of energy loss, but it seems probable that, by keeping all the resistances low, high efficiency can be obtained. A small variation of the speed of the alternator introduces only a very small component of the suppressed frequency. The point raised by Mr. Butterworth is important. When a telephone is used as detecting instrument, sometimes the higher harmonics are near the limits of audibility, and can be ignored even when increased in importance.

XXIII. *The Intensity at Points near the Principal Focus of an Object-glass with Symmetrical Aberration.* By JAMES WALKER, *M.A.*

RECEIVED FEBRUARY 6, 1912.

STREHL, in his "Theorie des Fernrohrs auf Grund der Beugung des Lichts,"* considers the case of spherical aberration, and, after a somewhat laborious calculation, obtains an expression for the intensity of the diffracted light, of which he says: "Man kann daher nach der Methode der unbestimmten Coefficienten die Entwickelung von M² "—the intensity—"vorstehenden Gesetzen gemäss beliebig weit forsetzen, und insofern halte ich das Problem der sphärischen Aberration für vollstandig gelöst." The laws in question are, however, not very obvious, and no proof is given that they hold beyond the few terms for which the calculation has actually been made.

By a modification of Strehl's method, a commodious expression for the general term in the expression for the intensity may be readily obtained, from which it is seen at once that the successive terms can be easily calculated by a simple process of derivation.

In the case considered by Strehl, the equation of the wave issuing from the object-glass is

$$z=\frac{r^2}{2p}+\frac{\tau^2 r^4}{8p^3}$$

and the approximate value of the length of path from the point $(r\cos\omega,\ r\sin\omega,\ z)$ on the wave to the point $(\xi,\ o,\ p+\Delta)$ is taken as

$$p+\Delta-\frac{r\xi}{p}\cos\omega-\frac{\Delta r^2}{2p^2}-\frac{ar^4}{8p^3},$$

where $a=\tau^2-1$, and $-ar^2/(2p)$ is the longitudinal aberration of the rays from points distant r from the axis.†

If R be the effective radius of the object-glass, and we write $r=\mathrm{R}\sqrt{\rho}$, the symbolical expression for the light-vector at the point $(\xi,\ o,\ p+\Delta)$ is, with the usual assumptions,

$$(\mathrm{C}+\iota\mathrm{S})\,\mathrm{Exp.}\left\{\iota\frac{2\pi}{\lambda}(vt-p-\Delta)\right\},$$

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* Barth, Leipzig, 1894.

† *Loc. cit.* pp. 15, 16.

where $$C+\iota S=\frac{1}{2}\frac{R^2}{\lambda p}\int_0^{2\pi}\int_0^1 \text{Exp.}\left\{\iota\frac{2\pi}{\lambda}\left(\frac{\xi R}{p}\sqrt{\rho}\,.\cos\omega+\frac{\Delta R^2}{2p^2}\rho\right.\right.$$
$$\left.\left.+\frac{aR^4}{8p^3}\rho^2\right)\right\}d\rho d\omega$$
$$=\frac{\pi R^2}{\lambda p}\int_0^1 J_0(x\sqrt{\rho})\,\text{Exp.}\{\iota(D\rho+A\rho^2)\}\,d\rho$$
$$=\frac{\pi R^2}{\lambda p}\int_0^1 J_0(x\sqrt{\rho})f(\rho)d\rho \quad \text{(say)},$$

where $$x=\frac{2\pi}{\lambda}\frac{R}{p}\xi,\quad D=\frac{2\pi}{\lambda}\frac{R^2}{2p^2}\Delta,\quad A=\frac{2\pi}{\lambda}\frac{R^4}{8p^3}a.$$

By successive integration by parts, using the relation

$$\int_0^\rho \rho^{\frac{n}{2}}J_n(x\sqrt{\rho})d\rho=2\rho^{\frac{n+1}{2}}\frac{J_{n+1}(x\sqrt{\rho})}{x}*,$$

we obtain

$$C+\iota S=\frac{2\pi R^2}{\lambda p}\left\{\frac{J_1(x)}{x}\,.f(1)-2\frac{J_2(x)}{x^2}f'(1)+\dots\right.$$
$$\left.+(-1)^n2^n\frac{J_{n+1}(x)}{x^{n+1}}f^{(n)}(1)+\dots\right\}$$
$$=\frac{2\pi R^2}{\lambda p}\left\{G_1(x)f(1)-\frac{G_2(x)}{\lfloor\underline{1}}f'(1)+\dots\right.$$
$$\left.+(-1)^n\frac{G_{n+1}(x)}{\lfloor\underline{n}}f^{n}(1)+\dots\right\},\dagger \quad . \quad (1)$$

where $G_n(x)=2^{n-1}\lfloor\underline{n-1}\,.\,J_n(x)/x^n$.

Now, if $f(\rho)=F(y)$, $y=\varphi(\rho)$,

$$f^{(n)}(1)=U_{n\,.\,1}F'(y)_1+U_{n\,.\,2}F''(y)_1+\dots\;+U_{n\,.\,n}F^{(n)}(y)_1,$$

where $F'(y)_1\dots$ are the values of $F'(y)\dots$, when ρ is written equal to unity, and

$$U_{n\,.\,k}=\frac{1}{k}\left[\frac{d^n}{d\sigma^n}\{\varphi(\rho+\sigma)-\varphi(\rho)\}^k\right]_{\substack{\sigma=0\\ \rho=1}}\ddagger,$$

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* Lommel, "Studien über die Bessel'schen Functionen" (Teubner, Leipzig, 1868), p. 19.

† In the case of the function $f(\rho)$ here considered this series is *absolutely convergent.*

‡ Schlömilch, "Compendium der höheren Analysis" (Vieweg, Braunschweig, 1881), 3 Aufl., Bd. II., p. 4. This method has been used by Straubel "Theorie der Beugungserscheinungen," Abh. d. k. bayer., Akad. der Wiss., II., Cl., xviii. Bd. I., Abth., p. 113 (1893).

and since, in the case under consideration,

$$F(y)=\text{Exp.}\{\iota(D\rho+A\rho^2)\},$$

we have $f^{(n)}(1)=\text{Exp.}\{\iota(D+A)\}\,.\,\{U_{n.1}\iota+U_{n.2}\iota^2+\dots+U_{n.n}\iota^n\}$

where, writing $D+2A=Z\,.\,A$,

$$\begin{aligned}U_{n.k}&=\frac{1}{\underline{|k}}\left[\frac{d^n}{d\sigma^n}\{D(1+\sigma)+A(1+\sigma)^2-(D+A)\}^k\right]_{\sigma=0}\\&=\frac{A^k}{\underline{|k}}\left[\frac{d^n}{d\sigma^n}(Z\sigma+\sigma^2)^k\right]_{\sigma=0}\\&=\frac{A^k}{\underline{|k}}\,.\,\frac{\underline{|k}}{\underline{|n-k}\,\underline{|2k-n}}\underline{|n}\,.\,Z^{2k-n}\\&=A^k\frac{\underline{|n}}{\underline{|n-k}\,\underline{|2k-n}}Z^{2k-n}\end{aligned}$$

and $U_{n.k}=0$, if $2k$ is less than n.

Substituting in (1), and arranging according to powers of ι, we have

$$\begin{aligned}C+\iota S&=\frac{2\pi R^2}{\lambda p}\,.\,\text{Exp.}\{\iota(D+A)\}\\&\quad\times\sum_0^\infty(-\iota)^n\left\{\frac{U_{n.n}}{\underline{|n}}G_{n+1}(x)-\frac{U_{n+1.n}}{\underline{|n+1}}G_{n+2}(x)+\dots\right.\\&\qquad\qquad\left.+(-1)^n\frac{U_{2n.n}}{\underline{|2n}}G_{2n+1}(x)\right\}\\&=\frac{2\pi R^2}{\lambda p}\,\text{Exp.}\{\iota(D+A)\}\\&\quad\times\sum_0^\infty(-\iota A)^n\left\{\frac{Z^n}{\underline{|o}\,\underline{|n}}G_{n+1}(x)-\frac{Z^{n-1}}{\underline{|1}\,\underline{|n-1}}G_{n+2}(x)+\dots\right.\\&\qquad\qquad\left.+(-1)^n\frac{1}{\underline{|n}\,\underline{|o}}G_{2n+1}(x)\right\}.\\&=\frac{2\pi R^2}{\lambda p}\,\text{Exp.}\{\iota(D+A)\}\times\sum_0(-\iota A)^n\frac{(Z\delta-\delta^2)^n}{\underline{|n}}\,.\,G_1(x)\\&=\frac{2\pi R^2}{\lambda p}\,\text{Exp.}\{\iota(D+A)\}\,.\,\text{Exp.}\{-\iota A(Z\delta-\delta^2)\}\,.\,G_1(x)\end{aligned}$$

where δ denotes a process of derivation, converting $G_n(x)$ into $G_{n+1}(x)$, and $G_m(x)G_n(x)$ into $G_{m+1}(x)G_n(x)+G_m(x)G_{n+1}(x)$.

Writing for shortness G_1 for $G_1(x)$, the intensity is given by

$$C^2+S^2=\left(\frac{2\pi R^2}{\lambda p}\right)^2 \text{Exp.}\{-\iota A(Z\delta-\delta^2)\}.G_1$$
$$\times\text{Exp.}\{\iota A(Z\delta-\delta^2)\}.G_1.$$

Omitting the factor $\left(\frac{2\pi R^2}{\lambda p}\right)^2$, the coefficient of A^{2m} in this expression is easily seen to be

$$\sum_{n=0}^{n=m}(-1)^n a\frac{(Z\delta-\delta^2)^{m-n}}{\underline{|m-n}}G_1.\frac{(Z\delta-\delta^2)^{m+n}}{\underline{|m+n}}G_1.$$

where $a=1$, if $n=0$, and $a=2$, if $n>0$.

Writing this in the form

$$M_0Z^{2m}-M_1Z^{2m-1}+\ldots+(-1)^rM_rZ^{2m-r}+\ldots$$

we have

$$M_r=\sum_{s=0}^{s=r}\frac{1}{\underline{|s}}.\frac{1}{\underline{|r-s}}\sum_{n=0}^{n=m-s}(-1)^n a\frac{\delta^{m-n+s}}{\underline{|m-n-s}}G_1.\frac{\delta^{m+n+r-s}}{\underline{|m+n-r+s}}G_1.^*$$

Since $2m-r=m+n-r+s+m-n-s$,

$$(2m-r)\delta\left\{\frac{\delta^{m-n+s}}{\underline{|m-n-s}}G_1.\frac{\delta^{m+n+r-s}}{\underline{|m+n-r+s}}G_1\right\}$$

$$=\frac{\delta^{m-n+s}}{\underline{|m-n-s}}G_1.\frac{\delta^{m+n+r-s+1}}{\underline{|m+n-r+s-1}}G_1+\frac{\delta^{m-n+s+1}}{\underline{|m-n-s-1}}G_1.\frac{\delta^{m+n+r-s}}{\underline{|m+n-r+s}}G_1$$
$$+\frac{\delta^{m-n+s}}{\underline{|m-n-s-1}}G_1.\frac{\delta^{m+n+r-s+1}}{\underline{|m+n-r+s}}G_1+\frac{\delta^{m-n+s+1}}{\underline{|m-n-s}}G_1.\frac{\delta^{m+n+r-s}}{\underline{|m+n-r+s-1}}G_1$$

and

$$\sum_{s=0}^{s=r}\frac{1}{\underline{|s}}\frac{1}{\underline{|r-s}}\sum_{n=0}^{n=m-s}(-1)^n a\left\{\frac{\delta^{m-n+s}}{\underline{|m-n-s-1}}G_1\frac{\delta^{m+n+r-s+1}}{\underline{|m+n-r+s}}G_1\right.$$
$$\left.+\frac{\delta^{m-n+s+1}}{\underline{|m-n-s}}G_1\frac{\delta^{m+n+r-s}}{\underline{|m+n-r+s-1}}G_1\right\}$$

$$=\sum_{s=0}^{s=r}\frac{1}{s}\frac{1}{\underline{|r-}}\left\{-\frac{\delta^{m+s}}{\underline{|m-s-1}}G_1\frac{\delta^{m+r-s+1}}{\underline{|m-r+s}}G_1+\frac{\delta^{m+s+1}}{\underline{|m-s}}G_1\frac{\delta^{m+r-s}}{\underline{|m-r+s-1}}G_1\right\}$$
$$=0.$$

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* In order that this may hold for all values of r we must suppose $\underline{|-1}=\infty$.

Hence

$(2m-r)\delta M_r$

$$=\sum_{s=0}^{s=r}\frac{1}{\underline{|s}}\frac{1}{\underline{|r-s}}\sum_{n=0}^{n=m-s}(-1)^n a\left\{\frac{\delta^{m-n+s}}{\underline{|m-n-s}}G_1\frac{\delta^{m+n+r+1-s}}{\underline{|m+n-r-1+s}}G_1\right.$$

$$\left.+\frac{\delta^{m-n+s+1}}{\underline{|m-n-s-1}}G_1\frac{\delta^{m+n+r-s}}{\underline{|m+n-r+s}}G_1\right\}$$

$$=(r+1)\sum_{s=0}^{s=r}\frac{1}{\underline{|s}}\frac{1}{\underline{|r+1-s}}\sum_{n=0}^{n=m-s}(-1)^n a\frac{\delta^{m-n+s}}{\underline{|m-n-s}}G_1\frac{\delta^{m+n+r+1-s}}{\underline{|m+n-r-1+s}}G_1$$

$$+\sum_{s=0}^{s=r}\sum_{n=0}^{n=m-s}(-1)^n a\left\{-s\frac{\delta^{m-n+s}}{\underline{|s}\,\underline{|m-n-s}}G_1\frac{\delta^{m+n+r-s+1}}{\underline{|r-s+1}\,\underline{|m+n-r+s-1}}G_1\right.$$

$$\left.+(s+1)\frac{\delta^{m-n+s+1}}{\underline{|s+1}\,\underline{|m-n-s-1}}G_1\frac{\delta^{m+n+r-s}}{\underline{|r-s}\,\underline{|m+n-r+s}}G_1\right\}$$

$$=(r+1)\sum_{s=0}^{s=r+1}\frac{1}{\underline{|s}}\frac{1}{\underline{|r+1-s}}\sum_{n=0}^{n=m-s}(-1)^n a\frac{\delta^{m-n+s}}{\underline{|m-n-s}}G_1\frac{\delta^{m+n+r+1-s}}{\underline{|m+n-r-1+s}}G_1$$

$$=(r+1)\ M_{r+1},$$

whence

$$M_{r+1}=\frac{2m-r}{r+1}\delta M_r=\frac{\underline{|2m}}{\underline{|r+1}\,\underline{|2m-r-1}}\delta^{r+1}M_0;$$

and since

$$M_0=\sum_{n=0}^{n=m}(-1)^n a\frac{\delta^{m-n}}{\underline{|m-n}}G_1\frac{\delta^{m+n}}{\underline{|m+n}}G_1,$$

the expression for the intensity may be written

$$\left(\frac{2\pi R^2}{\lambda p}\right)^2\sum_{m=0}^{m=\infty}A^{2m}(Z-\delta)^{2m}\sum_{n=0}^{n=m}(-1)^n a\frac{\delta^{m-n}}{\underline{|m-n}}G_1\frac{\delta^{m+n}}{\underline{|m+n}}G_1.$$

ABSTRACT.

Strehl has considered the diffraction produced by an object-glass with a symmetrical aberration proportional to the fourth power of the radius, and has pointed out how the terms of the development of the expression for the intensity can be calculated. But this method of calculation is not very obvious, and he has given no proof that the law of formation of the successive terms holds beyond the few terms for which he has actually made the calculation.

By introducing complex expressions, and integrating by parts, using an expression for the successive differential coefficients of a

function of a function of the variable due to V. Meyer, a general symbolical expression for the intensity is readily obtained in terms of functions connected with Bessel's functions by a simple relation, and it is then shown that the terms in the development of the expression for the intensity are obtained by a simple process of derivation.

DISCUSSION.

Mr. T. SMITH objected to the way in which a quotation from a German work was made the starting point of the Paper, without the addition of a translation for the benefit of those members who did not read German; it would also be desirable to give references to standard English works, as well as to the German originals, when quoting results such as the relation given for the Bessel functions. The Paper would be more easily followed if a short statement was made on the notation employed—*e.g.*, it might be said that the focal length is p, and that higher powers than the first of ξ and z are neglected. As regards the mathematics of the Paper, he would like to say that had certain steps been justified the treatment would have been very elegant and pleasing. He was surprised, however, to find that no word was mentioned as to convergency, although this was precisely one of those cases in which convergency is all important, and in which the results obtained can be considered to have no value without an investigation on this point. In investigations made in the past on the intensity when spherical aberration was neglected, he believed that one of the chief difficulties encountered was that the series obtained had only a short region of convergence, and that to obtain results that could be applied to any point except in the immediate vicinity of the focus the series had to be altered. Under these circumstances, there was no justification for publishing results without any mention of convergence. He would point out that the order of the terms in the double summation had been altered without a word of justification, though the terms were alternately positive and negative, and so the series might be semi-convergent. The Paper was left in an undesirably incomplete form; either diagrams or tables should be added showing at a glance what the results which had been reached meant. He hoped that Mr. Walker would be able to make such an addition.

The AUTHOR has communicated the following reply: In commenting on a statement made by an author it is essential that his exact words should be given; to append a translation is not a customary course, and I prefer to assume that members of the Physical Society will have no difficulty in apprehending the meaning of the passage. However desirable it may be to give references to English books, I do not know of any in which the formulæ occur as quoted. From the equation of the wave it is obvious at once that p is the radius of curvature at the origin and therefore the focal length. It is stated that the formula for the length of path is only approximate. With regard to the convergency of the series and the change in the order of its terms, it did not seem to me necessary to point out in a Paper intended for mathematicians that the series is absolutely convergent, as the proof is simple; but I have now done so in a footnote. If I understand aright, Mr. T. Smith's statement (made with some uncertainty) about the series when spherical aberration is neglected, I do not think that it is correct. Lommel, in his work on Fresnel's diffraction phenomena with a circular aperture, of which the Fraunhofer diffraction phenomena are a special case, shows that his series is convergent. It is, however, the case that the formulæ have a limited range of applicability, dependent upon the error that may be regarded as admissible in calculating the retardation. Though some interesting results might, perhaps, be obtained by the addition of diagrams or tables, they would in the present case only serve to draw away attention from the main purpose of the Paper.

XXIV. *Demonstration of a Method of making Capillary Filaments.* *By* H. S. Souttar, *F.R.C.S.*

The apparatus demonstrated was designed by the author in 1901 for the making by mechanical means of tubes for the capillary electrometer, and with these tubes an extensive series of experiments was carried out in Oxford by the late Sir John Burdon Sanderson. A somewhat similar apparatus has since been devised independently by Mr. Keith Lucas. Recently it has appeared that it could with advantage be applied to the construction of fibres for the string galvanometer, and it is to this application that attention is now directed.

The essential part of the apparatus is a small box of mica, between the upper and lower plates of which a fine platinum wire is strung backwards and forwards so as to form a small cage enclosing a cylindrical space 1 cm. in diameter and 2·5 cm. deep. On passing a current through the wire it can be heated to redness and the box is converted into a small furnace. A small glass tube is hung in the centre of the furnace and supports a small weight. As the glass softens the weight falls and the glass is drawn out into a fine capillary.

Heating is limited to a minute portion of the glass tube by a copper tube which extends down a short distance into the furnace and, surrounding the glass, protects it from the heat. Only the portion of tube immediately below the shield is fused.

The weight is a long light rod with a disc on its lower end which nearly fits a tube into which it falls. The tube is filled with a mixture of glycerine and water, with the result that the weight falls smoothly and evenly till it reaches a stop.

It is easy with this appliance to draw down a tube of 1 mm. external diameter into a tube of any length and of a diameter of 0·001 mm., with a wall less than 0·0001 mm. in thickness. As the thick ends remain attached to the filament it is easy to manipulate.

These filaments have the following advantages: They are exceedingly light, 10 cm. of such a thread weighing roughly 0·1 milligramme. They are exceedingly strong, such a thread supporting a weight of at least 0·05 gramme, corresponding to a tensile strength of, roughly, 100 tons to the square inch.

The tubular form exposes a large area for silvering, and the resulting conductor has a low resistance.

DISCUSSION.

Prof. C. H. **Lees** asked if the filaments obtained were uniform in diameter.

Dr. H. **Borns** inquired how the filaments were silvered.

Mr. R. S. **Whipple** stated that the Cambridge Scientific Instrument Co. had given up quartz fibres in favour of glass ones for use in string galvanometers. Glass fibres could be silvered much more easily than quartz ones. Quartz fibres, if it was desired to silver them at all, should be silvered immediately after blowing. They found that solid fibres could be drawn better than capillaries. There was not so much variation in diameter. A fibre 24 cm. long could be obtained having the extreme diameters of 0·5 and 1·5μ, giving a length of 12 cm. in the centre, in which the diameter only varied from 0·9 to 1·1μ. In a quartz fibre 12 ft. long the diameter might vary from about 1μ to 4μ, giving an average taper of about 0·3μ per foot.

Mr. A. **Campbell** inquired whether it was possible to fill the capillaries with mercury. Such would form a wire for a string galvanometer with very little rigidity. Some years ago Mr. Sinker exhibited some very fine capillaries filled with mercury, and suggested a double capillary tube filled with mercury for the suspension of a moving coil galvanometer, thus doing away with the lower suspension altogether.

Mr. W. **Duddell** said he was greatly interested in 1897 in string galvanometers in connection with wireless telegraphy, and constructed one with a narrow strip of gold leaf. The gold leaf was embedded in paraffin wax and cut in a microtome. Such a filament about 0·25μ thick by 2 or 3μ wide made a string of extremely low rigidity. In this way it would be possible to get a short string galvanometer of a lower resistance and considerably greater volt sensitivity than the Einthoven galvanometer. Great progress had now been made in the manufacture of Wollaston wire which could also be used for such galvanometers.

The **Author**, in reply, stated that the filaments were somewhat tapered. The upper end was the thinner. The extreme variation in diameter was 50 per cent. The method of silvering was extremely simple. The filaments were suspended in the limbs of a U tube, which must be perfectly dry and clean, and the silvering solution poured in. The lactose method was employed, the lactose solution being added to ammoniacal silver nitrate solution immediately before use. He had thought of filling the capillaries with mercury instead of silvering them. It was not the mass of the fibre that was troublesome, so much as the rigidity. Speaking from memory, he said it would take about 10 atmospheres to force mercury into a capillary tube of 2μ bore.

XXV. *The Equipment of the Spectroscopic Laboratory of the Imperial College of Science and Technology.* *By* A. FOWLER, *F.R.S.*

READ MARCH 8, 1912.

LABORATORIES for advanced instruction and research in spectroscopy are not very numerous, and it has been thought that a brief account of the laboratory at the Imperial College might be of interest. The laboratory itself is of ample size (70 ft. by 30 ft.), and is well provided with electric current, slate benches, firm piers, and other conveniences.

The equipment may conveniently be considered under two principal heads: (1) Instruments for the observation and photography of spectra; (2) Apparatus for producing luminous radiations and for the study of absorptions.

As regards the instruments, spectroscopes of small dispersion are as necessary as those of greater power, but as these are of ordinary types no description is called for. They include several spectroscopes with one prism, pocket spectroscopes, large direct-vision spectroscope, wave-length spectroscope, a small spectrograph with a replica concave grating, and two spectrographs with optical trains of quartz. The smaller quartz spectrograph is often extremely useful in preliminary work, on account of its portability and rapid action, and because it gives a record of the entire spectrum from λ2,000 to λ7,000 in a single exposure. When more accurate wave-lengths in the ultra-violet are desired, the larger quartz spectrograph is employed.

These quartz spectrographs give only small dispersion in the visible spectrum, and for greater dispersion in this region two spectrographs with glass prisms are utilised. One of them gives the visible spectrum a length of 6 in. and the other about 2 ft. Both are mounted on the Littrow plan, in which the same lens serves for collimator and camera, and the light passes a second time through the prism after reflection by a plane mirror. This arrangement is both economical and convenient, and stability is easily ensured even when the focal length is great, as it should be in order to secure full resolution on the granular film of a photographic plate. The larger instrument is provided with a prism of 60 deg., having a clear aperture of nearly 3 in., and the focal length of the lens is 12 ft. The slit is at the side of the long rectangular box enclosing the optical parts, and light entering it is reflected to the collimating lens by a small right-angled prism; on the return journey, the rays forming the spectrum pass slightly above the reflecting prism. The spectrum is photographed in two sections on plates 12 in. in length. In the region of the D lines the linear dispersion of this instrument is 8 tenth-metres per millimetre, and in the neighbourhood of H and K it is 1·8 tenth-metres per millimetre. The resolving power is 21,000 at D and about 70,000 at λ4,000. With a slit 0·04 mm. in breadth, the realised separating power, or purity of spectrum, is about 10,000 at D and 30,000 at λ4,000. Most of the direct light from the slit which is reflected in the direction of the photographic plate by the surfaces of the lens is brought to a focus near the plate, and is thrown above the aperture where the spectrum is photographed by a slight tilt of the lens. In the smaller instrument the lens surfaces are such that it is necessary to put an opaque bar across the middle of the lens so that no direct light from the slit may be reflected into the aperture which admits the spectrum to the photographic plate.

The most powerful instrument in the laboratory is a Rowland concave grating of 10 ft. radius. A special mounting for this was designed by Mr. Eagle and constructed in the College workshop. The spectrum photographed is that which is formed in the direction of the incident light, so that the grating and camera can be completely enclosed in a long rectangular box. A full account of this instrument has already been published.* As compared with Rowland's form of mounting, this arrangement has the disadvantage that more adjustments are necessary to change from one part of the spectrum to another, but this is counterbalanced by numerous advantages. The instrument requires no specially darkened room, is easily maintained at constant temperature, permits the observation of spectra of higher orders, and gives much less astigmatism. The spectrum is not normal, but the reduction of wave-lengths involves no greater labour than the investigation of the errors in photographs taken with Rowland's arrangement. The grating is of fine quality and the definition leaves nothing to be desired. In the first order, the linear dispersion is about 5 tenth-metres per millimetre, and the realised purity is very nearly equal to the theoretical resolving power of 48,000.

An echelon grating with 13 plates is loaned by the Science

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* "Astrophys. Jour.," Vol. XXI., p. 120 (1910).

Museum as required, and other instruments of high resolving power are also available.

Spectroscopy finds many of its most interesting applications in Astrophysics, and a good deal of research work has been suggested by observations of the sun and stars. A spectroscopic department would therefore be incomplete without an observatory. Provision for work on solar and stellar spectra at the Imperial College is at present very modest, but very good so far as it goes. There are two equatorial telescopes, one of 5 in. and the other of 6 in. aperture. The 5 in. is a thoroughly modern instrument, with Zeiss triple objective and electrically controlled driving clock. It is provided with two objective prisms, one of large angle for bright stars, and another of smaller angle for stars down to about sixth magnitude. Excellent photographs of the spectrum of a first magnitude star, $1\frac{3}{4}$ in. in length from F to K, are obtained with an exposure of about 20 minutes. There is not much scope now for original work on stellar spectra with such an instrument, except in the case of bright new stars, but it is a valuable part of the equipment for purposes of instruction.

In the case of the sun, there is so much light that a powerful spectroscope can be applied even to a small telescope, and original observations of value are possible. For solar work there is an Evershed spectroscope, giving a purity of about 25,000 in the neighbourhood of the *b* lines. Special attention has been given to the spectra of sunspots and solar prominences.

The arrangements for producing emissions and absorptions for the observation of spectra are fairly complete, as will be gathered from the following brief statement of the experiments shown in simultaneous operation to the Society.

1. Spark given by resonance transformer, designed and constructed by Mr. Eagle. The capacity of the condenser is $\frac{1}{5}$ mfd., and the spark is bright enough to permit the use of the large grating in the comparison of arc and spark spectra. Such investigations of the varying spectrum of the same substance lead to a more complete interpretation of solar and stellar spectra. An arrangement for passing the transformer spark in gases under pressure, for the investigation of the displacements of spectrum lines, has just been completed.

2. Zeeman effect on yellow line of helium, shown in second order of the large concave grating, a Du Bois magnet giving a field strength up to 30,000 C.G.S. units being used.

3. Spark spectrum of magnesium with and without self-

induction. The air-lines which appear in the ordinary condensed spark disappear with self-induction, and the metallic lines approximate to those of the arc spectrum.

4. The spectrum of magnesium hydride, produced by passing an arc between magnesium electrodes in an exhausted globe; that is, in an atmosphere of rarefied hydrogen given off by the heated metal. The bands are an important feature in the spectra of sunspots.

5. Spectrum of iron arc, showing enhanced (spark) lines on positive pole, observed with four-prism spectroscope.

6. Spectrum of iron arc, showing performance of concave grating replica.

7. Fluorescence spectrum of uranium phosphate, with blue light from arc lamp as exciting source.

8. Cathode ray fluorescence spectrum of Yttria, and other experiments showing rapidity of exhaustion by the Gaede pump.

9. Vacuum tube showing positive and negative band spectra of nitrogen, exhausted by Sprengel pump.

10. Vacuum tubes showing different band spectra of carbon monoxide. At very low pressures the bands are identical with those given by the tails of comets.

11. Electromagnetic electrodeless ring discharge, produced in an exhausted globe by discharge from 10 in. coil with adjustable plate condenser.

12. Selective exhaustion by charcoal cooled with liquid air, illustrated by preparation of a helium vacuum tube from gases evolved by heated cleveite.

13. Production of spectrum of cyanogen by the interaction of carbon tetrachloride and active nitrogen. Purified nitrogen at low pressure is allowed to stream through a vacuum tube in which a condensed discharge is passing, and thence into a side tube, where it comes in contact with the substance to be investigated. As Prof. Strutt has shown, this is a valuable addition to our resources for the production of the spectra of certain elements and compounds.

14. Spectrum of flame carbon arc, observed with Evershed solar spectroscope and large Littrow spectrograph.

15. Absorption spectrum of neodymium chloride. Other apparatus for absorption experiments also shown.

For the measurement of spectrum photographs there are two stage micrometers, reading to 0·001 mm. Reduction of the measures to wave-lengths is simplified by the use of a calculating machine.

XXVI. *A 2,000-Frequency Alternator.* *By* W. DUDDELL.

RECEIVED MARCH 15, 1912.

FOR many experiments and tests it is necessary to use a high-frequency alternating current. This is especially the case in connection with investigations on telephone apparatus. In order to facilitate the true interpretation of the results it is desirable that the current should have as near as possible a sine waveform.

The alternator described below was primarily designed for use in testing telephonic apparatus, for which purpose frequencies within a range of 100 to 2,000 periods per second are required. This large range of frequency entails, if the alternator is to give sufficient output at the lower frequencies, that it will give what may seem an unnecessarily large output at the higher frequencies.

As a basis for the design it was assumed that the alternator must be able to give an output of some 10 watts at any frequency within the working range, in order that one might always have a large margin of power beyond that actually necessary to actuate the apparatus. This margin of power is very desirable, as, unfortunately, the measuring instruments often consume considerably more power than the apparatus under test.

The alternator itself is designed along the well-known lines of what are commonly called flywheel alternators. The rotor has 30 salient poles which carry the field windings. The stator, in order to obtain a smooth waveform, is not slot-wound but consists of a smooth ring provided with a gramme ring winding. The gramme ring winding of the stator has many advantages when a smooth waveform is required. It is true that it entails a comparatively long air gap between the stator and rotor iron, which leads to a somewhat large exciting current being required. This might be looked upon as a defect of this type of machine, but this is not really the case, as the long air gap enables the armature reaction to be kept low, so that the waveform of the machine is but little distorted by the load. This is a very important point, as many small machines which give a good wave on open circuit give a very bad one when on load.

The problem of obtaining frequencies even as low as 2,000 without unduly narrow poles can only be solved by using comparatively high peripheral speeds. These high peripheral speeds can be obtained either by using a large diameter rotor running at a medium speed, or by using a small diameter rotor with a few poles running at a very high speed. There are a large number of constructional details which influence the choice. The design finally adopted for the present machine was a rotor having a diameter of 20 cm. running at a maximum speed of 8,000 revs. per min.

With running speeds of this sort great care has to be exercised in the design, construction and balancing of the rotating part. In the present machine the rotor disc has the poles on it milled out of the solid. The poles are provided near their outside ends with V-grooves. The field coils are former wound and slipped over the poles. In order to hold the field coils securely in place bronze wedges are driven into the V-grooves between the poles. Even the leads from the slip rings to the field coils require very firm anchoring.

To make certain that the balancing of the rotor would remain stable, sufficient current was passed through the field windings of the rotor, which had been previously impregnated with shellac varnish, to heat them to a high temperature. The rotor was then rapidly run up to about 30 per cent. over full speed and the field current stopped so as to allow the rotor to cool while running over speed. This resulted in the windings being firmly compacted against the inside of the bronze wedges in a position where there was no tendency for the windings to shift again and put the rotor out of balance.

The shaft is mounted in long brass bearings having spherical seats and provided with ring lubrication. So far no trouble has been experienced with the bearings.

The alternator and driving motor are bolted to a plank with means to allow for adjusting the belt. The alternator, motor and plank are supported on three points, two of which consist of indiarubber dog balls which are placed on a line parallel to the axis of the motor which passes nearly under the centre of gravity of the whole arrangement. The third point consists of a piece of rubber placed at the alternator end which only carries a light pressure depending on the position of the two dog balls. The object of this arrangement is to as far as possible remove any vibration from my springy wooden floor which might interfere with tuned instruments, and in this respect has been quite successful.

The alternator is driven by means of a V-shaped leather belt from an electric motor, plain V-grooved pulleys being used and no precautions are taken to remove the pull off the bearings. It is found advisable to run the machine with the belt just on the slipping point, as under these conditions a very much more uniform frequency is obtainable.

When running at the higher speeds somewhat curious effects are observable on the current supplying the driving motor. The motor behaves as if it had attached to it a very large flywheel and could not therefore readily alter its speed, so that any small change in the supply voltage is accompanied by violent changes in the current through the driving motor. If the voltage drops suddenly, say, 2 per cent., the current may even reverse and the motor become a generator.

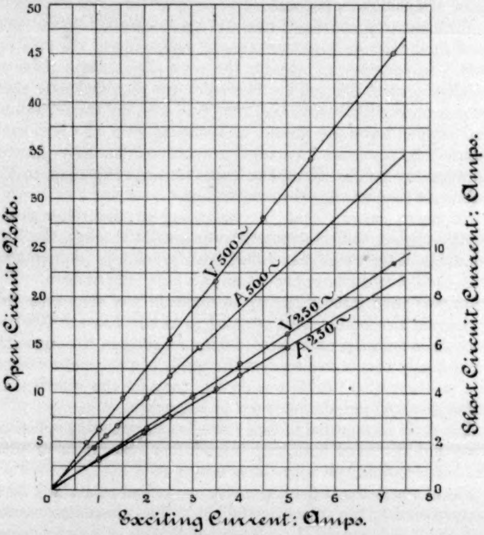

Fig. 1.

The open circuit characteristics of the machine and the short-circuit current curves for four different frequencies are shown in Figs. 1 and 2. The regulation on non-inductive load

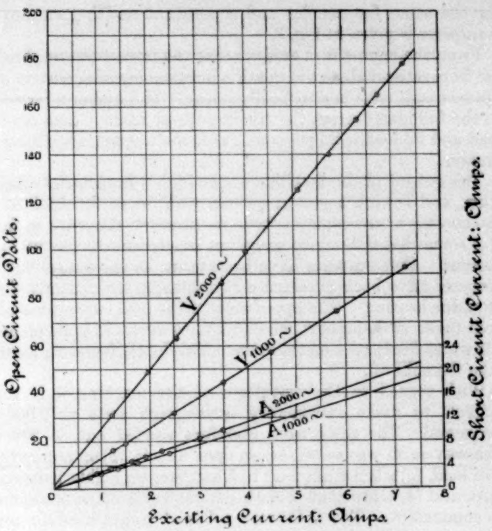
200
180
160
140
120
100
80
60
40
20
Open Circuit Volts.
V 2000 ~
V 1000 ~
A 2000 ~
A 1000 ~
24
20
16
12
8
4
0
Short Circuit Current: Amps.
0
1
2
3
4
5
6
7
8
Exciting Current: Amps.

FIG. 2.

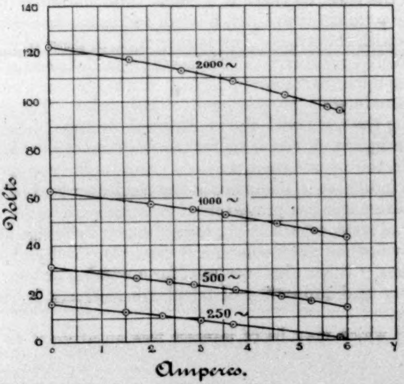
140
120
100
80
60
40
20
0
Volts
2000 ~
1000 ~
500 ~
250 ~
0
1
2
3
4
5
6
7
Amperes.

FIG. 3.

for the same frequencies and a constant exciting current of 5 amperes is given in Fig. 3.

From the curves it is evident that the iron of the machine is far from saturated, as the relation between exciting current and open-circuit volts is practically linear. If 5 amperes be taken as the full load current the rise in volts at 2,000 ∾ between full load and no load is 22 per cent.; at lower frequencies it is much greater.

The output of the machine at the higher frequencies attains ½ kw., and I think a greater output could be safely obtained as the cooling at the higher speeds is extremely efficient; in fact, it is somewhat difficult to assign an exact value to the full load current. Five amperes have been taken as the figure, but 10 amperes have been obtained on condensers without any signs of undue heating. The upper limit of the field current has also been taken at 5 amperes, which requires about 33 volts or, say, 165 watts dissipated on the rotor. This loss is found to hardly take the chill off.

With regard to the waveform of the machine it is very difficult to make satisfactory oscillograph tests at 2,000 ∾ frequency. The tests were therefore carried out at 500 ∾. The waveform was tested (*a*) on open circuit, (*b*) on non-inductive load, (*c*) when loaded up to 5 amperes on highly inductive coils, and (*d*) when loaded with various loads up to 5 amperes on condensers. The P.D. waves, Fig. 4, under the four conditions appear to the eye to be sine waves, but this is not really strictly the case, as there is a slight third harmonic present. In order to make it evident, a small condenser was connected in series with the strips of the oscillograph, in place of the usual non-inductive resistance. This has the effect of greatly magnifying the upper harmonics; in fact, the harmonics are magnified approximately proportional to their order. The result of repeating the tests with the oscillograph arranged in this way is shown in Fig. 5, which shows the presence of the third harmonic in the open circuit wave (*a′*) and in the non-inductive (*b′*) and inductive (*c′*), but not in the condenser load (*d′*).

The alternator has proved itself extremely useful for a large number of tests, especially the measurement by bridge methods with a vibration galvanometer of capacities, self and mutual inductions. It has also been used for a number of tests on the sensibility and properties of telephone receivers for wireless purposes.

A test which may be of interest was an attempt to repeat

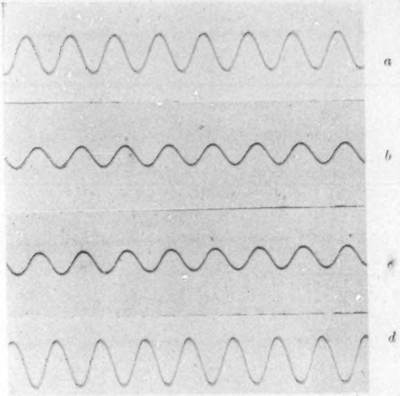

Fig. 4.—Oscillograph, Ordinary Connections.

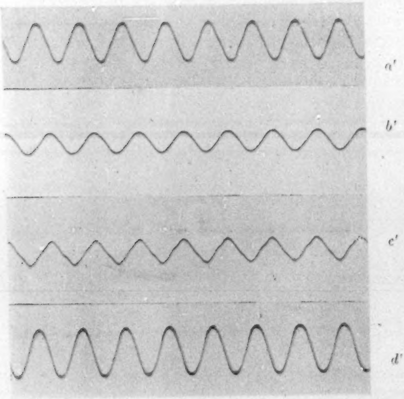

Fig. 5.—Oscillograph Strips in Series with a Condenser.

To face p. 176.]

Dr. Silvanus Thompson's experiment on the effect of alternating magnetic fields on the human being. According to one explanation of this effect, it is produced by E.M.F. induced in the head. If this is the case it should be easier to produce it the higher the frequency, so that one might expect that with 2,000 frequency it would only require $\frac{1}{40}$th of the field strength that is required at 50 periods to make the effect apparent. Dr. Thompson used some 200 to 400 amperes at 50 $\sim$ in a 60-turn magnetising coil, so that 5 to 10 amperes in a similar coil at 2,000 frequency might be expected to produce a similar result. A test was made with 7 amperes at 2,000 frequency and a 60-turn magnetising coil placed quite close to the head, 30 turns on each side, but neither I nor my assistant could perceive any effect whatever.

I wish in conclusion to express my indebtedness to my assistant, Mr. Swann, for making the design and all the tests of the alternator.

ABSTRACT.

The author described and exhibited a small 2,000-frequency alternator which he had designed and had constructed for testing purposes, mainly in connection with telephonic measurements. The alternator is of the salient pole revolving field type running at 8,000 revs. per min. The stator is smooth and wound like a gramme ring, the object being to obtain as near as possible a sine wave.

The author gave curves showing the open circuit characteristic, the short-circuit current and the regulation of the alternator at four different frequencies. The output of the alternator amounted to as much as ½ kw. at the higher frequencies.

Oscillograph curves of the alternator on open circuit and when loaded with non-inductive resistance, inductances and condensers were shown. These appeared to be practically sine waves. This is not, however, strictly the case, as there is a slight third harmonic present. To demonstrate this a further set of oscillograph curves of the P.D. of the alternator were exhibited. In taking these curves the non-inductive resistance in series with the oscillograph strips was replaced by a small condenser so as to accentuate the upper harmonics.

The author described an attempt at 2000 frequency, which had failed, to repeat Dr. Thompson's experiment on the effect of an alternating magnetic field on the brain.

DISCUSSION.

Mr. B. S. COHEN stated that Mr. Duddell had supplied one of his alternators to Telephone House, and it had given thorough proof of its reliability and usefulness. He exhibited characteristics and oscillographs taken from the machine. The third harmonic that was present was of no practical detriment. If the capacity of a microfarad mica condenser was found from the current taken by it and the voltage applied the error was less than 0·5 per cent. at a frequency of 1000. The only source of trouble had been in the driving belt, but that had been satisfactorily overcome. The instrument lent itself to a variety of interesting uses. If a telephone be put across the machine when run at the resonating frequency the amplitude of vibration was such that matches could be blown out at a distance of 3 or 4 metres.

Mr. CHURTON asked whether Mr. Duddell needed a constant voltage at different frequencies, as this would necessitate a very high flux at low frequencies and vice versa.

Mr. A. CAMPBELL expressed interest in the efficient manner in which Mr. Duddell had been able to avoid vibration in the mounting of the motor. Even a wire interrupter may disturb a vibration galvanometer (in tune with it) by vibrations mechanically transmitted across a large room. He mentioned that a vibration galvanometer of low ohmic resistance could be used to very largely suppress any harmonic to which the galvanometer was tuned, as the apparent resistance for this harmonic would be very high.

Mr. S. A. POLLOCK described a somewhat similar machine that had been constructed five years ago and was still running at the G.P.O. The rotor contained 60 poles which were wound zigzag with eight turns of wire. The stator was wound with four turns per pole. The machine was constructed from the castings of an ordinary small alternator, in which the stampings had been removed and replaced by Stelloid stampings wound as above. The machine had been run up to frequencies of 8,000, and was perfectly balanced at the highest speed. He exhibited oscillograms of the waveform, which were of the same general type as those from Mr. Duddell's machine. The form remained practically independent of the load. The machine was largely used for measuring the apparent resistance of loading coils. The coil was placed in series with a variable air condenser and an alternating-current ammeter. A Duddell galvanometer was placed across the coil and condenser and used as a voltmeter. The condenser was varied so as to obtain resonance, which was obtained by making C/E a maximum, in which case the reciprocal of this ratio was the apparent resistance of the coil. The method was so sensitive that a brass screw brought near to the coil appreciably altered its apparent resistance. The effective resistance of coils found by this means several years ago agreed to within ½ per cent. of the value found recently by Anderson's method. To keep the frequency constant a vibrating reed was placed in series with a split commutator fixed on the shaft of the motor and connected in series with the armature. If the motor ran out of phase with the reed either an accelerating or retarding current was sent through the armature. A counter and a chronograph always showed that the number of revolutions per minute was exactly the same as the vibrations of the reed. The wave-form was purified by means of sifting circuits.

Dr. W. E. SUMPNER commented on the small drop in voltage at the high frequencies. From a scientific point of view the chief points in the machine were the large air gaps and the unslotted armature. It was to these that the good regulation was due.

Mr. W. H. F. MURDOCH asked Mr. Duddell if, in order to increase the output of the alternator, artificial cooling might not be used here with advantage. For instance, since the winding might be a zigzag one of only one turn, this might consist of a copper tube through which oil or other cooling media might be passed. It seemed to him that in this way very large outputs could be obtained, speaking comparatively.

The AUTHOR, in reply, stated that the machine supplied to Mr. Cohen was one of the three original ones. In it the armature is wound in three sections, which can all be placed in parallel or in series. This was in order to obtain any required voltages without having to use a transformer, which might distort the wave-form. In reply to Mr. Pollock, he stated that the zigzag form of winding was an excellent one for the highest frequencies. He mentioned that Sir David Salomons had constructed an alternator with a zigzag winding giving a frequency of 10,000. This was a prototype of the modern alternators giving frequencies of 100,000. He also referred to the small machine he built many years ago at the Central Technical College, giving a frequency of 120,000. Mr. Pollock's method of getting resonance and then finding the losses was a good one, as was also the Anderson's method. In reply to Dr. W. E. Sumpner he stated that the good regulation possessed by the machine was helpful in avoiding distortions of all kinds. He was indebted to Messrs. Muirhead & Co. for having built the machine.

XXVII. *A Method of Comparing Capacities at Various Frequencies.* *By* ALBERT CAMPBELL, *B.A.* (*From the National Physical Laboratory.*)

RECEIVED MARCH 13, 1912. READ MARCH 22, 1912.
REVISED MARCH 28, 1912.

FOR the accurate determination of a capacity in terms of a resistance and a frequency Maxwell's commutator method (shown in Fig. 1) is well known.* It gives excellent results with condensers of all kinds, and two capacities can be compared with one another, by testing one after the other, by this method. But such a comparison cannot be made quickly, as the accurate determination of the frequency requires a run of eight or ten minutes on the chronograph in each case.

It would be possible to avoid this trouble by having two separate complete bridges (each as in Fig. 1) with the two commutators (C_1 and C_2) mounted on one spindle, thus making the frequency identical for each condenser.

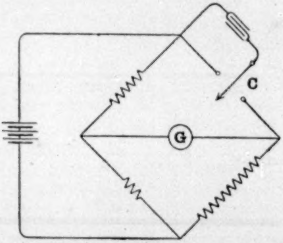

FIG. 1.—MAXWELL'S COMMUTATOR METHOD.

A simpler method, however, is shown in Fig. 2; it gives a comparison under much the same conditions of charge and discharge as hold in the Maxwell method.

A rotating double commutator is arranged, as shown, to charge and short-circuit each condenser simultaneously, the resistances R and S and the frequency being such that the charge and discharge are practically complete for both K_1 and K_2 in each cycle. The galvanometer must be of long period.

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* In Maxwell's original description a rocking commutator ("wippe") was mentioned. In 1888 Dr. Fleming brought the method into its present form by the introduction of a *rotating* commutator.

Let us first assume that the two commutators are so mechanically perfect that their corresponding contacts are made simultaneously. When K_1 and K_2 are of the same type as regards absorption and leakage, the current through the galvanometer will be practically zero at every instant if $R/S = K_2/K_1$.

If the condensers are not of the same type an alternating current will pass through the galvanometer, but, by suitable adjustment of the ratio R/S, the mean current can be brought to zero and an exact balance obtained. To a near approximation we may then take $K_2/K_1 = R/S$.

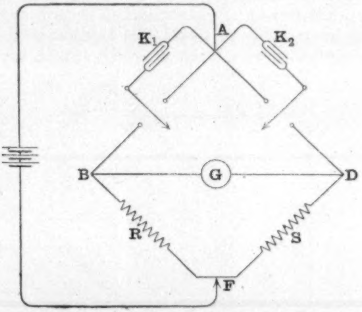

FIG. 2.—DOUBLE COMMUTATOR METHOD.

Especially at tolerably high frequencies it is not possible to ensure that the two commutators start the charging of each condenser at precisely the same instant,* and so, to eliminate any errors due to the difference between the two commutators, a second balance should be taken with the commutators interchanged. The mean of the two results gives the ratio of the capacities to a high degree of accuracy. The truth of this statement has been established by experimental evidence, as several condensers of widely different absorption have given results consistent with those of other methods even when the galvanometer resistance has been varied from 700 to 10,000 ohms, R and S being of the order of 500 to 1,000 ohms. When the condensers differ much in absorption the complete theory of what occurs during the periods of charge is very difficult, as the laws of absorption are still too obscure to give sufficient basis to work upon. Further experiments, however, are desirable. The advantages of the method are that alternating current is not required, and that, as a direct-current galvanometer is used, very high sensitivity can be attained. With condensers of 1 mfd. each, a reasonably sensitive galvanometer and a battery of 20 volts, a change of 1 part in 1,000,000 in one condenser gives a deflection of 0·5 mm. at 1 meter. Frequencies from 5 up to 200 ∿ per sec. have been used.

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* Mr. D. W. Dye has suggested that this could be ensured by arranging another part of the commutator to break the battery circuit at A just before, and make it just after, the condensers come into connection with B and D.

The methods (such as Wien's or Carey Foster's) in which alternating current is used are really preferable, although, when the highest accuracy is desired, the very small residual inductances of the resistances used have to be taken into account. The latter methods employ a sine wave voltage, the commutator methods a voltage of square-topped wave form. A long series of experiments which we have made appears to have established the following relation between the methods. *For condensers with negligible leakage the results of the two kinds of method are in agreement, provided that the respective frequencies are chosen so that the period of charge (or the equal period of discharge) with the square-topped wave is equal to* $2/\pi$ *times the half period of the sine wave.* When this condition is satisfied the results given by the two kinds of wave-forms come into agreement within one or two parts in 100,000 over a wide range of frequency. By "leakage" I here mean what is sometimes called "direct-current leakage."

An account of the experimental work here mentioned will appear in a paper shortly to be published on the determination of the Ohm in absolute measure.

ABSTRACT.

To compare the capacities of two condensers under conditions similar to those in Maxwell's commutator method, they are charged and short-circuited periodically in two adjacent arms of a bridge by a double rotating commutator, the corresponding contacts being made simultaneously. If the other arms of the bridge are resistances R and S, and a battery and direct-current galvanometer (of long period) are used, then a balance is obtained when $K_2/K_1 = R/S$. The method is highly sensitive. Methods (like the above) in which the

applied voltage has a square-topped wave-form give results in agreement with those using sine wave-form when the period of charge (and equal discharge) with the square-topped wave is $2/\pi$ times the half-period of the sine wave.

DISCUSSION.

Dr. J. A. **Fleming** remarked: Mr. Campbell's suggestion for using a double commutator for comparing capacities on a Wheatstone's bridge is not altogether new. Many years ago a double commutator was made in the Pender Electrical Laboratory at University College for this identical purpose, and the commutator part of this apparatus is exhibited by me on the table. Mr. Campbell speaks of the method of using a single commutator as "Maxwell's Commutator Method." As a matter of fact, the kind of commutator Maxwell suggested was a tuning fork, and this particular mode of commutation was mentioned in a Paper by me on "Problems on the Distribution of Electric Currents in Networks of Conductors," read before this Society in June, 1885. It was found, however, in practice that the tuning-fork commutator did not allow sufficient time for the contacts, and when this method of determining the capacity of a condenser was used by Sir J. J. Thomson in 1883 and by Dr. Glazebrook in 1884 at Cambridge they used the tuning fork to control another make-and-break contact of the vibrating-reed type. In the year 1888 I sent a short Paper to be read at Section A at the meeting of the British Association at Bath, in which a motor-driven revolving commutator was described for this purpose. In 1903, in conjunction with Prof. Clinton, I read a Paper before this Society in which a much more perfect motor-driven commutator was described for capacity measurements, and also a double commutator for inductance measurements. This motor-driven commutator has been used for nine years with the greatest success. To avoid the trouble of standardising the galvanometer, and for the sake of comparing two condensers by a null method, a double commutator on one shaft was made a few years later. Since a condenser plus a commutator behaves like a resistance, and lets through a certain quantity of electricity per second, it was an obvious application to insert in the two branches of a Wheatstone bridge two condensers and a double commutator, so arranged as to charge and discharge the condensers simultaneously. The method seemed so obvious that we did not trouble to describe it. The double commutator was also intended to be used in another way to measure the absolute capacity of a guard tube or guard ring condenser. It is well known that when a condenser of predetermined capacity with guard ring or tube is measured by the commutator method the guard tube must be discharged at the same instant as the main condenser, only the guard tube is discharged through a bye-pass circuit and the main condenser is discharged through the galvanometer. This method was a common subject of conversation in my laboratory some years ago. The double commutator method can also be used with advantage for comparing together by a bridge method the capacity of a small adjustable air condenser with the capacity of a wireless telegraph antenna. The small air condenser can have its capacity determined by the Carey Foster method, and it is then not necessary to carry about apparatus for standardising the galvanometer. In the case of our double commutator there is a counting mechanism which enables the operator to know the frequency of the discharges and to regulate them, and also special means for preventing the battery sending current directly through the galvanometer.

Dr. **Russell** queried the utility of finding the ratio of the effective capacities of two condensers having different absorptive properties by a double commutator method. He also asked whether the author's experimental result could not be expressed more simply by saying that the ratio of the effective capacities was the same with sine and square-shaped periodic waves when the charges given to the condensers were the same in the two cases.

Mr. E. H. RAYNER pointed out that in tests of condensers with alternating currents the polarisation was continually reversed, but in the present method the polarisation was always in the same direction.

The AUTHOR, in reply, said that the method did not appear to have been described before. He was interested to hear that Dr. Fleming had been using it for some time past. In reply to Dr. Russell, he pointed out that in the alternating-current methods the absorption was separated from the capacity, and hence the latter might reasonably be called the true capacity at the particular frequency used. He did not think Dr. Russell's remark about the curves having equal areas was quite valid. In reply to Mr. Rayner, he stated that when, in Maxwell's method, a reversing commutator was used, it gave with a mica condenser exactly the same results as the usual short-circuiting commutator.

XXVIII. *The Coefficients of Expansion of Fused Silica and Mercury.* *By* H. Donaldson, *B.A.*, *B.Sc.* (*The National Physical Laboratory*).

Received March 14, 1912. Read March 22, 1912.

Received in Revised Form June 1, 1912.

In a Paper read before the Society on November 24, 1911, Mr. F. J. Harlow gave values for the coefficient of cubical expansion of fused silica determined by observations of the apparent expansion of mercury in a silica envelope, assuming the absolute expansion of the mercury to be that given by Callendar and Moss.* The values so deduced differ somewhat widely from those calculated from the accepted values of the linear expansion, and Harlow suggested in explanation of this that these accepted values might be unreliable, as they had usually been obtained from observations on small specimens of silica whose actual expansions, over the range of temperature used, would be very small. For this reason, the writer's results for the linear expansion of silica, obtained from observations on the silica metre of the National Physical Laboratory, may be of interest. The method employed has been that of comparison, over a range of temperatures from about 0°C. to about 30°C., of the length of the silica metre with that of standard metal metres. The expansion coefficients of the three metal metres used, invar, nickel and nickel steel (43 per cent. nickel) were determined at the Bureau International, Sèvres, by comparison with their platinum iridium standard No. 13, the value of whose expansion coefficient was obtained from measurements in an absolute dilatometer and is not dependent on an optical method. Observations of the difference in length between the silica metre and the standard were taken, in each case, at six temperatures in the range above given in the Laboratory linear comparator, and the coefficient of linear dilatation of the silica was deduced in parabolic form by the method of least squares. The coefficients of linear dilatation of the standard metres, as given by the Bureau International, are

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* "Phil. Trans.," A Vol. 211, p. 1, 1911.

1. Invar: $L_t = L_0\{1+(1\cdot152t-0\cdot00061t^2)10^{-6}\}$.
2. Nickel: $L_t = L_0\{1+(12\cdot359t+0\cdot00925t^2)10^{-6}\}$.
3. 43 per cent. nickel steel:

$$L_t = L_0\{1+(8\cdot031t-0\cdot00269t^2)10^{-6}\}.$$

And those of the silica metre obtained from the comparisons are

1. From the invar metre

$$L_t = L_0\{1+(0\cdot38t+0\cdot0013t^2)10^{-6}\}$$

2. From the nickel metre

$$L_t = L_0\{1+(0\cdot35t+0\cdot0016t^2)10^{-6}\})$$

3. From the 43 per cent. nickel-steel metre

$$L_t = L_0\{1+(0\cdot41t+0\cdot0005t^2)10^{-6}\}.$$

The degree of concordance between these three determinations is shown in the following table of mean expansions over different ranges of temperature :—

Temperature range.	Mean coefficient of linear expansion of silica.		
	1	2	3
0°C. to 10°C.	$0\cdot39\times10^{-6}$	$0\cdot37\times10^{-6}$	$0\cdot41\times10^{-6}$
10°C. to 20°C.	$0\cdot41\times10^{-6}$	$0\cdot38\times10^{-6}$	$0\cdot42\times10^{-6}$
20°C. to 30°C.	$0\cdot42\times10^{-6}$	$0\cdot40\times10^{-6}$	$0\cdot42\times10^{-6}$

The comparison against the invar metre is probably affected to some extent by the thermal hysteresis inherent to invar, though this was kept quite small by slow heating and cooling. The observations taken of the length of the silica metre at the ordinary room temperature before and after the determination of the coefficient of expansion showed no evidence of hysteresis in that standard. Owing to the hysteresis of the invar metre it might be more just in deducing the final mean value for the silica dilatation to give the determination (1) only one-half the weight of the other two determinations, but whether this is done or not, the mean equation is

$$L_t = L_0 1+(0\cdot38t+0\cdot0011t^2)10^{-6}\},$$

which may be compared with Chappuis' equation from the Fizeau optical method

$$L_t = L_0\{1+(0\cdot385t+0\cdot00115t^2)10^{-6}\}.*$$

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* "Procès-Verbaux," Comm. Int. des Poids et Mesures, 1903, p. 75.

Kaye* has collected and reduced to a mean curve the results obtained by many observers for the coefficient of linear expansion of fused silica, and the agreement between this curve and the values from the silica metre is good, as may be seen from the table below:—

Temperature range.	Mean linear expansion coefficient.	
	From Kaye's curve.	From silica metre.
0°C. to 10°C.	$0{\cdot}40\times10^{-6}$	$0{\cdot}39\times10^{-6}$
10°C. to 20°C.	$0{\cdot}41\times10^{-6}$	$0{\cdot}40\times10^{-6}$
20°C. to 30°C.	$0{\cdot}42\times10^{-6}$	$0{\cdot}41\times10^{-6}$
30°C. to 40°C.	$0{\cdot}43\times10^{-6}$	$0{\cdot}42\times10^{-6}$

The formula obtained from the silica metre is capable of extrapolation, even up to 100°C., without appreciable divergence from the previously accepted value for the coefficient, for we have

Value for range 0°C. to 100°C. from Kaye's curve

$$=0{\cdot}49\times10^{-6}.$$

Value for range 0°C. to 100°C. from silica metre by extrapolation $=0{\cdot}49\times10^{-6}$.

This is fairly conclusive evidence that, in spite of the small expansions to be measured, and the consequent risk of complications from changes in surface conditions, due to heating, there is no serious error, such as suggested by Harlow's results, in the application of the Fizeau interference method to the measurement of the linear expansion of silica.

It is not to this, therefore, that we can look for the explanation of the difference between Harlow's values for the coefficient of cubical expansion and those deduced from the linear coefficient. These differences are great, both in the absolute value of the coefficient for the fundamental interval 0°C. to 100°C., and in its differentiation with respect to temperature, for we have

Temperature range.	Calculated mean cubical coefficient.	Harlow's mean cubical coefficient.
0°C. to 100°C.	148×10^{-8}	100×10^{-8}
0°C. to 184°C.	154×10^{-8}	145×10^{-8}

These errors may lie either in bad observations, which appears unlikely from the excellent concordance between

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* "Phil. Mag.," October, 1910.

Harlow's different series of experiments, or in the value assumed for the coefficient of cubical expansion of mercury, or in some want of agreement between the linear and radial expansions in a silica tube. Now the method of obtaining the value of a small quantity (the expansion of silica) by means of the difference between two large quantities (the absolute expansion of mercury, and its expansion relative to silica) must always be doubtful, and, in the case under consideration, it is rendered more so by the serious discrepancies which exist between the standard determinations of the absolute expansion of mercury.

The two most recent of these determinations are those of Chappuis* and of Callendar and Moss,† the latter of which Harlow has taken as the basis of his calculations. The method of Callendar and Moss was a more delicate modification of Regnault's hydrostatic method, and was used over a range of temperature extending from just below 0°C. to 300°C. Chappuis' observations were mainly confined to temperatures between 0°C. and 44°C., with one well-determined point near 100°C., and were obtained with an apparatus which was a combination of a weight thermometer and a graduated tube, the method involving a knowledge of the coefficient of cubical expansion of the glass of which the vessel was made. The value for this cubical expansion was calculated from the value for the linear expansion determined in a comparator. The results from the two methods show serious differences, the values for the cubical expansion coefficient for the range 0°C. to 100°C. being

Chappuis, 0·000,182,551‡
Callendar and Moss, 0·000,182,054.

The difference between these two values ($49{\cdot}7 \times 10^{-8}$) is of the greatest importance in dealing with the small expansion of fused silica ; so much so, in fact, that a use of Chappuis' value, instead of that of Callendar and Moss, in Harlow's calculations, leads to a mean cubical coefficient for silica, over the range 0°C. to 100°C., of $148{\cdot}6 \times 10^{-8}$, which is in very good agreement with the value, 148×10^{-8}, calculated from its known linear coefficient. Hence, the writer contends that Harlow's observations give no reason for doubting either the previously accepted values for the coefficient of linear expansion of silica or the application to that substance of the usual method of calculating the cubical expansion from the linear.

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* "Trav. et Mem. Bur. Int.," XIII., 1907.

† *Loc. cit.*

‡ In Chappuis' original memoir the calculated cubical coefficient, by an arithmetical error, differs slightly from that quoted here. The required correction has been applied throughout this Paper. *See* Eumorfopoulos, "Proc. Roy. Soc.," June, 1908.

On the assumption of the values for the expansion of silica given by Kaye,* Harlow's observations give the following values for the mean coefficient of expansion of mercury :—

0°C. to 100°C. 0·000,182,545.

0°C. to 184°C. 0·000,183,802.

The value at 100°C. agrees very closely indeed with that of Chappuis previously quoted, but Chappuis differs widely from Harlow's value, and indeed from all other values, at 184°C.

In this connection it is to be noted that, with the exception of the well-determined point at 100°C., all Chappuis' observations are limited to the region 0°C. to 45°C. In such a distribution, very small errors in the observations at the lower temperatures, such as might arise from any lack of homogeneity in the material of the bulb of the weight thermometer, would suffice to produce a considerable effect on the cubic term of the resulting equation, and so cause it to be unreliable for extrapolation to temperatures appreciably beyond the actual range of observation.

From these considerations it is probable that a parabola through Chappuis' observations, though giving rather larger residuals in the region 0°C. to 45°C., should extrapolate better than the cubic equation. The parabolic formula so obtained is

$$V_t = V_0(1 + 0{\cdot}000,181,317t + 0{\cdot}000,000,012,29t^2),$$

which gives, for the mean expansion between 0°C. and the higher Harlow point (184°C.) the value

0·000,183,578

as against 0·000,183,802 from Harlow's observations

and 0·000,185,044 from Chappuis' cubic formula.

It will immediately be seen how much nearer the extrapolated parabola is to Harlow's observation than is the extrapolated cubic equation. This fact, and the general type of agreement between Harlow's different sets of observations, lead one to consider his point at 184°C. to be a reliable one. A new cubic equation has, therefore, been calculated, by the method of least squares, to represent the combination of

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* *Loc. cit.*

Chappuis' observations, corrected as explained in the previous footnote, with those of Harlow. The cubic equation is

$$V_t = V_0(1 + 0{\cdot}000{,}181{,}385t + 0{\cdot}000{,}000{,}009{,}770t^2 + 0{\cdot}000{,}000{,}000{,}018{,}318t^3),$$

and the resulting curve passes practically through the Chappuis-Harlow points at 100°C., and through the Harlow point at 184°C., while over the range 0°C. to 100°C. it has a probable error considerably less than twice that of Chappuis' own cubic equation, with the additional advantage of increased reliability from the longer observational range.

The curves corresponding to the various formulæ are shown in the two figures. The scale is such that if a is the mean expansion coefficient between 0°C. and t°C. on any formula, then the ordinate corresponding to that formula at the temperature t°C. is equal to

$$t(a - 0{\cdot}000{,}182{,}545).$$

The maximum difference between the two cubic curves over the observational range from 0°C. to 45°C. is seen to be no greater than the range of variation of the experimental points, thus clearly indicating the unreliability which must attach to Chappuis' cubic when extra-polated. It may be noted that not only does that equation give values too high at temperatures above 100°C., but also begins to turn back between 0°C. and -25°C. In Fig. 2 it is also to be seen that the difference between Harlow's and Chappuis' observations at 100°C. is distinctly less than Chappuis' own experimental errors, so that, if the value of the silica expansion is that assumed in the present Paper, Harlow's determination may be taken as absolute confirmation of Chappuis' work.

The evidence for accepting Chappuis' observations as the best up to the present time has been clearly put forward during the writing of the present Paper by Scheel,* and by Scheel and Heuse,† from the Reichsanstalt and it is, therefore, unnecessary to repeat it here.

No observations of mercury expansion at temperatures above 184°C. have been used in calculating the curves in the present Paper. The only ones available are those of Callendar and Moss, and of Regnault. In the former case there seems

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* "Zeit. für Instrumentenkunde," Vol. 32, p. 14, 1912.

† "Phil. Mag.," March, 1912.

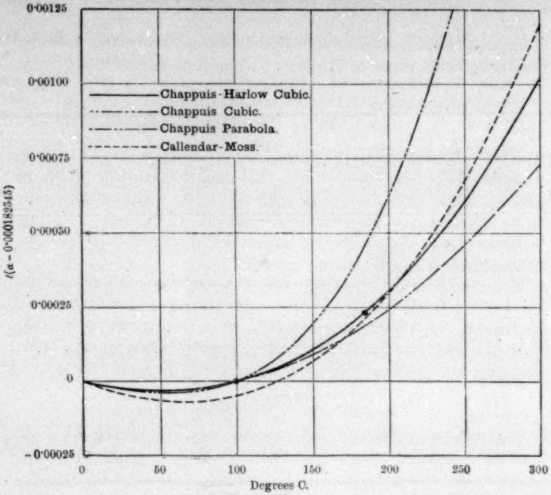
0·00125
0·00100
0·00075
0·00050
0·00025
0
−0·00025
$t(a - 0·000182545)$
Chappuis-Harlow Cubic.
Chappuis Cubic.
Chappuis Parabola.
Callendar-Moss.
0
50
100
150
200
250
300
Degrees C.

FIG. 1.

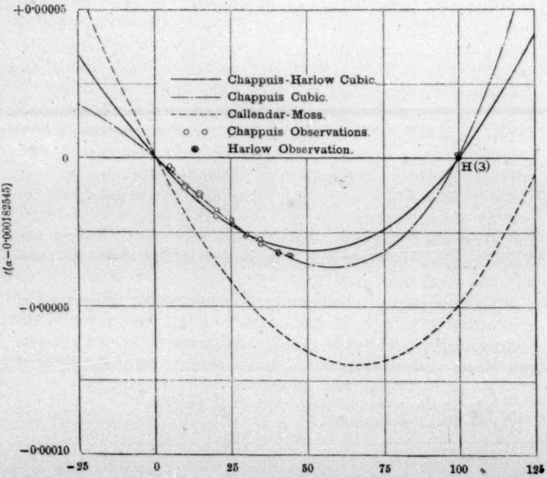
+0·00005
0
−0·00005
−0·00010
$t(a - 0·000182545)$
Chappuis-Harlow Cubic.
Chappuis Cubic.
Callendar-Moss.
○ ○ Chappuis Observations.
● Harlow Observation.
H(3)
−25
0
25
50
75
100
125

FIG. 2.

to be some unsuspected source of experimental error in the determinations at lower temperatures at any rate, and, in that of Regnault, the uncertainties in the various methods of reducing his observations, and the widely different values obtained by the different methods of reduction, make it impossible to draw any very definite conclusions from them.

In conclusion, it may be said that, although Harlow's work has been of considerable help in the difficulties connected with mercury expansion, there are still some points which require further investigation. The suggestion of this Paper is that such points could best be elucidated by further experiments with a silica weight thermometer. The close agreement between Chappuis and Harlow at 100°C. in mercury expansion using glass and silica bulbs, and between Chappuis' values for water expansion using glass and platinum-iridium bulbs, seems to show that, in these substances, at any rate, no serious error is introduced by calculating the cubical expansion from the linear. Still, a difference of 2 per cent. between the actual and the calculated values for glass would be sufficient to account for the whole difference between the results obtained for mercury by Chappuis and by Callendar and Moss, but in fused silica a difference of the same absolute magnitude would mean an error of 50 per cent. in the calculated cubical expansion.

The present work on the silica metre has been done on a specimen almost identical with that which would be used as a weight thermometer (the bulb of Chappuis' glass instrument was 1 metre long) and the reference lines were on planes *beyond* the sealed-off ends of the tube.

There is the further advantage in silica of a very definite zero-point, owing to the absence of thermal hysteresis, and, if this zero point were determined frequently, any secular change in the volume of an annealed specimen should certainly be less than the errors of experiment in weight thermometry.

The writer wishes very sincerely to thank Mr. J. E. Sears, in charge of the Metrology Division of the Laboratory, for much help freely given during the preparation of this Paper.

ABSTRACT.

It is shown by experiments on the silica metre of the National Physical Laboratory that the values for the coefficient of expansion of fused silica obtained by previous observers on small specimens by optical methods are confirmed by the comparator method with a long specimen. These values are then used to obtain, from Harlow's experiments (" Proc. Phys. Soc.," XXIV., p. 30, December, 1911),

values for the mean coefficient of expansion of mercury for the ranges 0°C. to 100°C. and 0°C. to 184°C. The use of these in conjunction with the previous observations of Chappuis on the expansion of mercury leads to the representation of that expansion by the equation:—

$$V_t = V_0(1 + 1{\cdot}81385 \times 10^{-4}t + 9{\cdot}770 \times 10^{-9}t^2 + 1{\cdot}8318 \times 10^{-11}t^3).$$

Other formulæ for the representation of the same quantity are quoted and examined, and it is suggested that further experiments with a silica weight thermometer would go far in elucidating the points of difficulty connected with them.

The discussion on this Paper was adjourned to the next meeting. Prof. Callendar's contribution to the discussion took the form of the following Paper.

XXIX. *On the Expansion of Vitreous Silica.* *By* H. L. Callendar, *M.A., LL.D., F.R.S.*

Prof. Callendar begged leave to re-introduce to the notice of members the method and apparatus for the investigation of the expansion of vitreous silica which he had described and demonstrated at a meeting of the Society on March 22, 1901. The late Mr. A. W. Shenstone, whose work as the pioneer of vitreous silica was so well known, had on that occasion kindly provided him with two rods of the material 1·5 mm. in diameter and 40 cm. long, and also with a tube 1 cm. diameter and 30 cm. long. The rods were electrically heated in a very thin and uniform platinum tube, 3 mm. in diameter, the resistance and expansion of which had previously been investigated with great care in connection with some experiments on the effusion and transpiration of gases at high temperatures. The method possessed special advantages for work at high temperatures because the rod could be raised to any desired temperature not exceeding 1,700°C. in three of four minutes with little or no disturbance of the surrounding conditions, and the mean temperature could be inferred with considerable accuracy either from the expansion or the resistance of the platinum tube. The preliminary results of the investigation were published by Mr. Shenstone in his lecture on "Vitrified Quartz" at the Royal Institution on March 8, 1901. The final results, communicated to the Physical Society on March 22, 1901, were reported in the "Chemical News" of March 29th and other papers, but were not considered to be of sufficient importance to merit more extended publication at that time. In view, however, of various misunderstandings which had arisen, and of the increased interest taken in the question at the present time, it might be worth while to give a brief summary of the old results. The expansion was found to be accurately reversible up to 900°C. with a nearly uniform coefficient, approximately $0{\cdot}58 \times 10^{-6}$ between 300°C. and 900°C. There was a well-marked flat or change-point in the curve near 1,000°C., above which the slope resumed its original value as far as 1,400°C. When maintained at a steady temperature in this region the rod showed a continuous increase of length, accompanied by marked superficial devitrification. In order to separate the permanent elongation due to this cause from the reversible expansion, a variable zero method was adopted. The current was switched off immediately after each reading at high temperatures, and the zero observed as soon as the apparatus had cooled to atmospheric temperature. The contraction gave the reversible effect, the difference of the zeros before and after, the permanent elongation. The latter change was found to occur more rapidly as the temperature was increased. Above 1,400°C. a reversible contraction was observed, which was demonstrated at the meeting on a greatly magnified scale by means of an optical lever. After repeated heatings to 1,600°C. or 1,700°C. the change point at 1,000°C. disappeared, and the expansion became more uniform, but the devitrification had by this time proceeded so far that the material would be valueless if annealed in this way. Further heating soon caused the alteration to extend in places through the core of the rod, which became very brittle and broke. The reversible expansion had by this time increased to nearly double its original value. The second rod similarly tested showed precisely similar effects, but the heating in this case was not pushed beyond the stage of superficial devitrification. The results differed widely from those previously obtained by Le Chatelier by comparison with a porcelain rod (probably owing to uncertainties in the expansion of porcelain), but had since been verified over the range 0°C. to 1,000°C. by the observations of Holborn and Henning and of Randall. The material appeared to be eminently suitable for accurate work at temperatures up to 800°C. or 900°C., but its irreversible elongation and devitrification at higher temperatures rendered it useless for high-temperature gas thermometry, which was the primary object of the investigation.

More recently the expansion of vitreous silica at ordinary temperatures had acquired special interest in connection with mercurial thermometry and standards of length and expansion. The majority of observers had used the Fizeau method with specimens 10 mm. to 15 mm. long. Somewhat different values had been found for different specimens with different standards of comparison. For a cylindrical specimen on a platinum-iridium tripod Chappuis found 50×10^{-6} for the expansion from 0°C. to 100°C. and $0{\cdot}385\times10^{-6}$ for the coefficient at 0°C. Scheel, for a similar specimen, tested against a quartz crystal ring, found $45{\cdot}5\times10^{-6}$ from 0°C. to 100°C. and $0{\cdot}217\times10^{-6}$ at 0°C. For a ring specimen tested in a vacuum by the absolute method he found values almost identical with Chappuis; but Randall, employing a similar ring specimen, also made by

Zeiss, found the mean coefficient from 16°C. to 80°C. (which is nearly the same as that from 0°C. to 100°C.) to be only $0{\cdot}424 \times 10^{-6}$. Such differences might be due to accidental errors or to differences in form and treatment of the specimens employed, or to differences in the standards of comparison. But since the whole expansion of 1 cm. of fused silica between 0°C. and 100°C. was only of the order of one wave-length of light, it was also possible that small constant errors might arise in so delicate an experiment from gas films or other surface effects variable with temperature. It seemed, therefore, desirable to measure the expansion of the long silica rods at low temperatures by a direct interference method in which such sources of error were excluded.

The free end of the silica rod in the platinum tube apparatus was accordingly fitted with a small lens giving Newton's rings with a fixed lens. An expansion of 50×10^{-6} with this arrangement would give about 50 rings, which could be easily counted in two or three minutes by suitably regulating the electric heating. The fractions of a half wave-length could be estimated with great precision by measuring the diameters of the first and last rings. The experiment was so easy that an error of the order of 1 per cent. appeared to be practically impossible, and an accuracy of 1 in 1,000 might reasonably be expected. The ring system itself was not exposed to changes of temperature, and the method was absolute in the sense that the result did not depend on assuming the expansion of some other material. This method gave a smaller and more rapidly diminishing value for the expansion of the silica rods than that obtained by other observers employing the orthodox Fizeau method with short specimens. On the other hand, Donaldson's relative comparisons of a silica metre over the range 0°C. to 30°C. with three metre standards of invar, nickel and nickel-steel respectively (the expansion of which relative to the platinum-iridium standard had been determined at the Bureau International) gave results the mean of which agreed closely with Chappuis' formula, in which the expansion of platinum iridium was also assumed. It would appear either that different specimens of silica differed in expansion or that the discrepancies were due to errors in the absolute determinations, which were of a higher order of difficulty than the relative comparisons.

The recent determinations of the relative expansion of mercury in silica bulbs by Harlow and Eumorfopoulos agreed well with that of Chappuis obtained with a *verre dur* bulb, when Chappuis' values of the expansion of silica and *verre dur* were assumed, but gave somewhat lower values for the expansion of silica and *verre dur* when the values of Callendar and Moss for the absolute expansion of mercury were assumed and the cubical coefficient was taken as three times the linear. This introduced the further question as to whether the expansion of silica or *verre dur* could be assumed to be isotropic, since neither could be perfectly annealed. In order to elucidate this point he had designed an arrangement for observing by the Fizeau method the differential expansion between a tripod and a ring cut from the same tube. The finished apparatus was promised in a day or two, and might be expected to afford some positive evidence on the subject. It was proposed to examine different specimens of glass as well as silica to see if any relation could be traced between the residual strains and the expansion, and whether differences could be eliminated by annealing. If his own observations of the expansion of silica and mercury were correct, the difference between the radial and axial expansion of a silica tube should be small in absolute magnitude, but well within the limits of sensitiveness and accuracy of the differential method. The absolute measurement of a linear coefficient was a difficult and delicate operation, and any errors were increased threefold in deducing the cubical coefficient ; but in the case of a material like silica, where the expansion was so small as to minimise the importance of exact measurement of temperature, the weight thermometer method afforded a most valuable check on the accuracy of the hydrostatic method, provided that the isotropy of the bulb employed were independently verified. It might even prove to be more accurate than the hydrostatic method at low temperatures, where the latter suffered from increased viscosity lag and other difficulties of manipulation.

Note added by Prof. Callendar.—With the assistance of Mr. A. Eagle, he had made some observations on the difference between the radial and axial expansion of a silica tube similar to that from which the bulbs of the mercury weight thermometers employed by Harlow and Eumorfopoulos had been constructed. The tube had been supplied by the Silica Syndicate, and the tripod and ring had been optically worked by Messrs. Hilger. Several independent determinations had been made by Mr. Eagle on different days with closely concordant results. The mean of these showed that the axial coefficient of expansion of the specimen tested exceded the radial coefficient by $0{\cdot}20 \times 10^{-6}$ over the range 18°C. to 90°C. The axial coefficient of the tripod had also been measured, and found to be $0{\cdot}46 \times 10^{-6}$ when reduced to correspond with the range 0°C. to 100°C. This would make the cubical coefficient of a bulb constructed from the same tube approximately $0{\cdot}98 \times 10^{-6}$, which agreed as closely as could be expected with the results of Harlow and Eumorfopoulos. The exact reduction of the observations had not yet been completed, owing to pressure of other work, but it appeared that the difference between the axial and radial coefficients increased, while the axial coefficient diminished more rapidly at lower temperatures. Thus, the cubical coefficient of a drawn silica bulb would vanish somewhere between 0°C. and 15°C., as had been found by Eumorfopoulos to be the case.

The investigation had so far proved to be a confirmation of the accuracy of the work of Callendar and Moss on the absolute expansion of mercury at low temperatures, and would appear to indicate that the values generally assumed for the expansion of water and mercury were appreciably too high. Having regard to the direct experimental proof of the anisotropy of a drawn silica tube, and the possibility of errors in the " accepted " values of the linear coefficient, it did not appear profitable to discuss further the speculations on the expansion of mercury at higher temperatures, which Donaldson and others had based on the assumption of the isotropic expansion of silica.

DISCUSSION ON THE TWO PRECEDING PAPERS.

Mr. F. J. HARLOW quoted some observations on the apparent expansion of mercury and silica from 0°C. to 44°C. that he had recently made. These gave a still lower value for the expansion of silica.

Dr. G. W. C. KAYE congratulated Mr. Donaldson on drawing from Prof. Callendar such an interesting account as they had just listened to. He was glad Prof. Callendar was taking up such an important subject, as the question of the expansion of silica was now an international one.

Dr. J. A. HARKER stated that Prof. Callendar suggested that Donaldson's coefficient for the expansion of silica ultimately depended upon the coefficient found for a small specimen of platinum-iridium found by the Fizeau method. The metre rods against which Mr. Donaldson had compared his silica meter had had their coefficients determined at the Bureau International relatively to their standard platinium-iridium metre, but this had had its coefficient determined absolutely, as well as by the Fizeau method.

Mr. H. DONALDSON, in reply, said that Prof. Callendar had raised several questions to which it was difficult to give any answer at the present time. He suggested that different specimens of silica might behave differently. Their standard metre bars had been compared against the standard platinum iridium bar at the Bureau International, but this had had its coefficient of expansion determined absolutely and not relatively. Mr. Sears and himself had also designed a method of finding the difference between the longitudinal and transverse coefficients of drawn silica by means of an optical lever method. He valued Prof. Callendar's remarks on the difficulty of annealing silica.

XXX. *The Solution of Network Problems by Determinants.* By ROLLO APPLEYARD, *M.Inst.C.E.*

RECEIVED MARCH 7, 1912. READ APRIL 26, 1912.

MAXWELL's method of solving problems relating to the distribution of electric currents in networks of conductors, by means of mesh equations and determinants, is of great service when the currents in a network are of sine form. This method was fully explained in a Paper read by Dr. J. A. Fleming to the Physical Society in 1885, and he also added a method for calculating the resistance of a network, which had not been given by Maxwell.* By a familiar device, capacities, inductances and leakances in networks containing such currents can all be expressed as resistances, and Maxwell's method is consequently applicable to the general case. It is proposed here to consider how mutual inductance can best be dealt with in forming the equations and the determinants, and to suggest some simplifications.

The Sign of Mutual Inductance.—As mutual inductance in a network may have a *plus* or a *minus* sign it is convenient to adopt a convention, and to represent the mutual inductance by two parallel coils or conductors, as shown in Fig. 1. The directions of the cyclic currents may be indicated in the usual way by bent arrows. It is desirable to retain capital letters for currents and small letters for resistances. Consider what happens in an adjacent mesh when a cyclic current in any given mesh, linked to the first by mutual inductance, is momentarily increased. If the current induced by this means in the adjacent mesh is in the same direction as the cyclic current of that mesh, the mutual inductance, as a convention, may be given a *plus* sign; and it may be given a *minus* sign if the induced current is opposed to the cyclic current in that adjacent mesh. For example, in Fig. 1, a momentary increase in the cyclic current A would induce in mesh B a momentary current opposed to A, and consequently in the same direction as the cyclic current B. The mutual inductance may therefore be given a *plus* sign. Again, in Fig. 2, a momentary increase of the cylic current X would induce in the adjacent mesh a momentary current opposed to the cylic current Y, and consequently the mutual inductance may be given a *minus* sign.

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* *See* "Problems on the Distribution of Electric Currents in Networks of Conductors, treated by the Method of Maxwell." (Dr. J. A. Fleming, "Proc." Physical Soc., Vol. VII., 1885–1886.)

Retention or Rejection of an M-term.—In writing down the mesh equations there is sometimes a doubt whether for a given mesh a mutual inductance term should be included or not in the expression for the sum of the E.M.F.s round it, when it is linked to an adjacent mesh by mutual inductance. In such a mesh there is, of course, an E.M.F. due to mutual inductance only when the current in an adjacent mesh fluctuates ; and fluctuation of its own cyclic current produces within the mesh itself no E.M.F. of mutual inductance, and consequently no M-term. It will presently be shown that in forming the determinant the M-terms can be made to take their proper places in this respect automatically.

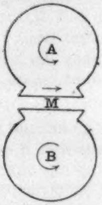

Fig. 1.

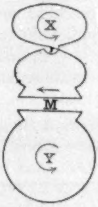

Fig. 2.

Numerical Order of Meshes.—It is the custom in forming the determinant to write down the terms of the leading diagonal by inspection, by considering the meshes one at a time, in any order—the sums of the resistances in each mesh, taken separately, are the successive terms of the leading diagonal. But it never seems to have been explained how the determinant can best be completed by inspection after this operation. There is a very simple means of doing this, and incidentally it removes all doubt regarding what terms should, and what terms should not, contain an M. It depends upon numbering the meshes in any consecutive order. If the cyclic currents are denoted by letters in alphabetical order the letters will be sufficient indication of the numerical order. There is some advantage in beginning to count from the mesh containing the generator. The sum of the resistances in mesh (1) then forms the top left-hand term of the determinant. The sum of the resistances in mesh (2) takes the next place along the leading diagonal, and similarly for the other meshes, in the numerical order, forming the remainder of that diagonal. Now as the influence of adjacent meshes is left out of account in forming every constituent of this diagonal, *there is never an M-term in the leading* diagonal. Constituents outside the leading diagonal, however, are all formed from considering the E.M.F.s arising from meshes adjacent to each mesh of the network, in succession. Hence, *there is always an M-term in a constituent outside the leading diagonal, provided that the constituent corresponds to a mesh linked by mutual inductance to a mesh in which there is a varying current.* The effect of retaining the numerical order in the leading diagonal is that the number of each column of the determinant agrees with the number of a given mesh, and each row can now be filled in by inspection. For example, beginning at the first mesh, write down the sum of the resistances in the *branch* that separates it from the second mesh. This is found to be the proper constituent for the second place of the top row. Similarly, the sum of the resistances in the *branch* that separates the first from the third mesh is the proper constituent for the third place of the top row, and so on for all the remaining constituents of that row. Next proceed to the second mesh, and observe that the second place in the second row is already filled by the constituent there of the leading diagonal. To fill up the first place of the second row write down the sum of the resistances in the *branch* that separates the second mesh from the first. To fill up the third place of the second row write down the sum of the resistances in the *branch* that separates the second mesh from the third, and so on for all the remaining constituents of that row. Then proceed to the third mesh, and find all the constituents of the third line, beginning at the diagonal constituent in that line, and observing in succession the sums of the resistances of the *branches* that separate it from the other meshes. This result can be generalised by stating that *the magnitude of the constituent of any given row and given column in a network determinant is represented by the sum of the resistances in the branch separating the mesh bearing the number of the given column from the mesh bearing the number of the given row. All constituents of the leading diagonal are to be given a plus sign, and all constituents outside that diagonal are to be given a minus sign.*

Null Condition.—In ascertaining the conditions that correspond to no current in a particular branch, Maxwell's device* of denoting the cyclic current in the mesh on one side of the branch by, say, X, and the cyclic current in the mesh on the other side of it by X+Y, requires a subsequent sorting out of the terms in X and Y, and the formation of a second determinant. This work can be simplified *by adding the constituents of the* X+Y *column to those of the* X *column to form a new* X *column, and by retaining the constituents of the* X+Y *column of the first determinant as the constituents of the* Y *column in the new determinant.* To illustrate this, consider the case of a Wheatstone bridge, Fig. 3. Taking the meshes in the order A, B (B+C), and writing down the sum of the resistances in each, the leading diagonal is found. In passing from A to B, branch r is crossed. Hence r is the constituent of the second place of the top row. In passing from A to (B+C), branch a is crossed. Hence a is the constituent of the third place of the top row. Similarly, in passing from B to A, branch r is crossed. Hence r is the constituent of the first place of the second row. By this simple means the determinant can be written down very quickly by inspection, from the foregoing rules, thus—

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* *See* "Handbook for the Electrical Laboratory and Testing Room," Dr. J. A. Fleming, Vol. I., p. 202.

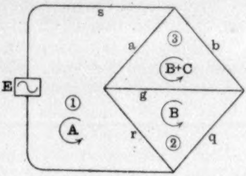

FIG. 3.

$$\begin{array}{cc} & \begin{array}{ccc} (A) & (B) & (B+C) \end{array} \\ \triangle = & \left| \begin{array}{ccc} a+r+s & -r & -a \\ -r & g+r+q & -g \\ -a & -g & a+b+g \end{array} \right| \end{array}$$

For clearness the corresponding cyclic currents have been placed at the head of each column. Now add the constituents under (B+C) to those under (B) to form a new (B) column, and retain the (B+C) column to form the new (C) column, of the required second determinant for A, B and C, and the result is—

$$\begin{array}{cc} & \begin{array}{ccc} (A) & (B) & (C) \end{array} \\ \triangle = & \left| \begin{array}{ccc} a+r+s & -r-a & -a \\ -r & r+q & -g \\ -a & a+b & a+b+g \end{array} \right| \end{array}$$

To find C we replace the third column by E, 0, 0, so that

$$C=E\frac{\triangle_{n-1}}{\triangle}=E\frac{\begin{vmatrix}-r & r+q\\ -a & a+b\end{vmatrix}}{\triangle}=E\frac{aq-rb}{\triangle}.$$

which is the required solution. In general, by this method, it would suffice to write down the first determinant, and from it to write down the $\triangle_{n-1}$ of the second determinant, also by inspection. If the cyclic currents on either side of a branch are represented by (B+C) and (B) respectively, it will be found that there is always advantage in placing the (B+C) column last, however complicated or extensive the network may be. Also, the (B) column should be placed next to the last. If this is done, $\triangle_{n-1}$ can the more easily be written down by inspection from the first $\triangle$, and the current C is determined by replacing the last column of $\triangle$ by E, 0, 0, 0, ... as in this example.

Linked Circuits.—The same method is applicable to an arrangement of circuits linked by mutual inductance, and containing resistances, capacities and self-inductance, as in Fig. 4.

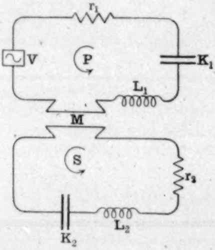

FIG. 4.

For example, if it is required to find the current S in the secondary corresponding to a current P in the primary, the determinant is formed as before, beginning with the leading diagonal. Each capacity is converted into a resistance by dividing its reciprocal by *ip*, where $p=2\pi\sim$; and each inductance is converted into a resistance by multiplying it by *ip*. The determinant corresponding to Fig. 4 is consequently

$$\triangle=\begin{array}{|cc|}\text{(P)} & \text{(S)}\\ \left(r_1+\frac{1}{K_1 ip}+L_1 ip\right) & -Mip\\ -Mip & \left(r_2+\frac{1}{K_2 ip}+L_2 ip\right)\end{array}$$

It is seen that the leading diagonal again contains no term in M; each of the terms outside that diagonal, however, contains an M term, because the assumption is that S as well as P is other than zero. The E.M.F. in the P mesh is V, but there is no generator in the S mesh. Replacing therefore the second column, *i.e.*, the S column, by V, O, the current in the secondary is seen to be

$$S=\frac{\begin{vmatrix} r_1+\frac{1}{K_1ip}+L_1ip & V \\ -Mip & O \end{vmatrix}}{\triangle}=\frac{VMip}{\triangle},$$

which is the required solution.

Hughes Induction Bridge.—To illustrate the advantage of operating in the manner here suggested the case of the Hughes

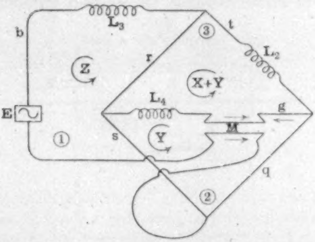

FIG. 5.

induction bridge may be considered. In Fig. 5 the meshes have been numbered, and small straight arrows have been added near M to accentuate the directions of the cyclic currents at the points where the arrows appear. To form the determinant the numerical order is retained, and the leading diagonal is written down by inspection. The need for a convention in regard to the sign of M is apparent in this case, because two of the constituents contain —M and two contain +M, and if the signs are disregarded the result is worthless. It is instructive to consider these four constituents. In passing from mesh (1) to mesh (2), the resistance *s* and the mutual inductance M are crossed. Now, following the convention, a momentary increase in Z will set up in the *g* branch a current in the reverse direction to Z, *i.e.*, in the *same* direction as Y, and hence as regards mesh (2) the sign of M is *plus*, and thus the proper constituent for the second place of the first row is, in magnitude $(s+Mip)$. Now consider mesh (3). A momentary increase in Z will set up in the g branch a current *opposed* to the cyclic current $(X+Y)$; hence as regards mesh (3) the sign of M is *minus*, and thus the proper constituent for the third place of the first row is, in magnitude $(r-Mip)$. Similarly the first constituent of the second row is, in magnitude $(S+Mip)$ because a momentary increase in Y produces in mesh (1) a current in the *same* direction as Z. The first constituent of the third row, however, is, in magnitude $(r-Mip)$, because a momentary increase in Z produces in mesh (3) a current opposed to its cyclic current $(X+Y)$. Each term outside the leading diagonal is finally to be given a *minus* sign, and the determinant is written, by inspection,

$$\triangle=\begin{array}{|ccc|} \multicolumn{1}{c}{(Z)} & \multicolumn{1}{c}{(Y)} & \multicolumn{1}{c}{(X+Y)} \\ (b+L_3ip+r+s) & -(s+Mip) & -(r-Mip) \\ -(s+Mip) & (q+s+g+L_4ip) & -(g+L_4ip) \\ -(r-Mip) & -(g+L_4ip) & (r+t+L_2ip+g+L_4ip) \end{array}$$

To find the required $\triangle_{n-1}$, add the third column to the second, to obtain a new second, and write down the resulting $\triangle_{n-1}$ of the new determinant. Hence, replacing the new third column by E, 0, 0,

$$X=E\frac{\triangle_{(n-1)}}{\triangle}=E\frac{\begin{vmatrix} -(s+Mip) & (q+s) \\ -(r-Mip) & (r+t+L_2ip) \end{vmatrix}}{\triangle}.$$

Equate the numerator to zero, and separate real from imaginary terms. The two equations corresponding to balance are thus found to be

$$ML_2p^2-st+rq=0$$
$$sL_2-M(r+s+t+q)=0.$$

The above formula for the Hughes induction balance was originally given by Lord Rayleigh. It was also deduced by Maxwell's method by Dr. Fleming and given by him in his "Handbook for the Electrical Laboratory," Vol. I., p. 285.

As a matter of history, it may be observed that Maxwell used cyclic currents and determinants for the solution of network problems in 1873. At p. 399 of Vol. I. of his "Treatise on Electricity and Magnetism," which was published in that year, he applied this method to find the degree of accuracy attainable with a Wheatstone bridge. His diagram, with its straight arrows, does not make it clear that the three mesh currents are cyclic, but the text leaves no doubt about it, for he says : "By considering the system as made up of three circuits OBC, OCA and OAB, in which the currents are x, y, z respectively, and applying Kirchhoff's rule to each cycle, we eliminate the values of the potentials . . . ," and in forming his equations he proceeds to regard the meshes as independent circuits. The introduction of the curved arrows, all having the same direction of rotation, to represent the cyclic currents, appears to have been introduced later.

CONCLUSIONS AND SUMMARY.

Let it be supposed that cyclic currents have been assigned to all the meshes of a given network, and that all capacities (K, in farads), inductances (L, in henries) and leakances (S, in mhos) have been converted into resistances in ohms $\left(\frac{1}{\mathrm{K}ip}, \mathrm{L}ip, \frac{1}{\mathrm{S}}, \text{ where } p=4\pi\sim\right)$ The general network problem then is to find the current, in amperes, in any given branch, corresponding to the application of an E.M.F. of sine form, between any two fixed points in the network. The procedure for solving such a problem may be summarised as follows :—

1. Denote the cyclic current in the mesh on one side of the given branch by, say, (X + Y), and the cyclic current in the mesh on the other side of it by, say, Y, so that X is the magnitude of the real current through the given branch.

2. Number the meshes in consecutive order, beginning at the generator mesh and ending at the (X + Y) mesh. Let the Y mesh be next to the last.

3. Write down successively the total resistance of each mesh in the order of their numbers—including capacities, inductances and leakances, converted into resistances as above—and make these values the successive terms of the leading diagonal of a determinant, beginning at the top. Observe that the order of the columns of the determinant is now the same as the order of the meshes. So also is the order of the rows.

4. To fill up any given row (or column), say, the nth, observe what are the resistances of the branches crossed in passing from the nth mesh successively to the 1st, 2nd, 3rd . . . . mesh, and make these values the respective constituents of the 1st, 2nd, 3rd . . . . places in the nth row. The nth constituent of the nth row will always be in the leading diagonal, and it is to be given a *plus* sign. All constituents outside the leading diagonal are to be given a *minus* sign.

5. Now replace the (X + Y) and (Y) columns of the determinant so formed by columns corresponding only to X and Y. To do this, add the (X + Y) column to the Y column to form a new Y column, and retain the (X + Y) column for a new X column. This is equiva-

lent to multiplying out the fundamental mesh equations, and bringing all terms in X inside one bracket and all terms in Y inside another bracket. The final column now corresponds with X, and the penultimate column with Y. The determinant Δ_n is thus completed.

6. Replace the final column by E, 0, 0, 0, 0 . . . ., putting E at the head of the column. Expand the determinant so formed, in terms of E, 0, 0, 0, 0 . . . ., and it becomes $E\,\Delta_{n-1}$. In general this operation may be omitted, since Δ_{n-1} can at once be written down when operation 5 is completed.

7. Then, by the well-known property of determinants, $X=E\frac{\Delta_{n-1}}{\Delta_n}$, and the resistance *r*, between the two points of the network to which E is applied, is $r=\frac{\Delta_n}{\Delta_{n-1}}$.

8. If the network contains mutual inductances, a convention must be adopted in respect to signs. Regard each mutual inductance as a pair of parallel straight conductors; then a sudden increase of current in one conductor will induce in the other a momentary current in a direction opposed to that in the first. If this induced current is in the same direction as the cyclic current of the mesh to which the second conductor belongs, then the mutual inductance term M*ip* is to be regarded as *plus*, and if the induced current is in the opposite direction to that cyclic current, M*ip* is to be regarded as *minus*.

9. Observe that there can never be an M-term in the leading diagonal. There is, however, always an M-term in a constituent outside the leading diagonal, provided that the constituent corresponds to a mesh linked by mutual inductance to a mesh in which there is a varying current.

10. This method of solution is based upon the assumption of a sine-form E.M.F. of definite frequency, and consequently all the constants of the meshes are effective values corresponding to the selected frequency. If the conditions of equilibrium in any particular case do not explicitly contain a frequency term, the current wave may be of any shape; and under these circumstances the method applies to a current impulse of any form whatever.

DISCUSSION.

Mr. F. E. SMITH stated that current in one arm of a network due to an E.M.F. in another was the same as the current in the second arm caused by the same E.M.F. in the first arm.

The CHAIRMAN (Mr. A. Campbell) stated that Heaviside had investigated almost every possible bridge, with mutual and self-inductances in every arm. The method of determinants was undoubtedly very powerful, but he preferred the simple method of denoting the current in the various arms by unknown variables and then writing down equations for the potentials of the different points in the network. A mutual inductance could always have a plus or a minus sign. Thus for every bridge another could be obtained by connecting the mutual inductance up the other way. Frequently, however, this rendered it impossible to find a point of balance.

XXXI. *A Method of Measuring Small Inductances.* By S. Butterworth, *M.Sc.* (*Manchester*). *Assistant Lecturer in Physics, School of Technology, Manchester.*

Received March 28, 1912. Read May 10, 1912.

The Paper describes a method of measuring inductance in which a single condenser may be used for all values of the self-induction.

Results are given showing the accuracy that may be expected in the case of inductances of the order of a few microhenries.

1.—*Development of Method.*

A self-induction may be measured in terms of a capacity by the method of Maxwell. The inductance is placed in the fourth arm of a balanced Wheatstone bridge (Fig. 1) and a condenser of capacity K is placed in parallel with the conjugate arm Q. The bridge will be balanced for variable currents, provided that

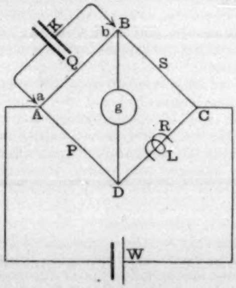

Fig. 1.—Maxwell's Bridge.

$$\frac{L}{K}=RQ=SP,$$

where L is the self-induction of the arm R.

Unless the condenser can be varied continuously, this method is tedious in operation. The following modifications have, therefore, been proposed :—

(*a*) If $\frac{L}{K} > RQ$, move the contact *a* along the battery arm. If V is the resistance intercepted between *a* and A, balance is attained when

$$\frac{L}{K}Q = S\{V(P+Q)+PQ\}.$$

This modification is due to Anderson.*

(*b*) If $\frac{L}{K} < RQ$, move the contact *b* along the S arm.† If U is the resistance intercepted between *b* and B, balance is attained when

$$\frac{L}{K}Q = P(Q+U)(S-U).$$

The former method is the more useful because the adjustment of V can be made without disturbing the steady current balance. Method (*b*) requires a slide wire in the arm S for rapid working. In order to avoid this the author has employed a combination of both methods. This is shown in Fig. 2.

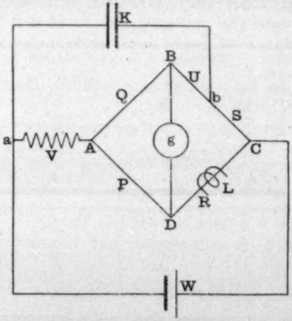

FIG. 2.—MODIFIED BRIDGE.

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* Anderson, " Phil. Mag." (5) 31, p. 329, 1891.
† Butterworth, " Proc." Phys. Soc., XXIV. (Part 2), p. 83, 1912.

With the notation of this figure the conditions for balance are

$$RQ = P(U+S),$$
$$\frac{L}{K}Q = S\{V(P+Q)+P(Q+U)\}.$$

By a proper choice of S *any inductance may be measured with a single condenser. Also, by adjusting* V, *the inductive balance may be made independently of the resistance balance.*

2.—*Sensibility.*

Since the sensibility is always proportional to the inductance to be measured, it is important, especially in the case of low inductances, to use that arrangement which gives the maximum sensibility. It may be shown that the sensibility when using a vibration galvanometer as detector is increased by diminishing the frequency. With frequencies of the order of 100 per second and a capacity of 0·1 microfarad, the condenser has practically no effect on the sensibility.

The source of E.M.F. may be treated as a battery of definite resistance (W) and the galvanometer as of definite resistance (g). The reactance of both these arms may also be made negligible.

The conditions for maximum sensibility therefore become those for the Wheatstone bridge.

Hence with a given battery and galvanometer we must make*

$$Q=\sqrt{Wg},\ P=\sqrt{Rg\frac{R+W}{R+g}},\ U+S=\sqrt{RW\frac{R+g}{R+W}}.$$

The resistance V is here included in W.

The value of U is immaterial, but V must be as low as possible.

3.—*Experimental Results.*

The following results show what accuracy may be expected if the ordinary precautions are taken.

The coil measured consisted of 10 turns of silk-covered copper wire wound on a rectangular wooden block 13·5 cm. by 7·5 cm. Its resistance was about 0·2 ohm. In obtaining the results of Tables II., III. and IV. non-inductive resistances were placed in series with this coil.

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* Heaviside, "Phil. Mag.," July, 1875; or Kempe, "Handbook of Electrical Testing," p. 175.

The source was a small induction coil whose primary was excited by a 10-volt battery, and the equivalent resistance of the secondary was found to be about 2,000 ohms.

The detector was a Duddell vibration galvanometer of about 200 ohms effective resistance.

A silvered mica condenser was used, its capacity being 0·1 mfd.

The values of Q and U+S were so chosen as to satisfy approximately the conditions of maximum sensibility and the steady current balance was obtained by adjusting P.

The resistances are all expressed in ohms and the inductances in microhenries.

TABLE I.

P=20·3		Q=600	R=0·203		U+S=6·0	
U	0	1	2	3	4	5
S	6	5	4	3	2	1
V	17·5	25	36	54·5	92	200
L	23·03	23·09	23·03	23·02	23·11	22·72

Mean self-induction=23·00±0·09 microhenries.

TABLE II.

P=47·4	Q=600	R=1·185		U+S=15·0
U	11	12	13	14
S	4	3	2	1
V	10	28·5	65	170
L	23·62	23·73	23·71	23·19

Mean self-induction=23·56±0·19 microhenries.

TABLE III.

P=65·1	Q=600	R=2·17		U+S=20
U	17	18	19	...
S	3	2	1	...
V	9	44	150	...
L	23·07	23·17	23·34	...

Mean self-induction=23·19 microhenries.

TABLE IV.

P=75·7	Q=600	R=3·16		U+S=25
U	...	23	24	...
S	...	2	1	...
V	...	33·5	135	...
L	...	23·27	23·08	...

Mean self-induction=23·17 microhenries.

The high value obtained in Table II. was traced to the residual inductance of the added resistance coil. Otherwise the differences are well within the experimental errors.

4.—*Conclusion.*

The results show that it is easily possible to compare an inductance of the order of a few microhenries with a capacity of 0·1 mfd. and obtain an accuracy of at least 1 per cent.

Much greater precision could no doubt be attained with additional precautions, viz., elimination of or correction for the residual inductances of the remaining arms, and for the leakage or absorption of the condenser; the use of a galvanometer of proper effective resistance and low ohmic resistance.*

Fleming† has used Anderson's method to measure small inductances by means of very low capacities, so that by combining the present method with that of Fleming it is possible to compare a low capacity (say, a Leyden jar) with the usual standard condensers.

I wish to thank Prof. Gee for the interest he has taken in the work, and the Principal and Committee of the School of Technology for the facilities placed at my disposal.

ABSTRACT.

The author shows how Anderson's method may be modified so that, while still retaining the usual standards of capacity, very small inductances may be measured.

As in Anderson's method, balance is attained by a simple resistance adjustment.

The conditions of maximum sensibility are indicated and experimental results are quoted in which an inductance of 20 microhenries is compared with a capacity of 0·1 mfd.

The method may also be employed to compare a very low capacity with the usual mica standards of capacity.

DISCUSSION.

Dr. A. Russell took it that the author had assumed no iron in the circuit in obtaining his formula.

Mr. A. Campbell pointed out that the author obtained his balance by means of a vibration galvanometer, so the method would correctly give the apparent self-induction at that particular frequency.

Dr. W. H. Eccles pointed out that the method would be rarely used for coils with iron cores since it was for small inductances. Since the periodicity of the alternating current did not occur in the equations for equilibrium the method could, if all the resistances were independent of the frequency, be used with an interrupted current and a telephone.

The President remarked that all who had had occasion to try it knew how difficult it was to measure a small self-induction, and would therefore welcome Mr. Butterworth's Paper.

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* Butterworth (*loc. cit.*).

† "Phil. Mag.," (6), 7, p. 586, 1904.

XXXII. *Demonstration of the Generation of Electricity by Carbon at High Temperatures.* *By* J. A. HARKER, *D.Sc., F.R.S., and* G. W. C. KAYE, *B.A., D.Sc.*

EXHIBITED MAY 10, 1912.

THE experiments described owe their origin to some contamination phenomena which were encountered when tubes of refractory rare earths were baked in carbon-tube resistance furnaces at temperatures from 1,500°C. upwards. It was found that the tubes often had their outer surfaces carbonised to an appreciable depth, while the inner surfaces, though freely exposed, were much less attacked. The blackening was presumably caused by particles shot from the carbon walls of the furnace with velocity high enough to penetrate the refractory material after crossing a few millimetres of air at atmospheric pressure.

The preliminary experiments on the nature of these particles were carried out by the use of two insulated exploring electrodes of carbon inserted into an alternating-current furnace. They were connected externally to a battery of cells and the potential-current curves were determined for the electrode gap in the furnace at a number of temperatures. No appreciable current could be detected at temperatures below about 1,400°C., but as the temperature rose it was found that quite small E.M.F.s gave rise to steady currents of relatively enormous magnitude.

For example, with 8 volts, currents up to 10 amperes have been obtained at a temperature of about 2,500°C.

The relation between current and temperature was found to be of an exponential character.

The magnitude of the currents made it evident that the atmosphere of the furnace was ionised to an ususual degree at high temperatures, and the authors were led to try the effect of temperature alone in the absence of any applied potential. Accordingly the battery was cut out, and while one of the electrodes remained stationary within the furnace the other could be suddenly displaced to a colder or hotter part of the furnace. The resulting difference of temperature manifested itself as a transient current in the circuit, which in some cases amounted to 2 amperes. The current died away when the two electrodes attained the same temperature.

In the apparatus shown the movable electrode was moved in and out of the hot region of the furnace by means of a clock-work mechanism. The pulsating electric current thus produced was large enough at high furnace temperatures to light up a nest of small glow lamps, the illumination waxing and waning as the movable electrode moved in and out. The experiment has been modified in later work by keeping both electrodes stationary and water cooling one of them. The cold electrode was made up of a brass tube, which in some cases was provided with a carbon sleeve. A continuous current can thus be generated. Its direction is such as would be produced by a discharge of negative particles from the hot electrode. As before, no potential is applied. At the lower temperatures small positive currents have also been detected; at high temperatures negative currents up to nearly an ampere have been measured. The whole of the experiments were conducted at atmospheric pressure and almost entirely with low-voltage alternating-current furnaces.

DISCUSSION.

Mr. G. D. West thought that it seemed from the above experiments that in a carbon filament lamp, if run at a high enough temperature, more current might be carried by the ionised gas than by the filament.

Prof. C. H. Lees remarked that the authors declined to give any theory of the effect they had observed. Was the emission one of electrons or was some chemical action taking place? Some Papers before the German Physikalische Gesellschaft had recently cast doubt on the emission of electrons from heated bodies observed by H. A. Wilson and O. W. Richardson, and it had been pointed out that a chemical reaction would follow Richardson's formula, which afforded, therefore, no proof that the effect was due to electrons.

Dr. Erskine Murray asked if the authors had found the emission to remain steady for several hours. The phenomenon was similar to that of the valves used in wireless telegraphy.

The President said he had listened with considerable interest. The authors had done on a large scale what had been done on a small scale a few years ago. He doubted the emission being electrons, and thought that some electrochemical effect should be sought for.

Dr. Harker, in reply, stated that the effect was not chiefly a chemical effect as it was the same whether the gas in the furnace was H_2 or N_2, nor did it matter what the carbons were made of. Specially pure carbons gave phenomena of just the same order of magnitude.

Dr. Kaye, in reply, stated that the feature which differentiated their experiments from others was that the effect occurred at atmospheric pressure and not *in vacuo*. It was hardly likely at this pressure that the effect could be confined to electrons, as the mean free path of such would be very small. The effect might be due to the transference of carbon itself, probably in ultra-microscopic particles and not molecules. When a cold tube was used for one of the electrodes it was found in half an hour to be coated with a deposit of carbon. There was no likelihood that the effect would decay with the time, as it had been kept constant for over three hours. It was also not necessary to use electric heating in the furnace. Heating by thermit and Méker gas furnaces both gave the same results.

XXXIII. *The Conversion of Starch into Dextrin by X-Rays.* By H. A. Colwell, *M.B.*, and S. Russ., *D. Sc.*, *Beit Memorial Research Fellow.*

Received April 4, 1912. Read May 10, 1912.

As the result of an attempt to detect any permanent physical or chemical changes in complex organic fluids after irradiation by X-rays it has been found that, among the numerous fluids experimented upon, solutions of starch give evidence of these changes to such a marked degree that it has been possible to show that starch under the influence of X-rays is gradually converted into dextrin and to establish this in a quantitative manner.

Experimental Methods.

Solutions of starch were made in the usual manner and contained 4 grammes in 250 c.cm. of distilled water. About 50 c.cm. of the solution were placed in a Petri dish, sealed with a very thin sheet of mica to prevent evaporation and placed about 2 cm. below an X-ray bulb, which was run under conditions giving a moderately soft type of radiation (about 8 cm. spark-gap). A thermo-electric junction placed on the mica and connected to a low-resistance moving-coil galvanometer measured the small rise of temperature occasioned by the gradual warming of the bulb. This was never allowed to exceed a few degrees.

Physical Changes.

After periods of irradiation, ranging in various experiments from 2 to 8½ hours, two changes in the fluid were manifest, namely, a marked diminution in its viscosity and a decrease in its opacity when compared with a portion of the original solution.

The viscosities of the experimental and control solutions were compared by means of an apparatus consisting of a horizontal capillary tube sealed to a vertical glass cylinder. The time taken for the liquid to fall between two fixed marks on the cylinder was measured.

After prolonged irradiation it was found that the viscosity of the starch solution was so much diminished that the rate of flow through the system was more than doubled, as may be seen from the data in Table I., which represents a typical series of observations.

TABLE I.
Comparison of Viscosities.

	Control. Time of flow.	Experimental. Time of flow.	
Temp. 18·3°C.	5′·02″	2′·08″	Temp. 18·8°C.
	5′·11″	1′·59″	
	5′·11″	1′·57″	
	5′·14″	1′·58″	

A calibration of the instrument with three liquids of known viscosities showed that the reduction in time (about one-half) observed for the irradiated starch corresponded to a rather larger diminution in the viscosity than this number indicates. The change in viscosity and opacity suggested the possible conversion of the starch into some of its cleavage products and accordingly experiments were undertaken to determine what, if any, chemical changes had occurred as a result of the irradiation.

CHEMICAL CHANGES.

The Iodine Reaction.—The control and irradiated solutions both gave a deep-blue colour with iodine.

No reduction was obtained on boiling with Fehling's solution or with alkaline safranin in the case of either the control or experimental fluid, showing that no conversion into reducing sugar had taken place.

Precipitation by Electrolytes.—It has been shown by Young* that starch and some of its early cleavage products are precipitable by certain electrolytes, among which are sodium sulphate and ammonium sulphate.

1. When a saturated solution of sodium sulphate is added to a solution of starch and left standing for some hours it precipitates the ordinary starch, but not soluble starch or dextrin.

Accordingly samples of the control and irradiated solutions were saturated with sodium sulphate and allowed to stand for some hours. A marked difference was observed in the two cases.

The control showed a well-marked precipitate while the irradiated solution gave only a small precipitate and remained turbid.

After centrifugalisation, the supernatant fluids were pipetted off. That from the control solution gave no coloration with iodine, the irradiated giving a deep-blue colour. This showed that a portion of the starch had been converted into soluble starch which is not precipitated by sodium sulphate.

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* Young, "Journal of Physiology, XXII., p. 401 (1898).

2. A saturated solution of ammonium sulphate, when added to an equal volume of starch solution (*i.e.*, half-saturation) and left for a few days, precipitates all the starch and any soluble starch present, but has not this action on dextrin.

To see whether this substance has been formed in the irradiated fluid, samples of the control and irradiated fluids were taken and to each was added its own volume of a saturated solution of ammonium sulphate. After being allowed to stand for two days the filtrates were examined. In the control no coloration was obtained on the addition of iodine, while the irradiated portion gave a port-wine colour indicating the presence of erythrodextrins.

Quantitative Estimation of the Dextrin Formed.

The procedure just detailed for the detection of dextrin in the irradiated starch solution was carried a stage further in order to obtain some idea of the percentage change occurring in the starch.

After irradiation for 8½ hours the dextrin formed in the solution was separated from the starch and the soluble starch in the manner indicated. The ammonium sulphate remaining in the filtrate was got rid of by dialysis. This was continued for several days until no precipitate was obtained with barium chloride, thus showing the complete absence of ammonium sulphate. The pure dextrin left was evaporated down and weighed. Starting with 15 c.cm. of starch solution (4 grammes in 250 c.cm. of distilled water), which contained 0·24 gramme of starch, the dextrin obtained after irradiation weighed 0·0108 gramme. Hence nearly 5 per cent. of the starch had been converted into dextrin. Owing to the intermediate formation of soluble starch, this indicates that a considerably larger percentage of the starch had been altered.

The Action of X-rays on Dextrin.

Having shown the conversion of starch into soluble starch and then to dextrin it was decided to see whether the sequence of changes was continued and could be detected at any further stage, possibly to the achro odextrins. A 5 per cent. solution of commercial dextrin was obtained and irradiated for 8½ hours in the manner already described.

Subsequent to the irradiation no change in the appearance or in the viscosity of the fluid was observed.

Two Nessler glasses were taken and in each were put 15 c.cm. of water and 0·1 c.cm. of iodine solution. To one was added 0·1 c.cm. of the control and to the other 0·1 c.cm. of the irradiated dextrin. No difference in tint was observed. On varying the quantities of iodine and dextrin the agreement of tint in the two cases was maintained.

Further on saturating equal volumes of the control and irradiated dextrin with crystals of ammonium sulphate a small precipitate of erythrodextrin was obtained in each case, apparently equal in bulk on centrifugalisation.

The filtrates gave a red colour with iodine, of equal intensity when the comparison was made under similar conditions. No evidence was obtained therefore of any conversion of the erythrodextrins to the achro odextrins.

It is interesting in this connection to find that M. Massol* was able to convert starch into glucose by exposing it to ultra-violet light for long periods.

THE NATURE OF THE ACTION UPON STARCH.

It is well known that solutions of starch may gradually be converted into the erythrodextrins and achro odextrins by hydrolysis and a simple explanation of the recorded results of irradiation might be put forward on these lines if hydrogen and hydroxyl ions were liberated from distilled water when irradiated by X-rays, but from the work of Kernbaum,† who exposed flasks of distilled water to the action of X-rays for 100 hours without any trace of gas being liberated, it seems clear that the part played by the water in the changes occurring in the starch solution must be a very small one and insufficient to account for them. The alternative is that the effect is due to a direct action upon the molecules of the starch caused by the X-rays or by the secondary rays (corpuscular or secondary X-rays) which may be produced by the primary X-rays in their passage through the solution.

The conversion of starch into dextrin is generally held to correspond to the change of a complex molecule into one of a more simple type (though still complex). Whether this actually corresponds to a more stable type, as the experiments upon dextrin seem to show, cannot be decided without more numerous agents being brought to bear upon the case and quantitative comparisons made of the effects observed. It is hoped that observations with different kinds of radiation may serve in this connection.

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* M. Massol, "Comptes Rendus," 152, p. 902.

† Kernbaum, "Comptes Rendus," Congrès International de Radiologie et d'Electricité, Tome I., p. 135.

CANCER RESEARCH LABORATORIES, THE MIDDLESEX HOSPITAL.

ABSTRACT.

When solutions of starch are irradiated for several hours by X-rays of moderate penetrating power, the opacity and viscosity of the solutions are markedly diminished. These physical changes are attended by chemical changes; there is a partial conversion of the starch into soluble starch and dextrin. A quantitative estimation of the amount of dextrin formed after the starch solution had been irradiated for 8½ hours showed that it corresponded to about 5 per cent. of the amount of starch initially present.

When solutions of dextrin were subjected to a similar exposure of X-rays, no conversion of this substance into glucose was obtained. From the fact that starch may be converted into dextrin by hydrolysis, a simple explanation of the effect would be afforded if water were decomposed by X-rays. From the negative results obtained by Kernbaum on these lines, however, it seems as if the effect were more likely attributable to a direct action upon the starch molecules, either by the X-rays or the secondary rays which they produce.

DISCUSSION.

The **PRESIDENT** thought the Paper one of considerable importance as definitely proving the chemical effect of X-rays.

Dr. A. D. **WALLER** also expressed his interest in the Paper.

XXXIV. *The Equi-Density Temperatures of Water and Other Liquids.* *By* Charles Robert Darling.

Received May 3, 1912.

At the meeting of the Society held at the Finsbury Technical College in March, 1911, the author called attention to the fact that aniline if dropped into water below 64 deg. would sink, whereas if the temperature exceeded 64 deg. the aniline would rise. This was accounted for by the coefficient of expansion of aniline being greater than that of water; hence its density diminishes more rapidly when heated, becoming equal to that of water at 64 deg. The present communication contains the results of experiments with a number of liquids which, like aniline, are either denser or less dense than water, according to temperature.

Equi-density Temperatures for Water and Several Liquids.—For the purposes of the experiments to be described, liquids insoluble or slightly soluble in water were chosen. The densities of each were determined at different temperatures with a pyknometer, and recorded graphically as shown in Fig. 1. A

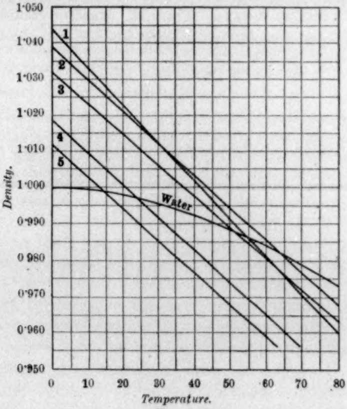

Fig. 1.

temperature-density curve for water was drawn on the same sheet, and the equi-density temperature for each liquid obtained from the intersections. The results were as under :—

Number	Liquid.	Equi-density temperature with water.	Density when equal.
1	Aceto-acetic ether…………	54°	0·9865
2	Aniline ………………………	64°	0·9815
3	Iso-butyl benzoate ………	50°	0·9880
4	Orthotoluidine ……………	24°	0·9973
5	Anisol………………………	15°	0·990

The following points are noteworthy : (1) All the liquids examined expand practically uniformly between 0 deg. and 80 deg. ; (2) the curve for aceto-acetic ether cuts that of aniline at 31 deg., and that of butyl benzoate at 62 deg., which represent equi-density temperatures for the liquids named. It is necessary to mention that the liquids employed were purchased as " refined," and were probably in no case perfectly pure.

Experiments with Liquids at or near the Temperature of Equal Density with Water.

1. *To Demonstrate the Existence of an Equi-density Temperature.* A reservoir containing the liquid under trial is made to communicate with two glass tubes, each furnished with a tap. One tube terminates near the surface of cold water contained in a beaker whilst the other passes to the bottom of a beaker containing hot water, the end of the tube being bent so as to discharge upwards. On opening the taps the liquid will be observed to fall through the cold water, and to rise through the hot. For a lecture experiment a mixture of equal volumes of aniline and orthotoluidine answers well, the hot water being at 70 deg. or more.

2. *The Automatic Formation of Drops of Liquids in Water.*—If a sufficient quantity of aniline be added to a mass of water in a beaker, and the temperature be maintained at 70 deg. or more, it is noticed that the aniline, which at first rises to the surface, parts with a large drop which falls to the bottom of the beaker, but which, after a short time, rises to the surface and joins on to the residual aniline ; after which a second drop is detached and rises as before, and so on indefinitely. A beaker 15 cm.

long and 7·5 cm. in diameter is suitable, filled to $\frac{2}{3}$ of its height with distilled water, and 40 cc. of aniline added. The same results may be obtained with aceto-acetic ether, which, however, is more soluble in hot water than aniline, and therefore not so satisfactory.

Reference to Fig. 1 will show that if the temperature of the water be 81 deg. that of the aniline on the surface must fall below 75 deg. in order that it may sink; and, similarly, for water at 75 deg. the temperature of aceto-acetic ether must be below 65 deg. for a drop to fall. The existence of such temperature differences has been verified by the aid of thermal junctions; and it is evident that these liquids when floating on hot water cool more rapidly than the adjacent water. This may be due to a lower specific heat, but it is doubtful whether this entirely explains the sudden cooling observed after the drop has risen to the surface.

This experiment is well adapted to show the formation of drops, the long neck which forms before partition in the case of aniline frequently giving rise to three or more secondary drops of varying size.

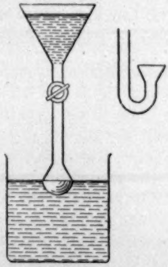

Fig. 2.

3. *The Formation, Under Control, of Drops of Liquid in Water.*—For this purpose orthotoluidine is the best liquid to employ. If discharged into water from a funnel possessing a stem of 1cm. diameter at 23 deg., spheres of any desired size may be formed, and may be prevented from falling to the bottom of the vessel by placing a 3 per cent. salt solution beneath the water. To produce large falling drops the apparatus shown in Fig. 2 is suitable, the orthotoluidine being allowed to run gradually from

a funnel, the stem of which is widened to a diameter of 3 cm. or 4 cm. at the end, which is placed in contact with the surface of water. The temperature of the water should be about 4 deg. below that of equi-density. Ascending drops may be formed by attaching to a funnel the bent tube shown to the right in Fig. 2, and discharging the liquid into water at about 35 deg. By controlling the tap of the funnel the shapes assumed by the drops durring formation may be examined at leisure. Figs. 3A

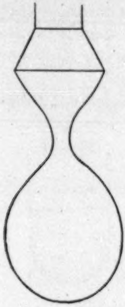

FIG. 3A.

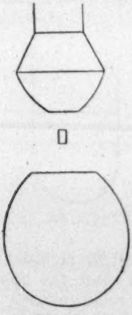

FIG. 3B.

and 3B show a large drop of orthotoluidine in two stages: 3A just before breaking, and 3B immediately after rupture has occurred; the secondary drop formed from the constricted neck being cylindrical in shape.

Similar effects may be obtained with the other liquids mentioned, which at the equi-density temperature will form spheres, at lower temperatures falling drops, and rising drops at higher temperatures.

4. *The Formation of Supported Drops, or Liquid Columns, in Water.*—When a falling drop is formed in a shallow layer of water it may reach the bottom of the vessel before rupture takes place, in which case a stable liquid column may form. A simple method of producing such columns is to pour butyl benzoate into a test-tube of 3 cm. diameter, a small quantity of water having previously been placed in the hemispherical end. The liquid is added until the pendant drop reaches the bottom of the tube, at which stage the appearance presented is as shown in Fig. 4A. If a small quantity of water be now allowed to run down the side of the tube, the column of liquid is elongated, the constricted part being narrowed, as in Fig. 4B. A further addition of water causes the drop to break, the appearance of the narrow part of the column immediately before rupture being as shown by the dotted lines in Fig. 4C. Similar

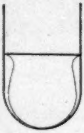

FIG. 4A.

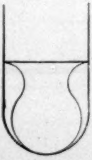

FIG. 4B.

effects may be obtained with aceto-acetic ether and aniline, each liquid, however, giving columns of different shape, depending upon the area of the surface of the water covered by the liquid.

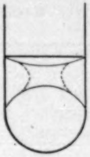

FIG. 4C.

XXXV. *Demonstration of the Movements of Semi-oily Liquids on a Water Surface.* *By* CHARLES ROBERT DARLING.

THE effect produced when a drop of liquid is placed upon a clean water surface is considerably modified if the liquid be slightly soluble. Whereas a drop of oil spreads and forms a permanent film, slightly-soluble liquids form films which afterwards break into globules, which, if a certain minimum size be exceeded, break up into smaller globules, until a state of equilibrium is reached. The division of the films or globules is produced by indentations which spread until partition has taken place. This indentation gives to globules a reniform shape; and in certain cases the distorted globules are projected violently across the surface of the water. The effects differ in intensity with different liquids, and phenomena peculiar to a given liquid may also be noted. The following are typical examples:—

1. *Aniline*, after spreading into a film, collects into one or more large globules, which become indented round the edges, after which the gloubles recover their shape; the recovery, however, being accompanied by the partition of small globules. This continues until the globule is reduced to a very small size, after which no further movements are observed.

2. *Dimethylaniline* forms a film which breaks up into small globules with great rapidity. The indentations, commencing at the edges, bifurcate and spread through the film, dividing it into small parts which become globular in shape. The partition is assisted by the formation of holes in the film, from which indentations spread and meet those which commence at the edges.

3. *Quinoline* behaves similarly to (2), but the action is slower, enabling the breaking-up process to be followed easily by the eye. The small globules which are formed finally become rings, a hole appearing at the centre of each, causing the globule to spread on all sides. These rings are quite permanent, and do not appear with the other liquids mentioned.

4. *Xylidine, 1 : 3 : 4*, forms globules which become indented on one side, and then move rapidly across the surface, and then recover their shape; after which indentation and projection are repeated until the globules are broken up into smaller ones,

when the action ceases. The behaviour of xylidine is similar to that of orthotoluidine, to which the author has previously called attention.

In explanation of these actions it is suggested that the surface tension of the water, which causes the drop to spread into a film, is weakened by the solution of the liquid. The opposing tensions (air-liquid and water-liquid) then overcome that of the water, with the result that the film is drawn back and indented. The strength of the air-water tension is afterwards partially restored by the sinking or diffusing of the dissolved part, so that the drawn-up mass again tends to spread. Indentations in a film or globule would, therefore, occur at the point at which the air-water tension had become most weakened, and if the opposing tensions were strong, the globule would be drawn across the surface. Equilibrium would then be produced when the air-water tension had become uniformly weakened, and the opposing tensions (now acting on small globules of greater curvature than the larger masses, and therefore at a larger angle to the water surface) possessed a resultant tension equal to that of the soiled water.

XXXVI. *The Maximum Sensibility of a Duddell Vibration Galvanometer.* By H. F. HAWORTH, *Ph.D., M.Sc., B.Eng., A.M.I.E.E.*

RECEIVED MAY 21, 1912. READ JUNE 14, 1912.

IN order to try and improve the sensibility of a Duddell vibration galvanometer,* particularly at high frequencies, the permanent magnets were removed and replaced by a field-magnet system after the manner of the Duddell oscillograph. Some experiments were made to determine how the sensibility of the instrument, as an ammeter and as a voltmeter, varied with the strength of the flux in the gap.

The connections to the galvanometer were as follows:—*ab*, Fig. 1, is a double-pole change-over mercury switch; with this switch to the left, a_1a, b_1b, and K_2 closed, current from the alternator AC passes through the high resistance R_2 (from 10^4 to 10^6 ohms) and through the galvanometer. A low-reading Ayrton-Mather electrostatic voltmeter, EV (max. 8 volts), is shunted across R_2 and so the galvanometer current is measured, the galvanometer deflection is noted, and from these data the current sensibility is determined.

On moving the double-pole switch over, closing contacts aa_2 and bb_2, the current flows through a resistance, R_1, of about 100 ohms and through the low resistance R_3 (from 0·1 to 1 ohm) across which the galvanometer is now shunted. The electrostatic voltmeter is now shunted across R_1, thus determining the current through R_3 and so the voltage across the galvanometer. Hence, the galvanometer deflection being noted, its sensibility as a voltmeter may be determined.

The field coils of the instrument were connected through a potential divider to the 200-volt mains. The total flux was measured by means of a search coil of one turn wound on the field magnet near the gap, comparison being made with a Hibbert standard.

The direct-current sensibility was measured by putting the double-pole switch to the left, opening the key K_2 and closing K_1.

The current through the strips was adjusted by means of a resistance, R_4, and measured with a sensitive milliammeter, A. In most of the experiments the maximum flux had the same value—namely, 93,600 lines, the gap flux density for this maximum flux being 14,000 lines per square centimetre—and the direct current through the galvanometer was adjusted until a steady deflection of 70 divisions on the scale was produced with this flux. Readings of volts and deflection, amperes and deflection were then taken with this value of the flux. The field current was then varied, and the flux was measured by pressing the key K_1, K_2 being open with the double-pole switch to the left, the flux being practically proportional to the deflection as the control was strong compared with any magnetic control which might be set up due to magnetic particles in the phosphor bronze strip.

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* "Proc. Phys. Soc.," Vol. XXI., pp. 774-787.

Knowing the direct-current sensibility the magnification with alternating currents, due to the synchronous motion,

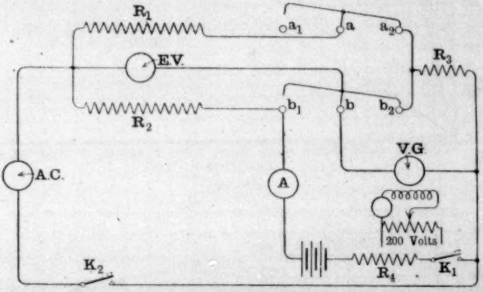

FIG. 1.

may be calculated. The following results (Table I. and Figs 2, 3, 4 and 5) show the effect of flux alterations on the sensibility of the galvanometer.

As would be expected, the sensibility of the instrument when used as an ammeter increases as the flux increases, and the limitation of sensibility is simply the limitation of the flux due to saturation of the iron.

On turning to the curves of sensibility as a voltmeter we have a different state of affairs.

Up to frequencies of 500 or 600, depending on the length of the wire, the sensibility increases from zero as the flux increases, until it reaches a well defined maximum, after which any increase of flux decreases the sensibility of the instrument.

If the apparent resistance V/A of the galvanometer be calculated from the voltmeter and ammeter flux curves, it is found

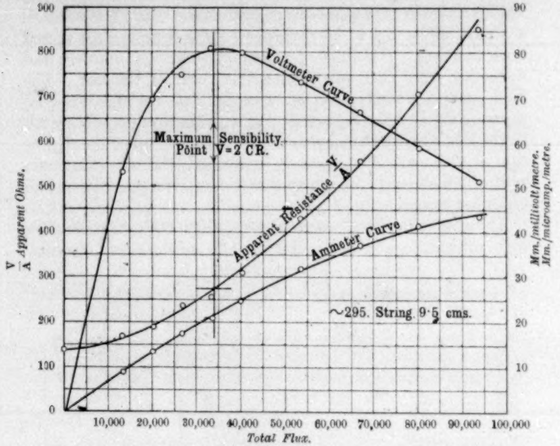

FIG. 2.

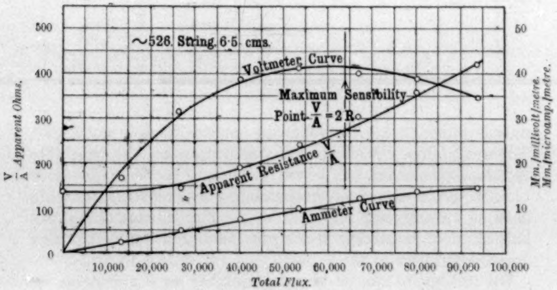

FIG. 3.

that the maximum sensibility of the galvanometer used as a voltmeter is reached when the apparent resistance is equal to twice the ohmic resistance of the instrument.

Treating the galvanometer as a motor of resistance R, and having a back E.M.F., E_b, if a forward E.M.F., E, is applied we

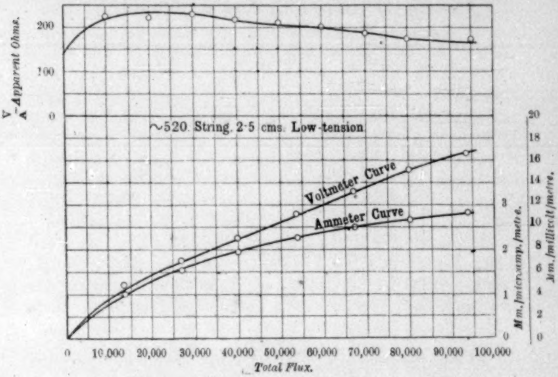

FIG. 4.

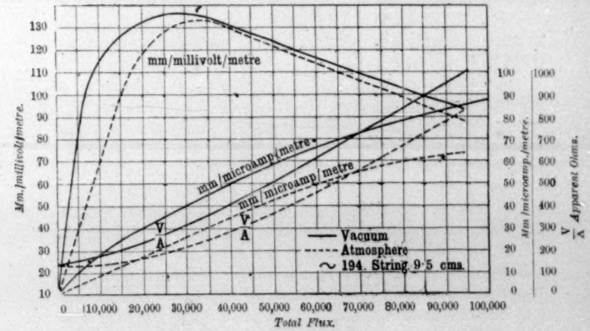

FIG. 5.

produce the greatest transference of electrical into mechanical energy when $E_b=\frac{E}{2}$—*i.e.*, when half the input is transformed

TABLE I.

Length of string, cm.	Tension.	Frequency.	Maximum voltmeter sensibility, mm./millivolt/metre.	Total flux for this maximum sensibility.	Apparent resistance for this maximum sensibility.	Max. mm. per micro-amp. at metre for flux 93,600 lines.	D.C. sensibility, micro-amp. per mm. at a metre for flux 93,600.	Magnification ratio of A.C. to D.C. sensibility.	Curves.
9·5	Low	237	95	30,800	275	46	6·75	310	...
	Medium ...	295	81	35,500	275	44	10·7	470	Fig. 2.
	Full.........	320	74	38,500	275	38	13·8	525	...
6·5	Low	310	73	55,000	275	26	13·2	343	...
	Medium ...	410	53	57,500	275	22	26·2	576	...
	Full.........	526	41	61,000	275	14·5	44·2	640	Fig. 3.
			No maximum sensibility. Value at flux 93,600		Varies up to				
2·5	Low	520	17	93,600	200	3	90·5	271	Fig. 4.
	Medium ...	740	14·4	93,600	200	2	179	358	...
	Full.........	900	10	93,600	160	1·6	265	424	...

into mechanical work and the other half into heat; then $C=\frac{E}{2R}$.

The back E.M.F. of the galvanometer is approximately 180 deg. out of phase with the current and corresponds to the back E.M.F. of the motor above; as such it may be treated as an additional resistance to the galvanometer, and so we obtain the greatest transference of electrical energy into mechanical energy, or motion, in the galvanometer, when the apparent resistance of the instrument is twice its ohmic resistance, or, in other words, when the back E.M.F. of the galvanometer is equal to its ohmic resistance drop. In this case the current and applied E.M.F. are in phase.

Mr. Butterworth ("Proc. Phys. Soc.," XXIV., p. 83, 1912) has shown mathematically that if a vibration galvanometer is placed in a circuit of self-induction L and resistance R, and if the various constants of the circuit are adjusted, a maximum deflection is obtained under certain conditions, and the current when this absolute maximum has been obtained is given by $I=\frac{E}{2R}$, and it is in phase with the applied E.M.F.

A very convenient way of altering the constants of the circuit so as to obtain maximum sensibility is to vary the flux through the gap of the instrument until, for a certain want of balance in the bridge, if used for bridge work, a maximum deflection is obtained. The back E.M.F. of the galvanometer is then equal to its ohmic resistance drop and is in phase with it.

If the length of the strings is kept constant the flux for maximum sensibility increases as the tension increases.

For high values of the frequency obtained by using short strings the flux can be increased to saturation without reaching a maximum sensibility, so it is advantageous to use an electromagnet system with a vibration galvanometer, because (1) for low frequencies and long strings a weak flux can be employed, as this is the condition for maximum sensibility, and (2) for high frequencies with short strings a very much stronger flux can be used than could be obtained with a permanent magnet, thus giving greater sensibility both as an ammeter and as a voltmeter. The sensibilities at high frequencies were only limited by the saturation of the magnet, and the back E.M.F. of the instrument was always less than its CR drop.

The sensibility of a Duddell vibration galvanometer throughout its scale would be much more uniform if some flux-squeezing device could be arranged, so that as the strings were shortened the total flux could be concentrated on the parts of the strings between the bridges.

In the table three sets of readings are given, each taken with a different length of string, and for each length of string three tensions are used, (1) with the string nearly slack, called the low tension; (2) a medium tension, somewhere about 10 grammes weight; (3) a full tension, about 20 grammes weight. It will be noticed that as the tension increases the flux for maximum sensibility increases and the magnification (*i.e.*, the ratio of the alternating-current sensibility to the direct-current sensibility) also increases.

When the strings have been shortened to a certain length the sensibility as a voltmeter does not pass through any maximum value even up to the greatest flux possible, so, with these and shorter string lengths, it is advantageous to work with the maximum magnetic field. In Fig. 2 it will be noticed that the maximum sensibility is obtained with the 9·5 cm. string with only one-third of the maximum flux, and that the maximum is sharply defined; with the higher frequency and shorter string length of Fig. 3, the maximum sensibility is reached with about two-thirds of the maximum flux, but it is much less sharply defined than in Fig. 2; in Fig. 4, with a very short string, the sensibility does not pass through any maximum value, and so the full excitation may be used.

The experiment of running the galvanometer in a vacuum was tried, but the results obtained were not very encouraging considering the trouble involved. The results of this experiment are shown in Fig. 5. At 194 ∼ and maximum flux the current sensibility was increased from about 65 mm. per micro-ampere at a metre to about 85 m./m./m., apparently showing that the greater part of the mechanical work produced was used in overcoming the molecular friction of the system, and that a much smaller part was used to overcome the air damping. The voltmeter sensibility reaches its maximum value in a vacuum for a smaller value of the flux than required in air.

At a 1,000 ∼ the increase of sensibility on running the instrument in a vacuum was somewhat greater than at 200 ∼ (as might be expected), being about 40 per cent., as against a 30 per cent. increase at 200 ∼.

Conclusion.—In a moving coil vibration galvanometer the maximum sensibility as a voltage detector will be obtained if the flux through the coil is so adjusted that at any time the back E.M.F. of the coil is equal to and in phase with its ohmic resistance drop (CR).

ABSTRACT.

The maximum sensibility of a moving coil vibration galvanometer as a voltage detector is obtained when the flux through it is so adjusted that the back E.M.F. of the coil is equal to its CR drop; then the back E.M.F. is equal to half the applied voltage, and the current is equal to V/2R and is in phase with the applied voltage.

Increases of current sensibility of about 30 per cent. at 200 ∾ and 40 per cent. at 1,000 ∾ were obtained on running the instrument in a vacuum, thus showing that a large part of the mechanical work produced was used in overcoming the molecular friction of the system. Tables and curves are given showing the variations of voltage and current sensibility with alteration of flux, &c.

DISCUSSION.

Mr. A. Campbell remarked that he had convinced himself that the chief energy loss in his vibration galvanometer was in the elastic hysteresis of the wires. In Duddell's galvanometer the moving wires are so light that he thought it might be otherwise, but Dr. Haworth's Paper showed that the damping was still chiefly due to the same cause.

XXXVII.—*An Accurate Examination of the Steinmetz Index for Transformer Iron, Stalloy and Cast Iron.* By F. Stroude, *B.Sc.*

Communicated by Prof. C. H. Lees. Received May 24, 1912.
Read June 14, 1912.

Synopsis.

A careful series of determinations of hysteresis loss on these materials are described. The method employed was the method of "uniformly varying flux." A set of hysteresis measurements by a ballistic method is also included, the results obtained being compared with those from the "slow cyclic" tests. The figures for the loss are reduced to the form of the Steinmetz law $W_h = \eta B^\varepsilon$, and it is shown that for each material the index "ε" is sensibly constant over a fairly wide range of B, but that its value differs considerably for different materials (cast iron $\varepsilon = 1{\cdot}8$. Transformer iron $\varepsilon = 1{\cdot}7$. Stalloy $\varepsilon = 1{\cdot}66$). For the purpose of comparison, results of previous experimenters are included and discussed.

The testing of iron for magnetic hysteresis by the ordinary ballistic methods is open to the objection that the magnetic treatment given to the specimen differs widely from that which it will receive in practice. Wattmeter tests, though free from the above objection, are liable to considerable inaccuracies, due to the small amount of power to be measured, and the necessity of correcting, by methods which are only approximate, for that portion of the power which represents the energy lost in eddy currents. The uncertainty concerning the maximum values of the magnetising force and the flux density, due to an imperfect knowledge of the wave forms, also tends to reduce the accuracy of the results obtained.

The "uniformly varying flux," or "slow cyclic" method of testing iron,* appears to be particularly suited to this work, since it applies to the iron a cyclical treatment, throughout which the induction is steadily varying at a constant rate, thus offering conditions which do not differ from those usually prevailing in practical work except in the speed (and also to a slight extent in the wave form) of the variation. This method is also capable of a degree of accuracy which is practically unattainable by other methods, whilst the apparatus is capable of rapid manipulation.

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* D. K. Morris and G. A. Lister, I.E.E., Vol. XXXVIII., pp. 282-294.

The experiments described below were undertaken with a view to determining what differences, if any, exist between the hysteresis loss of a specimen of iron as measured by one of the usual ballistic methods and the loss measured by the method of "uniformly varying flux." They are also intended to form the first of a series of accurate experimental figures, suitable for mathematical analysis with a view to the discovery, if possible, of some relation between magnetic hysteresis and induction density which will better accord with practical results than the universally adopted empirical law due to Steinmetz. For purposes of comparison the work of other experimenters has been consulted, and their results are included in the Paper.

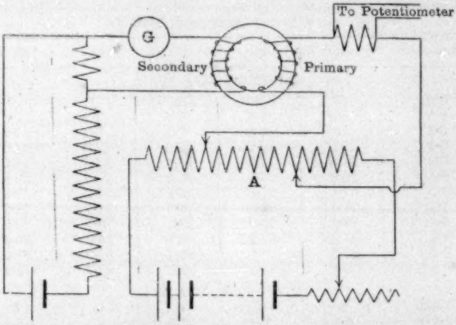

FIG. 1.—CONNECTIONS FOR SLOW CYCLIC METHOD.

The diagram of connections for the "slow cyclic" test is shown in Fig. 1, where A represents a special form of adjustable rheostat described in a previous Paper.* The current through the primary winding on the specimen was varied at such a rate as to keep the voltage induced in the secondary winding constant at a predetermined value, and the rate of variation of the current was recorded by means of a potentiometer working in conjunction with a pendulum operating a Morse recorder. By plotting current and time values the hysteresis loops were obtained, and their areas measured by means of a planimeter.

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* J. T. Morris, R. M. Ellis and F. Stroude, THE ELECTRICIAN, LXI., p. 40.

For each value of the primary current at least four determinations were made, the mean of these results being used for computing the area of the loop. The agreement obtained was very good, being well within 1 per cent. of the total time taken on the loop. Specimens figures are given in Table I.

Table I.

0·8*a* *loop.*

Current.	Time from start.				
	I.	II.	III.	IV.	Mean.
			(*a*) *Stalloy Ring* A.		
0·8	0	0	0	0	0
0·5	3·5	3·4	3·5	3·4	3·5
0·3	7·4	6·8	7·1	7·3	7·2
0·1	13·0	12·7	12·9	12·3	12·7
0·0	17·5	17·5	17·4	17·7	17·5
0·1	26·6	26·6	26·3	26·7	26·6
0·2	57·4	57·2	57·1	57·3	57·3
0·3	71·4	71·3	70·9	71·7	71·3
0·4	79·0	79·0	78·5	79·5	79·0
0·5	83·9	83·2	83·6	84·3	83·8
0·6	87·7	87·1	87·6	88·0	87·6
0·8	93·4	93·1	93·3	93·7	93·4
			(*b*) *Cast Iron Ring* Q.		
1·830	0	0	0	0	0
1·098	5·3	5·2	5·4	5·3	5·3
0·732	8·6	8·2	8·4	8·3	8·4
0·366	12·4	12·2	12·3	12·0	12·2
0·0	15·9	16·0	15·9	15·7	15·9
0·366	24·2	24·0	24·0	23·6	24·0
0·549	29·2	28·8	28·7	28·6	28·8
0·732	37·3	36·8	37·0	36·8	37·0
0·915	47·3	47·2	47·0	46·8	47·1
1·098	57·4	57·0	57·1	56·8	57·1
1·280	65·5	65·3	65·4	65·2	65·3
1·468	72·0	71·7	71·9	71·6	71·8
1·640	76·5	76·5	76·4	76·3	76·4
1·830	80·6	80·7	80·5	80·4	80·5

Before each set of experiments the specimens were carefully demagnetised. For this purpose the primary current was increased to a high value, and then gradually reduced to zero, being continually and rapidly reversed all the time.

The specimens were given a uniform treatment of 100 current reversals, and 12 continuous reversals at each current value to ensure that the iron had attained a cyclical state before observations were made.

Three specimens of iron were tested, viz. :—

I.—Sankey's best (1900) transformer iron.
II.—Stalloy (3 per cent. silicon iron).
III.—Cast iron.

All the specimens tested were in the form of rings. 6 in. external diameter and 5 in. internal diameter. In the case of materials I. and II. these rings were built up of stampings. Two rings of each material were tested. Particulars of these rings are given in Table II. The magnetisation curves obtained by joining the summits of the hysteresis loops are given in Fig. 2.

TABLE II.—*Particulars of Rings.*

Material.	Ring letter.	Sectional area (sq. cm.).	Mean length of path (cms.).
Transformer Iron (1900)	A B	1·976 1·904	43·92 43·93
Stalloy	A B	1·648 1·775	43·88 43·89
Cast Iron	Q A	1·59 1·593	43·89 43·98

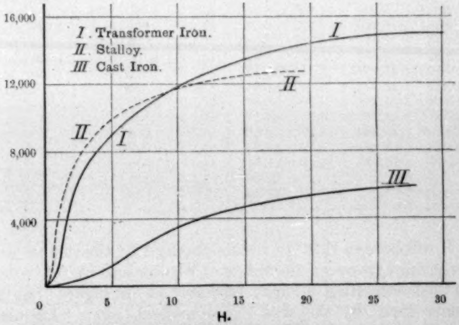

FIG. 2.—MAGNETISATION CURVES.

RESULTS.

I. *Transformer Iron* (1900).—The first set of experiments were made on Ring A in 1907 by the author working in conjunction with Mr. R. M. Ellis, the slow cyclic method being used.

To corroborate the results obtained, a further set of experiments were made on Ring B in 1908-9 by the author, and Mr. T. H. Langford, hysteresis loops being taken both by the slow cyclic method and by the ballistic "step-by-step" method. The results of these experiments are tabulated below (Table III.).

TABLE III.—*Transformer Iron—Results.*

Ring A.			Ring B.					
Slow cyclic method.			Slow cyclic method.			Ballistic method.		
H	B	W_h	H	B	W_h	H	B	W_h
0·0286	13·19	0·00068	0·4205	182·2	1·47	0·4205	225	3·22
0·0715	36·82	0·0341	0·843	798	49·03	0·841	665	39·4
0·1430	73·20	0·238	1·264	2,705	461	1·27	2,705	476
0·2861	136·9	0·990	1·685	4,395	860	1·66	4,200	982
0·4005	242·7	2·93	2·101	5,910	1,588	2·09	5,600	1,560
0·5722	356·7	8·80	2·524	6,580	1,942	2·53	6,330	1,900
0·715	545·4	22·4	2·945	7,500	2,312	2·95	7,200	2,240
0·858	880·2	59·4	3·365	8,220	2,700	3·35	7,740	2,720
1·001	1,362	190·2	3·790	8,960	3,176	3·80	8,350	3,080
1·245	2,415	338	4·205	9,260	3,340	4·22	9,000	3,440
1·660	4,306	856	5·050	10,190	3,950	5·05	9,460	3,870
2·074	5,018	1,193	5·900	10,840	4,420	5·90	10,220	4,430
2·904	6,850	2,027	6·74	11,800	5,040	6·74	10,870	4,900
4·15	8,017	2,760	7·568	11,920	4,930	7·55	11,330	5,120
5·81	9,458	3,595	8·41	12,270	5,160	8·40	11,730	5,510
8·30	11,155	5,110	9·25	12,580	5,630	9·22	12,120	5,860
10·79	12,127	5,650	10·09	13,090	5,860	...	...	...
14·01	13,362	6,260	10·91	13,350	6,060	...	...	...
25·31	14,950	8,310	11·77	13,600	6,540	...	...	...
33·19	15,980	11,170	14·59	14,200	6,390	...	...	...
41·48	16,485	10,120	16·68	14,590	6,790	...	...	...
58·07	16,735	10,420	20·81	15,130	7,070	...	...	...
...	...	...	25·00	15,510	8,310	...	...	...
...	...	...	28·19	15,900	9,150	...	...	...

It will be seen that the results obtained by the two methods are in fair agreement, the values of W_h obtained by the ballistic method exhibiting a slight tendency to be higher than the values given by the slow cyclic method, more particularly at the higher values of B. Comparative loops by the two

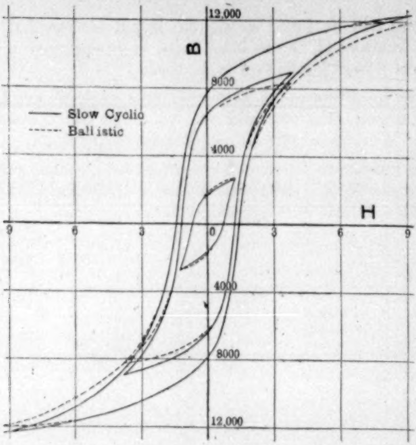

FIG. 3.—BALLISTIC AND SLOW CYCLIC LOOPS.

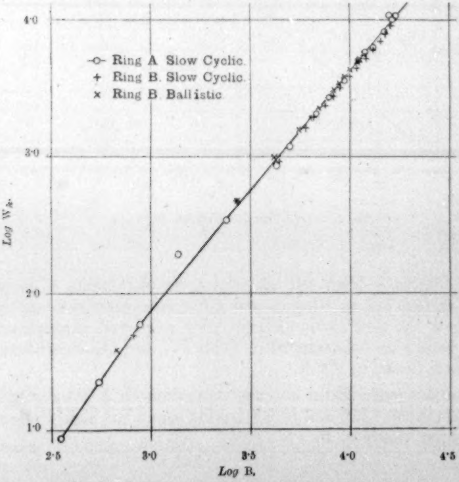

FIG. 4.—LOGARITHMIC CURVES—TRANSFORMER IRON.

methods will be found in Fig. 3. It will be seen that, for a given value of H, the ballistic loops, in general, give a lower value of B than do the slow cyclic tests.

From these figures the curves connecting Log W_h and Log B were drawn (Fig. 4), and the values of the "Steinmetz index" ε in the equation $W_h = \eta B^{\varepsilon}$ deduced therefrom.

The index was found to have an almost constant value of approximately 1·7 between the limits B=1,000 and B=17,000, the three distinct tests agreeing remarkably well.

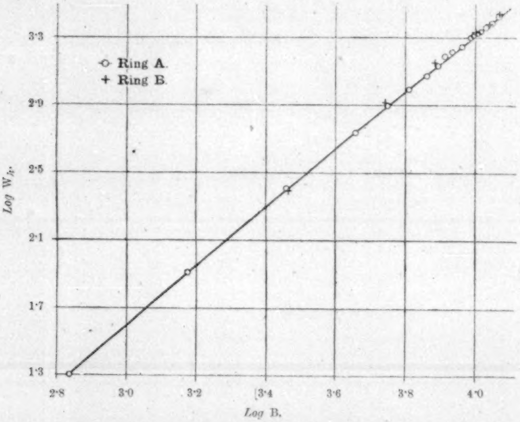

FIG. 5.—LOGARITHMIC CURVE—STALLOY.

II. *Stalloy.*—With this material a complete series of loops was carried out on Ring A, and a few corroborative loops on Ring B, the slow cyclic method being employed throughout. The results are summarised in Table IV., and the logarithmic curve is shown in Fig. 5.

For this material the index is approximately 1·66 for values of B between 4,000 and 12,000 having somewhat higher values for lower values of B.

TABLE IV.—*Stalloy—Results.*

Ring A.			Ring B.		
H	B	W_h	H	B	W_h
0·372	688	19·7	0·762	2,892	250·6
0·521	1,500	82·1	1·523	5,560	797
0·744	2,880	256	2·665	7,740	1,402
1·117	4,520	555	5·330	10,120	2,183
1·489	5,600	777	7·66	11,000	2,410
1·861	6,470	992	9·75	11,670	2,672
2·233	7,290	1,193	...	...	...
2·605	7,820	1,354	...	...	...
2·977	8,270	1,523	...	...	...
3·350	8,630	1,625	...	...	...
3·722	9,150	1,751	...	...	...
4·466	9,900	2,036	...	...	...
4·760	9,760	2,018	...	...	...
5·440	10,130	2,120	...	...	...
6·130	10,390	2,160	...	...	...
6·810	10,700	2,297	...	...	...
7·48	10,950	2,395	...	...	...
8·17	11,200	2,408	...	...	...
8·86	11,400	2,460	...	...	...
9·53	11,680	2,660	...	...	...

III. *Cast Iron.*—The same procedure was adopted, a complete series of loops being taken for Ring Q, and a few corroborative loops for Ring A.

The tabulated results will be found in Table V., and the logarithmic curve in Fig. 6.

TABLE V.—*Cast Iron—Results.*

Ring Q.			Ring A.		
H	B	W_h	H	B	W_h
4·592	1,145	424	5·51	1,550	838
5·54	1,765	1,140	8·72	3,220	2,602
6·429	1,990	1,262	15·12	4,520	3,700
6·72	2,120	1,429	17·25	4,900	6,340
7·55	2,460	1,888	25·85	5,900	8,930
8·40	3,000	2,590	...	...	...
9·22	3,360	3,210	...	...	...
10·06	3,566	3,574	...	...	...
10·9	3,630	3,760	...	...	...
11·74	3,770	4,335	...	...	...
16·06	4,710	6,080	...	...	...
18·25	5,040	6,940	...	...	...
22·8	5,560	8,390	...	...	...
27·3	5,960	9,240	...	...	...

The index will be seen to be practically constant at 1·82 for values of B between 2,000 and 6,000, rising for lower values of B.

In Table VI. will be found the value of ε at different flux densities deduced from the above results. Similar figures obtained from Papers by other experimenters are also given, the method employed for the determination being given in each case. The figures given for soft iron wire under "Steinmetz" are of peculiar interest, since it was from these figures that Steinmetz in the year 1890 evolved his so-called "law" of hysteresis. The experimental data for these figures were obtained by Ewing.

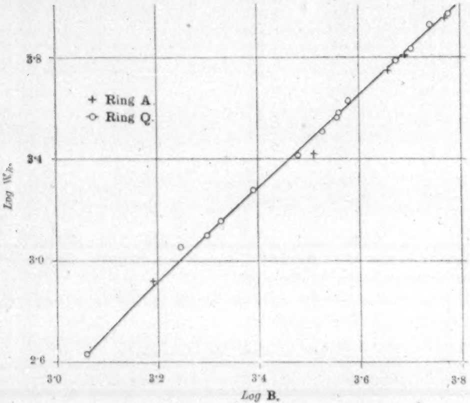

Fig. 6.—Logarithmic Curve—Cast Iron.

Steinmetz, in his Papers before the American I.E.E., gives a large number of tests on various materials. The indices obtained vary from 1·59 to 1·7. Unfortunately many of the materials were only tested at three flux densities, and the index values derived from them are consequently of doubtful value. In many cases, also, hysteresis tests were made with unsymmetrical flux changes, and used for computing the "index," Steinmetz claiming to have proved that "the energy dissipated by hysteresis depends only upon the difference of the limiting values of the magnetic induction . . . . but not upon the absolute values." M. Rosenbaum has recently shown this to be untrue ("Jour." I.E.E., XLVIII., p. 534), the hysteresis loss in one case being nearly doubled by unsymmetrical flux variation.

TABLE VI.

Authority.	Method.	Iron tested.	"Steinmetz index" between the limits:—										
			200 to 500	500 to 1,000	1,000 to 2,000	2,000 to 4,000	4,000 to 6,000	6,000 to 8,000	8,000 to 10,000	10,000 to 12,000	12,000 to 14,000	14,000 to 16,000	16,000 to 18,000
Present Paper	Slow cyclic R	Transformer iron A	2·27	2·0	1·71	1·71	1·71	1·71	1·71	1·71	1·71	1·71	1·71
	,, ,, R	,, ,, B	2·4	2·0	1·82	1·68	1·68	1·68	1·68	1·68	1·68	1·68	...
	Ballistic R	,, ,, B	2·3	1·72	1·72	1·72	1·72	1·72	1·72	1·72	...	...	...
	Slow cyclic R	Stalloy A and B	...	1·82	1·78	1·72	1·66	1·66	1·66	1·66	...	...	...
	,, ,, R	Cast iron Q and A	...	...	1·96	1·82	1·82	...	...	...	...	...	...
Steinmetz, "Elec. Eng." N.Y. X., p. 137	R	Soft iron wire	...	...	...	1·56	1·45	1·58	1·58	1·58	2·0	1·15	...
Steinmetz, Amer. I.E.E., Vol. IX.	A.C. wattmeter	Iron filings	1·96	1·65	...	...	...	...	...	...	...	...	...
Ewing and Miss Klaassen, "Phil. Trans." 1893	Ballistic R	Fine iron wire I.	2·0	1·9	1·68	1·68	1·68	1·68	1·60	1·43	1·43	1·43	...
	,, R	Fine steel wire II.	2·36	2·18	1·69	1·69	1·63	1·57	1·57	1·57	1·57	1·57	1·57
	,, R	Thick sheet iron III.	...	...	1·56	1·56	1·68	1·80	1·80	1·80	1·80	1·80	...
	,, R	Thin sheet iron IV.	1·9	1·68	1·55	1·475	1·475	1·475	1·70	1·70	1·70	...	...
	,, R	,, ,, ,, V.	2·02	1·80	1·64	1·64	1·64	1·64	1·64	1·64	...	...	...
	,, R	Charcoal iron wire VI.	...	...	1·65	1·60	1·55	1·55	1·55	1·55	...	...	...
	,, R	Soft iron wire VII.	...	1·84	1·74	1·62	1·62	1·62	1·62	1·62	1·62	1·62	...
	,, R	"Hedgehog" wire VIII.	...	...	1·85	1·54	1·40	1·50	1·62	1·62	1·62	1·62	...
		,, reannealled VIII.	...	2·0	1·90	1·75	1·60	1·57	1·54	1·54	1·54	1·54	...
J. Epstein, E.T.Z., XXI., 1900	A.C. wattmeter S	Transformer iron	...	...	...	...	...	1·62		1·72			...
Gumlich and Rose, E.T.Z., 1905.	Ballistic S	Transformer iron I.	...	...	...	1·525	1·58	1·65	1·79	1·82	2·03	2·38	...
	A.C. wattmeter S	,, ,, I.	...	...	...	...	1·55	1·60	1·81	1·83	1·89	2·33	...
	Ballistic S	,, ,, II.	...	...	...	1·55	1·55	1·60	1·66	1·67	1·76	...	...
	A.C. wattmeter S	,, ,, II.	...	...	...	...	1·52	1·56	1·60	1·74	1·73	...	...
	Ballistic S	,, ,, III.	...	...	...	1·67	1·62	1·66	1·75	1·66	1·87	2·8	...
	A.C. wattmeter S	,, ,, III.	...	...	...	...	1·77	1·60	1·72	1·69	1·82	2·47	...
E. Wilson, Winson and O'Dell, Proc. Roy. Soc. A., 80 pp. 548, 552.	Ballistic R	Stalloy	1·9	1·85	1·7	1·6	1·6	1·7	1·8	1·8	2·35	...	...
	,, R	Lohys	1·9	1·74	1·7	1·65	1·5	1·5	1·7	1·87	2·5	...	...
Lloyd and Fisher, Bull. B. of S., V., p. 453.	A.C. wattmeter S	Transformer Steel P	...	...	...	1·51	1·53	1·68	1·75	2·12	...	...	...
	S	,, ,, K	...	...	...	1·46	1·50	1·57	1·75	2·11	...	...	...
	S	,, ,, W_1	...	...	...	1·59	1·59	1·61	1·71	1·96	...	...	...
	S	,, ,, R	...	...	...	1·69	1·65	1·58	1·68	1·71	1·88	...	...

NOTE.—The letters in the second column indicate the form of specimen used in the test. R meaning ring and S = Built up "magnetic square."

To face p. 246.

In connection with Ring IV. (Ewing and Klaassen) Ewing states: " The changes in the index indeed correspond to the passage from one to another of the familiar successive stages in the process of magnetisation." In no other of the results included in Table VI. is any interconnection of this nature indicated.

In Fig. 7 are shown a few of the " index "—B curves plotted from the results in Table VI. It will be seen that Ewing's results on rings V. and VII. both show constant " indices " over a considerable range of B, and agree in that respect with the author's figures on Transformer Iron and Stalloy.

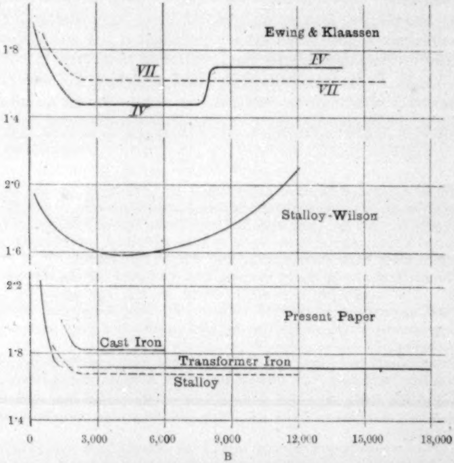

FIG. 7.—INDEX—B CURVES.

A rather interesting point in connection with the author's figures is the fact that the material with the highest maximum permeability has the lowest index, and vice versa.

Further experimental data are, of course, necessary before it will be possible to arrive at any general conclusion regarding the laws relating to magnetic hysteresis, and the author hopes before long to publish measurements of the hysteresis losses in iron at the lower and upper extremes of magnetisation. In view of the long time during which the research has been in progress, it was thought, however, preferable to publish the existing results relating to the practical range of magnetic induction rather than to wait for the completion of the other portions of the work which have scientific rather than practical value.

The above experiments were carried out in the electrical engineering department of the East London College (University of London). In conclusion, the author would like to record his best thanks for assistance received from the Research Fund established by the Governors of the College. He has also much pleasure in acknowledging the invaluable assistance given him by Mr. C. R. Turner during the latter part of the work, and finally would like to express his deep sense of indebtedness to Prof. J. T. Morris for much kindly advice and encouragement.

ABSTRACT.

These experiments were undertaken to provide a sound experimental basis, suitable for mathematical analysis, with a view to discovering, if possible, some relation connecting hysteresis loss and flux density which will accord with results obtained practically to a greater extent than the empirical law due to Steinmetz.

The method of uniformly varying flux was used for the determinations. The magnetic treatment given to the specimen by this method approximates to that received by iron employed in alternating-current work, whilst the method admits of a very high degree of accuracy.

Experiments were made with transformer iron stalloy (3 per cent. silicon iron) and cast-iron, two rings of each material being tested.

The rings were carefully demagnetised before testing and even reduced to a cyclical state for each current value before making observations.

A set of comparative tests on one of the transformer iron rings was made by the ballistic method, and these tests show that, in general, for a given value of B the hysteresis loss and the value of H for the ballistic tests are higher than the corresponding values for the slow cyclic tests.

The results were expressed in the form of the Steinmetz equation $W_h = \eta B^{\epsilon}$.

For transformer iron ϵ has the value 1·7 between the limits $B = 1{,}000$ and $B = 17{,}000$. Stalloy gives a value of $\epsilon = 1{\cdot}66$ between $B = 4{,}000$ and $B = 12{,}000$, having somewhat higher values for lower values of B. In the case of cast iron ϵ is equal to 1·82 between the limits $B = 2{,}000$ and $B = 6{,}000$, rising for lower flux densities. It will be seen that the material with the highest maximum permeability has the lowest constant " index " and vice versa.

The work of previous experimenters has been consulted, and

values of "ϵ" at different flux densities deduced from their experimental results.

It is hoped before long to publish results obtained at the lower and upper extremes of magnetisation.

DISCUSSION.

Dr. **Russell** thought the results in this paper would be most useful.

Mr. A. **Campbell** suggested that the author should try the effect of annealing the transformer iron, which will then sometimes give an index of nearly 2. The losses at low values of B were very important, and the results so far were very discordant. It is of importance to know the energy loss in telephone cables that are loaded by winding iron strip round them. Here the value of B would be very small, probably 50 or 100.

Prof. C. H. **Lees** remarked that there were interesting scientific problems raised by the constancy of the index over so wide a range.

Prof. G. W. O. **Howe** remarked that the index seemed to go off to infinity for very low values of B, indicating that the hysteresis fell very rapidly.

Prof. J. T. **Morris** drew attention to the accuracy of the present work. He pointed out that in order to find explanations for the losses experiments should be made at high temperatures, above 700°C., or as where the B and H curves are of simple character. The hysteresis loss is then zero for values of B less than a certain amount, and constant for all values greater than another fixed value, and a linear function of B between these two ranges.

XXXVIII. *The Calibration of Wave-Meters for Radio-Telegraphy.* *By Prof.* G. W. O. Howe.

Read May 31, 1912.

The most common form of wave-meter or frequency meter for use in radio-telegraphy is a simple oscillatory circuit consisting of a variable air-condenser connected across the terminals of an inductive coil. The latter is loosely coupled with the aerial or other circuit under examination, and the capacity of the variable condenser is altered until resonance is obtained, *i.e.*, until the natural frequency of the wave-meter corresponds to the frequency of the oscillatory current in the aerial. The attainment of resonance is indicated either by an ammeter in series with the wave meter or by a vacuum tube placed across the condenser terminals; the vacuum tube can sometimes be advantageously replaced by a low-reading electrostatic voltmeter. From the calibration curve of the variable condenser and the known inductance of the wave-meter coil the approximate frequency can be easily calculated. To cover a large range of wave-lengths a set of coils will be necessary, the number depending on the range and on the ratio of the maximum capacity of the variable condenser.

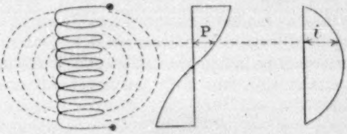

Fig. 1.

The dielectric being air, the capacity is the same for high-frequency alternating as for static pressures. If fine wire or specially stranded wire be employed in winding the coil the inductance will also be independent of frequency. The main source of error or uncertainty in the calculation of the frequency is the capacity between neighbouring portions of the coil. Even when the variable condenser is removed the coil itself has a definite natural frequency, the ends acting as the plates of a condenser and the central portion as an inductance, the two functions being distributed, however, over the whole coil and gradually merging into one another. The distribution of potential P and the current *i* throughout the coil will be somewhat as shown in Fig. 1, the width of the diagrams representing the amplitude at each point. When a condenser is connected across the coil as shown in Fig. 2 the distribution of the potential will be modified, owing to the smaller variation of the current amplitude throughout the coil.

The natural frequency of a cylindrical single-layer coil oscillating freely can be calculated by means of empirical formulæ and curves established by Drude from a large number of experiments.* It can, of course, be determined experimentally by bringing the coil near a circuit in which high-frequency currents are maintained, the frequency of which can be adjusted until a vacuum tube near the coil indicates that resonance is produced. The frequency of the oscillatory current can then be found by means of a calibrated wave-meter. This was essentially the method employed by Drude.

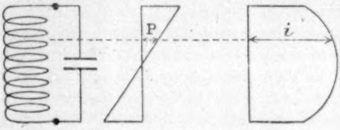

FIG. 2.

Fig. 3 represents an imaginary oscillatory circuit of the same frequency as the coil, but with concentrated capacity and inductance. The current would have the same amplitude at the ends as at the middle. An infinite number of such imaginary circuits could be made with different values of K and L, but with the same product KL and, therefore, of the same frequency. Can anyone of these imaginary circuits, as represented in Fig. 3, be said to represent or be equivalent to the actual freely oscillating coil ? If so, we should be justified in calling

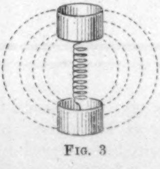

FIG. 3

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* Annalen der Physik, 1902. Vol. IX., p. 322.

the value of the inductance in Fig. 3 the effective inductance of the actual coil, and the capacity between the rings in Fig. 3, the effective or self-capacity of the actual coil. Drude * implies the existence of such an equivalent arrangement, and, because of the current distribution shown in Fig. 1, assumes that the effective inductance may be taken as $\frac{2}{\pi}$ of the total inductance of the actual coil. Knowing the effective inductance and the natural frequency, the effective or self-capacity of the coil is easily calculated. When a coil is used in conjunction with a variable condenser to form a wave-meter the question which arises is: How must the calculation of frequency be modified in order to allow for the effect of this self-capacity of the coil? No attempt is here made to give a rigorous answer to this question, but it is shown that for practical purposes the effective inductance of the imaginary circuit in Fig. 3 can be taken as equal to the total inductance of the coil, and that if the self-capacity, calculated from the natural frequency on this assumption, be added to the capacity of the variable condenser when calculating the frequency the error in the resulting calibration curve will be very small.

Experiments were made on a coil of 169 turns on a glass cylinder 14·5 cms. diameter and 21·3 cms. long. Its inductance was 2·16 millihenries and its natural wave-length when freely oscillating in space was 185·5 metres. Its terminal capacity was increased by small amounts by means of brass spheres various distances apart, and the resonant frequency was determined in each case. Whether we plot the wave-length against the root of the added capacity as in Fig. 4, or the square of the wave-length against the added capacity, as in Fig. 5, the dotted straight line through the origin would be obtained were it not for the self-capacity of the coil. The experimental results are plotted as crosses in Figs. 4 and 5. The latter figure shows clearly that for practical purposes the effect of the self-capacity of the coil is to add a constant capacity of 4·5 micro-microfarads to that of the variable condenser. This 4·5 is the self-capacity obtained from the natural frequency of the coil on the assumption that the full inductance is effective.

When the wave-meter has a set of coils, the ranges of which slightly overlap, another method can be used to determine experimentally the self-capacity of the coils. This may be

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* *Loc. cit.*

illustrated by means of an actual example. The wave-meter consisted of an air-condenser in a glass vessel lined with tinfoil, the latter being connected to one terminal and preferably

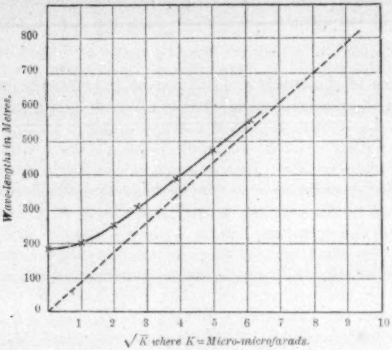

FIG. 4.

earthed, and a set of four coils of silk-covered copper wire on wooden drums. From the calibration curve, Fig. 6, it will be seen

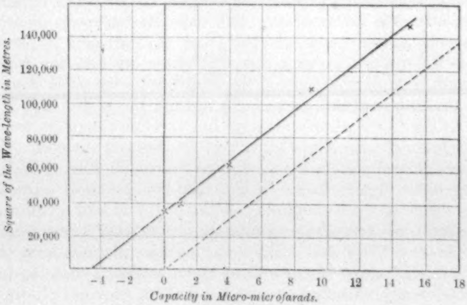

FIG. 5.

that the condenser has a minimum capacity of 76 and a maximum capacity of 1,042 micro-microfarads. The inductances

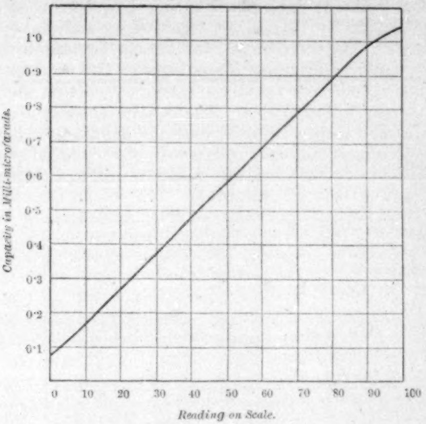

FIG. 6.—CALIBRATION CURVE OF VARIABLE AIR CONDENSER.

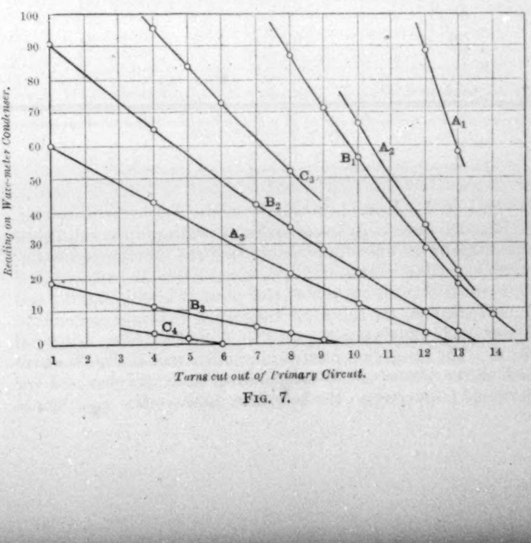

FIG. 7.

of the four coils, as determined at low frequencies were as follows :—$L_1=23{\cdot}7$; $L_2=51{\cdot}9$; $L_3=194{\cdot}9$; and $L_3=1{,}673$ microhenries. The wave-meter was loosely coupled with an oscillatory circuit containing a spark-gap. The frequency of the oscillatory currents could be varied by cutting out the inductance one turn at a time. The variable condenser of the wave-meter was always adjusted until the helium tube placed across its terminals indicated resonance. Over certain ranges it was possible to obtain readings with two different coils, and this was always done when possible. The results are plotted in Fig. 7, as A_1, A_2, &c., the numbers referring to the coils used. A, B and C refer to different series of readings, using different capacities in the spark-gap circuit, and are therefore not comparable, but all curves with the same letter are comparable. Thus with 10 turns cut out of the primary circuit $A_3=12$ and $A_2=67$, that is, for the same frequency, the wave-meter condenser reads 12 when using coil No. 3, and 67 when using coil No. 2. The results are plotted in another way in Fig. 8, where each curve shows the corresponding readings obtained for the same frequency on the two wave-meter coils. Each curve

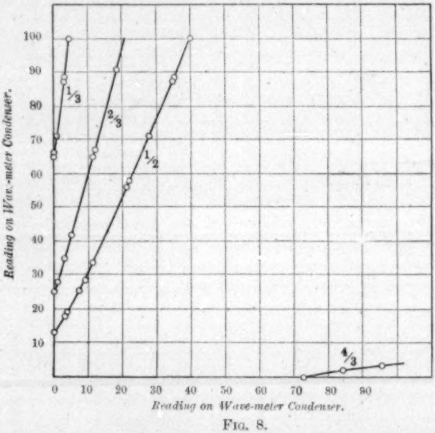

FIG. 8.

$\frac{1}{2}$ means that the ordinates are the readings obtained with Coil No. 1, and the abscissæ those obtained with Coil No. 3 for the same frequency.

in Fig. 8 contains points taken promiscuously from series A, B and C and their close agreement shows the accuracy of the wave-meter setting. It is seen that for a reading near the zero end of the condenser, where a knowledge of the self-capacity of the coil is essential, there is always a corresponding reading, obtained with a smaller inductance, with the variable condenser at the other end of the scale, where the effect of the self-capacity of the coil is almost negligible. From two such readings the self-capacity in the first case can be determined with sufficient accuracy for all practical purposes, as a large error in its assumed value when its effect is small causes a small error in the calculated value when its effect is large. This can best be shown by an actual example : Let K_a be the added capacity as read off the calibration curve of the variable condenser, K_s the self-capacity of the coil, and K_e the total effective capacity K_a+K_s.

All capacities are in micro-microfarads.

Now from Fig. 8, a wave-length which gave a reading 0 with L_4, gave a reading of 72·5 with L_3. Calling the capacities of the condenser for these readings K_{a4} and K_{a3}, we see from Fig. 6 that $K_{a4}=76$ and $K_{a3}=810$. It is known that the self-capacity of coil No. 3 is about 15, but let us assume firstly that it is 10, and secondly that it is 20, and see how this affects the calculated value of K_{s4}, the self capacity of coil No. 4.

We have

$$\frac{K_{e3}}{K_{e4}}=\frac{L_4}{L_3}=\frac{1{,}673}{194\cdot 9}.$$

(*a*) If $K_{s3}=10$, $K_{e3}=810+10=820$, $K_{e4}=95\cdot 5$ and $K_{s4}=K_{e4}-K_{a4}=95\cdot 5-76=19\cdot 5$.

(*b*) If $K_{s3}=20$, $K_{e3}=810+20=830$, $K_{e4}=96\cdot 6$ and $K_{s4}=K_{e4}-K_{a4}=96\cdot 6-76=20\cdot 6$.

Hence it may be safely assumed that the self-capacity of coil No. 4 is about 20. Similarly it was found that K_{s3} was 14 and K_{s2} about 18 micro-microfarads.

The natural frequency of coil No. 4 when oscillating freely, but with the earthed terminal still connected to the condenser, was found to be 347 metres. Taking the whole inductance of the coil, viz., 1,673 microhenries, this gives a self-capacity of 20·3 micro-microfarads, which agrees substantially with the value found above.

In the case here considered the error introduced by neglecting the effect of self-capacity reaches about 12 per cent. ; by allowing for the self-capacity of the coil in the way indicated the error can easily be reduced to less than 1 per cent.

By suitably designing the coils of wave-meters the effect of self-capacity may be greatly minimised. It will have been noticed that whereas coil No. 4, with an inductance of 1·673 millihenries, has a self-capacity of 20 micro-microfarads, the coil on the glass cylinder, with an inductance of 2·16 millihenries has a self-capacity of only 4·5 micro-microfarads. This difference is due to the fact that coil No. 4 has a diameter 7·6 times its axial length, whereas the diameter of the other coil is only 0·7 of its axial length.

As an example of the effect of shape on the self-capacity, in the following table are given four different designs for a coil of 2 millihenries, wound with wire of such a diameter that there are 20 turns per centimetre. The inductances are calculated from the formulæ of Rayleigh and Niven, and Lorenz, and the natural frequencies from Drude's curves for single-layer coils on ebonite tubes.

Diameter	Axial length.	Natural wave-length.	Self-capacity in micro-microfarads
5 cms.	22·5 cms.	102 metres	1·46
10 ,,	8·0 ,,	148 ,,	3·08
15 ,,	5·15 ,,	194 ,,	5·3
20 ,,	3·8 ,,	229 ,,	7·38

The effect of the self-capacity of the coils being to reduce the range of wave-length covered by each coil, it is necessary when calculating the inductances required for the various coils of a set, to allow considerable overlap, if self-capacity be neglected in the preliminary calculations, as otherwise there may be gaps between the ranges of the various coils when actually tested.

ABSTRACT.

Wave-meters consisting of a variable air-condenser and a set of coils can be calibrated approximately by calculation from the known capacity of the condenser and the inductance of the coils. The most probable source of error is that due to the capacity from turn to turn of the coil. This can be allowed for, with sufficient accuracy for all practical purposes, by finding the natural frequency of the coil alone and calculating its effective or self-capacity on the assumption that the whole steady current inductance of the coil is effective even when it is oscillating freely. This capacity is then added to the capacity

of the air-condenser. Another method of finding the required correction by comparison of the results obtained on the overlapping portion of the ranges of two coils is also described.

The correction can be made small by suitably designing the coils.

DISCUSSION.

Dr. W. E. SUMPNER remarked that it could not be considered satisfactory to find the self-capacity of a coil from its free period, taking the whole of its self-induction as effective, until this had been shown to give the same result as that deduced experimentally with a large number of different coils.

Mr. W. DUDDELL remarked that Prof. Howe's experiment showed that for practical purposes the self-capacity of a coil could be considered quite constant, for he had used much smaller external capacities than usual, and still could detect no difference. It was very important for a wave-meter to have a very definite free period of its own, and therefore L/R must be large. Its coefficient of damping should be less than that of the antenna circuit, as the usual method of finding this latter really gives the sum of the two. It was thus more important to keep the high frequency resistance of a coil small than its self-capacity. Dielectric losses were an important item in the high-frequency resistance, hence the voltage from one turn to the next should be kept down.

Prof. T. MATHER pointed out, with reference to a table Prof. Howe had put on the board, giving dimensions of different coils having the same self-induction but different self-capacities, that the length of wire necessary was almost the same in each case.

Dr. W. H. ECCLES pointed out that theoretically the self-capacity could not be constant when the external capacity was small, and cited the somewhat analogous case of a weight vibrating from a heavy spring when one-third of the weight of the spring was added to the weight provided the weight was relatively large. There might be some compensating influence at work such as the variation of the self-induction with frequency.

Mr. A. CAMPBELL stated that the method of obtaining the working value of a wave-meter coil by finding the frequency giving resonance on open circuit was used by Dieselhorst some years ago. At the National Physical Laboratory it is customary to plot λ^2 against the condenser capacity; over the working range the line obtained is straight, and from its slope the high frequency value of the inductance of the coil can be immediately deduced by dividing by $4\pi^2$. It was interesting to learn that Prof. Howe found by experiment that such a line passes through the point given by Dieselhorst's method. The capacity of a coil could be diminished by winding it in slices like the secondary of an induction coil.

Dr. ERSKINE-MURRAY pointed out that at high frequencies the current would go straight up through the coil, and not round it, if it were wound as Mr. Campbell suggests.

The AUTHOR, in reply to Dr. Sumpner, stated that he had only experimented on three or four coils, but he had purposely used specially small added capacities in order to detect any variation of the self-capacity, so for practical purposes it could well be assumed to be constant.

XXXIX. *Application of Heaviside's Resistance Operators to the Theory of the Air-core Transformer and Coupled Circuits in General.* *By* W. H. Eccles, *D.Sc., A.R.C.Sc.*

Read May 31, 1912.

The purpose of this Paper is to lay down a simple scheme of calculation whereby the solutions of very many of the mathematical problems arising in the theory of low and high frequency transformers may be written down from reference to a few tabulated special integrals. The work constitutes a development of the examples given by Heaviside of the utility of his "expansion theorem" on page 127 and the following pages of his "Electromagnetic Theory," Volume II.

In ordinary electrical parlance the term transformer is used to indicate a pair of circuits which are so arranged that the magnetic flux due to a current in either circuit passes wholly or partially through the other. The differential equations for the currents and voltages in such a system possessing invariable electrical dimensions are linear with constant coefficients. The solution of these equations when the primary E.M.F. is harmonic may be found in every text-book dealing with the ordinary transformer. In the present discussion the term transformer will be enlarged to include every apparatus in which the current in one circuit of the pair acts upon, and is reacted upon, by the other circuit.

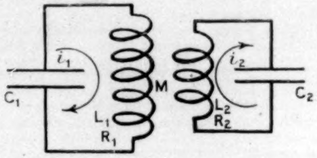

Fig. 1a.—Magnetic Coupling.

The linkage between the circuits is commonly called the "coupling" between them. When the coupling is effected without material conducting connection it is said to be "indirect," but when effected otherwise it is called "direct." Thus indirect coupling includes two very distinct varieties: magnetic coupling (Fig. 1a) and electric coupling (Fig. 1b); direct-coupling includes cases in which part or all of the

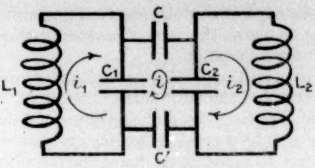
C
L_1
i_1
C_1
i
C_2
i_2
L_2
C'

FIG. 1B.—ELECTRIC COUPLING.

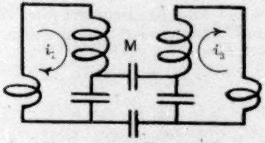

FIG. 1C.—MAGNETIC AND ELECTRIC COUPLING.

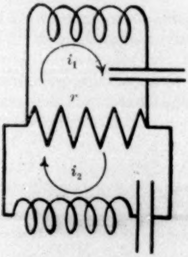

FIG. 2A.—RESISTANCE COUPLING.

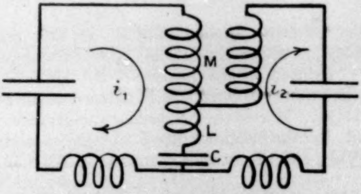

FIG. 2B.—MIXED COUPLING.

inductance, the capacity, or the resistance, is common to both circuits. Fig. 2A shows the case of resistance coupling. Figs. 1C, 2B show mixed coupling.

In order to avoid too early complication, two specially interesting examples, the two indirect couplings, will be dealt with separately before proceeding to the general case.

The reader may be reminded that the resistance operator of a condenser is $1/CD$, and of an inductance coil is LD. And since the symbol D, which stands for d/dt, obeys formally the fundamental laws of combination of algebraic symbols * the resistance operators may be used for the formation of the differential equations of an electrical problem just as if they were resistances.

Magnetic Coupling.

Let the circuit be as in Fig. 1A, and let L_1, R_1, C_1 refer to the primary, L_2, R_2, C_2 to the secondary. Let i_1, i_2 be the currents, e_1, e_2 the applied electromotive forces. The E.M.F. actually acting on the primary is e_1-MDi_2, on the secondary is e_2-MDi_1 when M is the mutual inductance between the circuits, and may be positive or negative according to the relative directions of the winding and the relative positions of the coils. Let the resistance operator of the primary, namely, $L_1D+R_1+(CD)^{-1}$ be called Ω_1, that of the secondary Ω_2. Then

$$i_1=\frac{e_1-MDi_2}{\Omega_1},\quad i_2=\frac{e_2-MDi_1}{\Omega_2}.$$

Therefore, on solving for i_1 and i_2,

$$i_1=\frac{e_1-(MD/\Omega_2)e_2}{\Omega_1-M^2D^2/\Omega_2},$$

$$i_2=\frac{e_2-(MD/\Omega_1)e_1}{\Omega_2-M^2D^2/\Omega_1}.$$

Thus the effective applied E.M.F. in the primary is $e_1-(MD/\Omega_2)e_2$; which shows that the secondary applied E.M.F. as "reduced to the primary" is $-(MD/\Omega_2)e_2$, and, similarly, the primary applied E.M.F. reduced to the secondary is $-(MD/\Omega_1)e_1$. Moreover, the primary resistance operator is changed by the neighbourhood of the secondary into $\Omega_1-M^2D^2/\Omega_2$, and the secondary into $\Omega_2-M^2D^2/\Omega_1$.

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* Boole's "Differential Equations," Chap. XVI.

Usually there is not any applied E.M.F. in the secondary; that is, $e_2=0$, and then

$$i_1=\frac{e_1/\Omega_1}{1-\mathrm{M}^2\mathrm{D}^2/\Omega_1\Omega_2}=\frac{i_{10}}{\Sigma},$$

where i_{10} is the "open circuit" current—that is, the current in the primary when the secondary circuit has no current in it or is removed—and Σ is an operator which may be called, from analogy with the well-known coefficient of magnetic leakage, the leakage operator. The term $\mathrm{M}^2\mathrm{D}^2/\Omega_1\Omega_2$ may then be called the coupling operator.

Further, $$i_2=-\frac{\mathrm{MD}/\Omega_1}{\Omega_2-\mathrm{M}^2\mathrm{D}^2/\Omega_1}e_1=-\frac{\mathrm{MD}}{\Omega_2}i_1$$

in the present case.

The equations $$i_1=\frac{1}{\Sigma}\cdot i_{10},\quad i_2=-\frac{\mathrm{MD}}{\Omega_2}i_1$$

thus comprise the whole theory of the relations between the currents—transient or alternating—in the ordinary transformer.

When sine wave alternating current of frequency $\omega/2\pi$ is being supplied to the primary, we integrate the equations by substituting $\iota\omega$ for D wherever it appears in the symbolic operators, where $\iota\equiv\sqrt{-1}$. And when the primary is traversed by a long-continued slowly-damped train of oscillatory current of the form $\epsilon^{-bt}\cos\omega t$, where $b=\omega/2\pi$ times the logarithmic decrement per period, the integration is effected by substituting $-b+\iota\omega$ for D in the operator. In this way we get

$$\Omega_1=\left(\mathrm{L}_1-\frac{1}{\mathrm{C}_1(\omega^2+b^2)}\right)\iota\omega+\mathrm{R}_1-b\mathrm{L}_1-\frac{b}{\mathrm{C}_1(\omega^2+b^2)},$$

$$=l_1\iota\omega+r_1,\text{ say},$$

and $\Omega_1\Omega_2=r_1r_2-l_1l_2\omega^2+(r_1l_2+r_2l_1)\iota\omega=\xi+\iota\eta$, where the symbols l_1, l_2, r_1, r_2 explain themselves.

Thus $$\frac{1}{\Sigma}=\frac{\xi+\iota\eta}{\xi+\mathrm{M}^2(\omega^2-b^2)+\iota(\eta+2\mathrm{M}^2\omega b)},$$

which is of the form $$\frac{\xi+\iota\eta}{\xi+\alpha+\iota(\eta+\beta)}.$$

An important theorem arises out of the elimination of R_2 between

$$\begin{aligned}\xi&=r_1r_2-l_1l_2\omega^2,\\ \eta&=(r_1l_2+r_2l_1)\omega.\end{aligned}$$

Here R_2 is contained only in r_2. Eliminating r_2, we get

$$l_1\xi-r_1\eta=-(r_1^2+l_1^2\omega^2)l_2,$$

which is linear in ξ and η. Now putting

$$x+\iota y=\frac{\xi+\iota\eta}{\xi+a+\iota(\eta+\beta)},$$

and, for generality, writing the linear relation as

$$\xi m+\eta n=1,$$

we have, separating the parts independent of ι from those including ι in the former equation,

$$\begin{aligned}&\xi(x-1)-\eta y+ax-\beta y&&=0,\\ &\xi y\quad+\eta(x-1)+\beta x+ay&&=0,\\ &\xi m\quad+\eta n\quad-1&&=0.\end{aligned}$$

$$\text{Hence}\quad\begin{vmatrix}x-1 & -y & ax-\beta y\\ y & x-1 & \beta x+ay\\ m & n & -1\end{vmatrix}=0,$$

$$\text{or,}\quad\left(x-\frac{am+\beta n+2}{2(am+\beta n+1)}\right)^2+\left(y-\frac{an-\beta m}{2(am+\beta n+1)}\right)^2,$$

$$=\frac{(a^2+\beta^2)(m^2+n^2)}{4(am+\beta n+1)^2}.$$

The interpretation of this result is that if the instantaneous values of the primary currents i_1 and i_{10}, when the secondary is in operation and when it is not, are represented by the vectors OP and OA, shown in Fig. 3, then the locus of P as R_2, the load on the secondary, is varied, is the circle whose equation is given above. The centre of the circle may have any position according to the values of a, β, m, n. The theorem, which is well known for the case of the transformer used with simple alternating current*, is thus established for the *quasi*-steady conditions of a slowly damped train of oscillations.

Moreover, it is clear from the equations for ξ and η that a linear relation beteeen ξ and η is obtained not only by the elimination of r_2, but also by the elimination of r_1, of l_1, or of l_2, separately. That is to say, a circle diagram similar to the above is available for expressing the relative magnitude and phases of i_1 and i_{10}, when r_1, l_1 or l_2 are separately varied.

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* Russell, "Alternating Currents," Vol. II., p. 229.

The equation, $$i_1=\frac{1}{\Sigma}i_{10},$$

may be written $$i_1-\frac{M^2D^2}{\Omega_1\Omega_2}i_1=i_{10},$$

and using $$i_2=-\frac{MD}{\Omega_2}i_1,$$

we have $$i_1+\frac{MD}{\Omega_1}i_2=i_{10}.$$

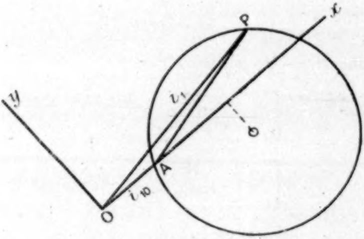

FIG. 3.

In the case of a damped oscillation the coefficient of i_2 becomes

$$\frac{M(\iota\omega-b)}{l_1\iota\omega+r_1},\text{ or }M\sqrt{\frac{\omega^2+b^2}{l_1{}^2\omega+r_1{}^2}}\,\epsilon^{\iota\left(\pi-\tan^{-1}\frac{\omega}{b}-\tan^{-1}\frac{l_1\omega}{r_1}\right)}.$$

Therefore, in Fig. 3, the length PA which completes the triangle POA, is $M\sqrt{\frac{\omega^2+b^2}{l_1{}^2\omega^2+r_1{}^2}}$ times the instantaneous magnitude of the secondary current, and is $\pi-\tan^{-1}\frac{\omega}{b}-\tan^{-1}\frac{l_1\omega}{r_1}$ ahead of that current in phase. This is an extension of a well-known result.

Let v_1, v_2 be the P.D.'s at the terminals of the primary and secondary condensers, so that

$$v_1=(C_1D)^{-1}i_1\text{ and }v_2=(C_2D)^{-1}i_2,$$

and let v_{10} be the primary P.D. when the current i_{10} is running so that $v_{10}=(C_1D)^{-1}\,i_{10}$.

Then, $$v_1=\frac{1}{\Sigma}\,v_{10},\quad v_2=-\frac{C_2}{C_1}\cdot\frac{MD}{\Omega_2}\,v_1,$$

and $$v_1+\frac{C_2}{C_1}\frac{MD}{\Omega_1}\,v_2=v_{10}.$$

That is to say, since v_1, v_2 and v_{10} are connected by equations similar to those connecting i_1, i_2 and i_{10}, the instantaneous relative values of the condenser P.D.'s may be represented by circle diagrams.

Returning to the equation

$$i_1=\frac{e_1}{\Omega_1-M^2D^2/\Omega_2},$$

and putting $D=-b+\iota\omega$, we get

$$\Omega_1-\frac{M^2D^2}{\Omega_2}=\left[l_1-\frac{M^2}{l_2{}^2\omega^2+r_2{}^2}\{l_2(\omega^2-b^2)-2r_2b\}\right]\iota\omega$$
$$+\left[r_1+\frac{M^2}{l_2{}^2\omega^2+r_2{}^2}\{\omega^2-b^2+2l_2b\omega^2\}\right],$$

say $$=l'_1\iota\omega+r'_1.$$

Thus, if $$e_1=E\epsilon^{-bt}\cos\omega t,$$

$$i_1=\frac{E}{\sqrt{l'_1{}^2\omega^2+r'_1{}^2}}\,\varepsilon^{-bt}\cos\left(\omega t-\tan^{-1}\frac{l'_1\omega}{r'_1}\right).$$

The effect of the secondary is therefore (unless b is very large) to increase the apparent resistance and decrease the apparent inductance of the primary. The above formulæ for these alterations are extensions of Maxwell's formulæ for the simple alternating-current transformer. The voltages at the condenser terminals may be similarly expressed, and the ratio v_2/v_1 is given by

$$\frac{v_2}{v_1}=M\frac{C_1}{C_2}\sqrt{\frac{\omega^2+b^2}{l_2{}^2\omega^2+r_2{}^2}}\cdot\varepsilon^{\iota\left(\pi-\tan^{-1}\frac{\omega}{b}-\tan^{-1}\frac{l_2\omega}{r_2}\right)}.$$

In the particular case of a resonance transformer with primary condenser, operating on a sine wave current from an alternator, we have $b=0$, and therefore,

$$\frac{v_2}{v_1}=\frac{C_1}{C_2}\cdot\frac{M\omega}{\sqrt{\left(L_2-\frac{1}{C_2\omega}\right)^2\omega^2+R_2{}^2}}\,\varepsilon^{\iota\left(\frac{\pi}{2}-\tan^{-1}\frac{l_2\omega}{r_2}\right)},$$

when the secondary is not tuned to the alternator. But when the transformer is tuned by varying L_2 or C_2 we have $l_2=0$, that is,

$$L_2=\frac{1}{C_2\omega^2}$$

and

$$\frac{v_2}{v_1}=\frac{C_1}{C_2}\cdot\frac{M\omega}{R_2}\cdot\varepsilon^{\iota\cdot\frac{\pi}{2}},$$

that is, the voltage ratio attained at the condensers in the steady state is a maximum and the secondary voltage is leading the primary by a right angle.

The ratio of secondary condenser P.D. to applied E.M.F. in the primary is in this case

$$\frac{v_2}{e_1}=-\frac{M}{C_2(\xi+\iota\eta)}.$$

For resonance $\xi=R_1R_2$, $\eta=R_2l_1\omega$; therefore

$$\frac{v_2}{e_1}=-\frac{M}{C_2R_2}\,\frac{1}{\sqrt{R_1{}^2+l_1{}^2\omega^2}}\cdot\varepsilon^{-\iota\tan^{-1}\frac{l_1\omega}{R_1}}$$

in the steady state.

Electric Coupling.

The connections for pure electric coupling are represented in Fig. 1B. This scheme of circuits is sufficiently general to include every case likely to arise in practice.

Taking the cyclic currents as marked and supposing E.M.F.s e_1, e_2 to be applied independently in the primary and secondary circuits, we obtain from the closed circuits taken in turn

$$(L_1D+R_1)\,i_1+\frac{1}{C_1D}(i_1-i)=e_1,$$

$$(L_2D+R_2)\,i_2+\frac{1}{C_2D}(i_2-i)=e_2,$$

$$\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{c}+\frac{1}{c'}\right)i-\frac{1}{C_1D}i_1-\frac{1}{C_2D}i_2=0.$$

Put
$$\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{c}+\frac{1}{c'}=\frac{1}{\gamma},$$

$$L_1D+R_1+\frac{1}{C_1D}\left(1-\frac{\gamma}{C_1}\right)=\Omega_1.$$

$$L_2D+R_2+\frac{1}{C_2D}\left(1-\frac{\gamma}{C_2}\right)=\Omega_2,$$

eliminate i and solve for i_1 and i_2. Then

$$i_1=\frac{\Omega_2e_1+\frac{\gamma}{C_1C_2D}e_2}{\Omega_1\Omega_2-\left(\frac{\gamma}{C_1C_2D}\right)^2}$$

$$i_2=\frac{\Omega_1e_2+\frac{\gamma}{C_1C_2D}e_1}{\Omega_1\Omega_2-\left(\frac{\gamma}{C_1C_2D}\right)^2}.$$

If, as is common, $e_2=0$,

$$i_1=\frac{e_1/\Omega_1}{1-\left(\frac{\gamma}{C_1C_2D}\right)^2/\Omega_1\Omega_2}=\frac{i_{10}}{\Sigma}, \text{ say };$$

and
$$i_2=\frac{\gamma/C_1C_2D}{\Omega_1\Omega_2\Sigma}e_1=\frac{\gamma/C_1C_2D}{\Omega_2}\,.\,i_1.$$

It will be seen that the equations connecting i_1, i_2, i_{10} differ from the magnetic coupling equations by the replacement of $-MD$ by γ/C_1C_2D. Thus when $D=-b+\iota\omega$ the term $-M(\iota\omega-b)$ in the former magnetic problem must be replaced by a term $-\frac{\gamma}{C_1C_2}\cdot\frac{\iota\omega+b}{\omega^2+b^2}$ in the electric problem ; or, if there is no damping, $M\iota\omega$ by $\frac{\gamma\iota}{C_1C_2\omega}$. Since the terms are of the same form the broad theory of the electric transformer is precisely the same, for damped or uniform alternating current, as that of the magnetic transformer.

Mixed Magnetic and Electric Coupling.

In order to examine this case we may suppose that in Fig. 1c the coils L_1, L_2 possess mutual inductance, and then rewrite the equations. Using the notation of the last paragraph we obtain

$$i_1=\frac{\Omega_2 e_1+(\gamma/C_1C_2D-MD)e_2}{\Omega_1\Omega_2-(\gamma/C_1C_2D-MD)^2}$$

in general, with a similar equation for i_2. In the particular case $e_2=0$, these equations become

$$i_1=\frac{i_{10}}{\Sigma},\qquad i_2=\frac{\gamma/C_1C_2D-MD}{\Omega_2}\,.\,i_1,$$

where i_{10} is the current in the primary when the secondary circuit is cut outside the coupling.

Whether the electric coupling acts with or against the magnetic coupling depends on the sign of M. But in any event the theory for the transformer with magnetic coupling and damped oscillations holds good for the transformer with electric and magnetic coupling.

Direct Coupling.

We turn now to apply the same methods to auto-transformers, in which there is complete conducting connection between the circuits. Since there occur practical cases where indirect magnetic coupling is utilised along with direct coupling, this rather wider case will be treated at once to save space. Let the circuits be as defined by Fig. 2B. Let L_1, R_1, C_1 be the resultant inductance, resistance and capacity of the primary circuit when the secondary is cut, and L_2, R_2, C_2 relate to the secondary. Let L, R, C be the portions of the above quantities that are common to the circuits. Thus, from the diagram,

$$\left(L_1D+R_1+\frac{1}{C_1D}\right)i_1-\left(\overline{L-M}\,.\,D+R+\frac{1}{CD}\right)i_2=e_1,$$

$$\left(L_2D+R_2+\frac{1}{C_2D}\right)i_2-\left(\overline{L-M}\,.\,D+R+\frac{1}{CD}\right)i_1=e_2.$$

Write these as

$$\Omega_1 i_1-\Omega i_2=e_1,$$
$$\Omega_2 i_2-\Omega i_1=e_2,$$

where Ω, the coupling operator, refers to the portion common to the two circuits, and Ω_1, Ω_2 are the operators of the separate circuits when the other is cut outside the coupling.

Solving for i_1, i_2,

$$i_1=(\Omega_2 e_1+\Omega e_2)/(\Omega_1\Omega_2-\Omega^2),$$
$$i_2=(\Omega e_1+\Omega_1 e_2)/(\Omega_1\Omega_2-\Omega^2).$$

If, as is common, $e_2=0$, these results become

$$i_1=e_1/\Omega_1\Sigma, \quad i_2=(\Omega_1/\Omega_2)\, i_1,$$

where $$\Sigma\equiv 1-\Omega^2/\Omega_1\Omega_2.$$

Thus we again find that when $D=-b+\iota\omega$, the theory of the case of simple magnetic coupling applies also to this more general case.

As a rule there is no coupling condenser C, and there is insignificant coupling resistance R in a practical transformer, and in this event $\Omega=(L-M)D$. If the windings are side by side and in the same sense M is positive and opposes L.

It is often convenient to work problems involving mutual inductance by imagining the M replaced by equivalent common self-inductance, and using the familiar formula for currents in branched circuits to get the operational equation.

Normal Vibrations.

When the applied E.M.F. is transient or sudden a system possessing two degrees of freedom is, in general, started into a vibration compounded of its two normal modes. These normal modes can be found in any of the problems on hand, by factorising the denominator of the fractional operator in the equation for i_1 or i_2, after arranging in powers of D. Thus in the equation

$$i_1=(\Omega_2 e_1+\Omega_1 e_2)/(\Omega_1\Omega_2-\Omega^2)$$

the denominator multiplied by D^2 is

$$\begin{aligned}&\{L_1L_2-(L-M)^2\}D^4+\{L_1R_2+L_2R_1-2(L-M)R\}D^3,\\&\quad+\{L_1/C_2+L_2/C_1-2(L-M)/C+R_1R_2-R^2\}D^2\\&\quad+\{R_1/C_2+R_2/C_1-2R/C\}D\\&\quad+\{1/C_1C_2-1/C^2\}=0.\end{aligned}$$

Now we may take it as well known that a single circuit L_1, R_1, C_1 if started into oscillation, vibrates with "angular velocity" $p_1=\sqrt{\dfrac{1}{L_1C_1}-\dfrac{R_1^2}{4L_1^2}}$, and with damping ε^{-b_1t}, where $b_1=R_1/2L_1$. Thus $2b_1=R_1/L_1$, $1/L_1C_1=p_1^2+b_1^2=g$, say.

Using this more concise and appropriate system of symbols, we obtain after division by L_1L_2,

$$(1-k^2)D^4+2\left(b_1+b_2-\frac{(L-M)R}{L_1L_2}\right)D^3$$

$$+\left\{g_1+g_2+4b_1b_2-\frac{2(L-M)/C+R^2}{L_1L_2}\right\}D^2$$

$$+2\left\{b_1g_2+b_2g_1-\frac{R}{L_1L_2C}\right\}D$$

$$+g_1g_2-1/L_1L_2C^2=0,$$

where $k=\sqrt{(L-M)^2/L_1L_2}$ is the coefficient of inductance coupling, $\sqrt{R^2/L_1L_2}$ is the coefficient of resistance coupling, and $\sqrt{1/L_1L_2C^2}$ is the coefficient of condenser coupling. $1-k^2$ is often written σ.

The equation being of the fourth degree must have two quadratic factors, and therefore, in the oscillatory case, two normal periods. The coefficients, which are functions of the electrical constants of the circuits, are not connected by any mathematical relations, and the biquadratic is therefore general. Its accurate solution must be approached through a reducing cubic.

The biquadratic

$$\sigma D^4+a_1D^3+a_2D^2+a_3D+a_4=0,$$

is equivalent to

$$\sigma(D^2+2BD+G)(D^2+2B'D+G')=0,$$

if $B=(a_1-\sqrt{x_3})/\sigma,\ B'=(a_1+\sqrt{x_3})\sigma,$

$$G=\{(a_1-\sqrt{x_3})^2-(\sqrt{x_1}+\sqrt{x_2})^2\}/\sigma^2,$$

and $G_1'=\{(a_1+\sqrt{x_3})^2-(\sqrt{x_1}-\sqrt{x_2})^2\}/\sigma^2,$

where x_1, x_2, x_3 are the roots of Euler's reducing cubic

$$x^3+3Hx^2+(3H^2-\tfrac{1}{4}\sigma^2I)-\tfrac{1}{4}G_0{}^2=0,$$

and where $H=\sigma a_2-a_1{}^2,\ I=\sigma a_4-4a_1a_3+3a_2{}^2,$

$$G_0=\sigma^2a_3-3\sigma a_1a_2+2a_1{}^3.$$

It will be found that this method of solving leads to grea arithmetical labour. Various approximations have been devised therefore, by Wien[*] and by Drude[†] and others, appropriate to the numerical values likely to arise in practice. The following method of approximation is easy :—

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[*] Wien, "Annalen d. Phy. and Chimie." Band 61, p. 151.

[†] Drude, "Annalen d. Phy." Band 13, p. 512.

Write the equation

$$\sigma D^4+2(b_1+b_2-k_1^2)D^3+(g_1+g_2+4b_1b_2-k_2^2)D^2,$$
$$+2(b_1g_2+b_2g_1-k_3^2)D+g_1g_2-k_4^2=0.$$

If this is equivalent to $(D^2+2BD+G)(D^2+2B'D+G')=0$, we have, by equating coefficients of powers of D,

$$\sigma(B+B')=b_1+b_2-k_1^2,$$
$$\sigma(G+G'+4BB')=g_1+g_2+4b_1b_2-k_2^2,$$
$$\sigma(BG'+B'G)=b_1g_2+b_2g_1-k_3^2,$$
$$\sigma G_1G'=g_1g_2-k_4^2.$$

Now b_1, b_2, B, B′ are in fact usually small compared with g_1, g_2, G, G′. Thus, the second and fourth equations may be taken, in a first approximation, as

$$\sigma(G+G')=g_1+g_2-k_2^2,$$
$$\sigma GG'=g_1g_2-k_4^2.$$

Hence $$G, G'=\frac{1}{2\sigma}(g_1+g_2-k_2^2\pm s),$$

where $$s=\{(g_1+g_2-k_2^2)^2-4\sigma(g_1g_2-k_4^2)\}^{\frac{1}{2}}.$$

The first and third equations now give

$$B, B'=\{(G, G')(b_1+b_2-k_1^2)-(b_1g_2+b_2g_1-k_3^2)\}/\pm s$$
$$=\frac{1}{2\sigma}\Big[b_1+b_2-k_1^2+\frac{1}{\pm s}\{(g_1+g_2-k_2^2)(b_1+b_2-k_1^2)$$
$$-2\sigma(b_1g_2+b_2g_1-k_3^2)\}\Big].$$

In arithmetical work a second approximation may be proceeded to by using these values of B, B′ in the second and fourth equations and solving again for G, G′.

When the frequencies fulfil the condition $g_1=g_2=g$, the so-called resonance condition,

$$s=2\sqrt{k^2g^2-k_2^2g+\sigma k_4^2},$$

and therefore

$$G, G'=(g-\tfrac{1}{2}k_2^2\pm\sqrt{k^2g^2-k_2^2g+\sigma k_4^2})/\sigma,$$

and

$$B, B'=\frac{1}{2\sigma}\left[b_1+b_2-k_1^2+\frac{(g-\frac{1}{2}k_2^2)(b_1+b_2-k_1^2)-\sigma(\overline{b_1+b_2}\,.\,g-k_3^2)}{\pm\sqrt{k^2g^2-k_2^2g+\sigma k_4^2}}\right].$$

It is unusual to meet a problem in which all the *k*'s appear.

The most frequent case is that in which only k^2 appears. Then for any but extreme values of k^2, we have approximately, in the case of resonance,

$$s=2kg,$$

and therefore $G=g/(1-k), \quad G'=g/(1+k),$

and $B=\frac{1}{2}(b_1+b_2)/(1-k), \quad B'=\frac{1}{2}(b_1+b_2)/(1+k)$

as first approximations.

For the second approximations for G and G′ we proceed :

$$4b_1b_2-4BB'=-(b_1-b_2)^2\,;$$

inserting in the G equations and solving as before

$$G,\ G'=\{2g-(b_1-b_2)^2\pm s'\}/2\sigma,$$

where $s'=\{(2g-\overline{b_1-b_2}^2)^2-4\sigma g^2\}^{\frac{1}{2}}.$

When a circuit containing inductance, resistance and capacity is coupled to one without capacity or without inductance, a cubic equation has to be solved. The approximate formulæ for yielding the factors $D+B'$, $D^2+2BD+G$, are

$$G=(g_1-k^2b_2^2)/\sigma,$$
$$B'=b_2+k^2b_2^3/g_1,$$
$$B=\tfrac{1}{2}(k^2b_2+2b_1)/\sigma-\tfrac{1}{2}k^2b_2^3/g_1.$$

Currents and Voltages produced by Sudden Changes of E.M.F. or by Electromotive Impulses.

Under this heading come most of the important problems of wireless telegraphy—in particular, those problems arising out of the breaking down of the spark-gap in the primary of a sending transformer.

The fundamental type of sudden change of E.M.F. is that in which the force is zero during all negative time and is unity during all positive time, as happens when a cell of one volt is switched into a circuit at the time $t=0$. This function of t, which might, of course, be expressed as a Fourier integral, we may call Υ. Fig. 4A is its graph. On the other hand, $1-\Upsilon$ represents the sudden removal of a unit E.M.F. from a circuit, and has the graph 4B.

It is necessary to examine the effect of various symbolic operators on this function. First $D\Upsilon$ is plainly an impulsive function which is zero during all negative time, infinite and positive at $t=0$, and zero during all positive time. From physical considerations, as well as from more rigorous considerations, it may be shown that $D\Upsilon$ is an infinity of lower order than $\underset{t\to 0}{Lt}\ t^{-1}$. In all our work we have to take $\underset{t\to 0}{Lt}\ tD\Upsilon=0$, with the consequence that, therefore, $t\,.\,D\Upsilon$ is zero for all values of t. With this understanding

$$D\Upsilon t = t\,.\,D\Upsilon + \Upsilon\,.\,Dt = \Upsilon,$$

$$D\Upsilon(1-\varepsilon^{-bt}) = (1-\varepsilon^{-bt})D\Upsilon + b\varepsilon^{-bt}\Upsilon = b\,\varepsilon^{-bt}\Upsilon,$$

and so on.

Again $\qquad D^{-1}\Upsilon = \Upsilon t$, and $D^{-1}\Upsilon t = \Upsilon t^2/2$,

as is obvious on considering the areas of the graphs, and, generally $\qquad D^{-n}\Upsilon = \Upsilon t^n/\underline{|n}$.

From above $D(1-\varepsilon^{-bt})\Upsilon = b\varepsilon^{-bt}\Upsilon$; add $b(1-\varepsilon^{-bt})\Upsilon$ to both sides then

$$(D+b)(1-\varepsilon^{-bt})\Upsilon = b\Upsilon,$$

or

$$(D+b)^{-1}\Upsilon = \Upsilon(1-\varepsilon^{-bt})/b.$$

This important formula may also be obtained by expanding $(D+b)^{-1}$ in inverse powers of D, or by solving the corresponding differential equation under the conditions here implied by Υ.

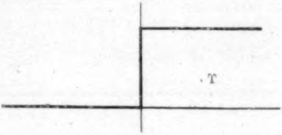


FIG. 4A.

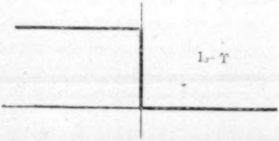


FIG. 4B.

By differentiating with respect to b we get

$$\{D/(D+b)^{-2}\}\,\Upsilon = t\Upsilon\varepsilon^{-bt}.$$

To find $(D+b)^{-1}\Upsilon\varepsilon^{-at}$, note that the exponential function can be passed to the left of the operator if $D-a$ be written for D throughout the operator (Boole, *loc. cit.*).

Thus $(D+b)^{-1}\Upsilon\varepsilon^{-at} = \varepsilon^{-at}(D-a+b)^{-1}\Upsilon = \Upsilon(\varepsilon^{-at}-\varepsilon^{-bt})/(b-a)$.

By putting $a=b'-\iota\omega$, separating real and imaginary parts, we get the results for $(\mathrm{D}+b)\Upsilon\,\varepsilon^{-b't}\cos\omega t$ and $(\mathrm{D}+b)\Upsilon\varepsilon^{-b't}\sin\omega t$, as tabulated later.

Operators of the form $(\mathrm{D}+b)^{-1}$ arise from circuits containing inductance and resistance. In circuits containing inductance, resistance and capacity the denominator of the operator is of second degree in D when cleared of fractions. To provide for this case fractional operators of the form

$$(\mathrm{AD}+\mathrm{B})/(\mathrm{D}^2+2b\mathrm{D}+g)$$

must be examined. This may be split into two partial fractions of forms $\mathrm{A}'/(\mathrm{D}+a)$, $\mathrm{B}'/(\mathrm{D}+\beta)$, a and β being in general complex. The integrals of these latter operators have already been found, and, therefore, the integral corresponding to the general quadratic fraction can be written down. They are given in the tables on the next page.

In the general case of the transformers discussed in this Paper, fractional operators with quartic denominators arise. These fractions may be split into partial fractions which are integrable separately; in all practical cases these are two irreducible quadratic fractions. In such cases the following formulæ are useful:

Let

$$\frac{\alpha\mathrm{D}^3+\beta\mathrm{D}^2+\gamma\mathrm{D}+\delta}{(\mathrm{D}^2+2\mathrm{BD}+\mathrm{G})(\mathrm{D}^2+2\mathrm{B}'\mathrm{D}+\mathrm{G}')}=\frac{\mu\mathrm{D}+\nu}{\mathrm{D}^2+2\mathrm{BD}+\mathrm{G}}+\frac{\mu'\mathrm{D}+\nu'}{\mathrm{D}^2+2\mathrm{B}'\mathrm{D}+\mathrm{G}'}.$$

Then

$$\mu=\frac{\alpha\{4\mathrm{B}(\mathrm{B}'\mathrm{G}-\mathrm{BG}')-\mathrm{G}(\mathrm{G}-\mathrm{G}')\}-2\beta(\mathrm{B}'\mathrm{G}-\mathrm{BG}')+\gamma(\mathrm{G}-\mathrm{G}')-2\delta(\mathrm{B}-\mathrm{B}')}{4(\mathrm{B}-\mathrm{B}')(\mathrm{B}'\mathrm{G}-\mathrm{BG}')-(\mathrm{G}-\mathrm{G}')^2},$$

and

$$\nu=\frac{\alpha\,.\,2\mathrm{G}(\mathrm{B}'\mathrm{G}-\mathrm{BG}')-\beta\,.\,\mathrm{G}(\mathrm{G}-\mathrm{G}')+\gamma\,.\,2\mathrm{G}(\mathrm{B}-\mathrm{B}')-\delta\{4\mathrm{B}(\mathrm{B}-\mathrm{B}')-(\mathrm{G}-\mathrm{G}')\}}{4(\mathrm{B}-\mathrm{B}')(\mathrm{B}'\mathrm{G}-\mathrm{BG}')-(\mathrm{G}-\mathrm{G}')^2},$$

and μ', ν' are obtained by interchanging B and B′, G and G′ in the formulæ for μ and ν.

Approximately, under resonance conditions with inductive coupling,

$$\mu=\frac{1-k^2}{2gk}\left\{\frac{ag}{1-k}-\gamma+\delta\,.\,\frac{b_1+b_2}{g}\right\},$$

$$\nu=\frac{1+k}{2gk}\left[\beta g-\gamma(b_1+b_2)+\delta\left\{\frac{(b_1+b_2)^2}{g}+k-1\right\}\right],$$

and μ', ν' are obtained from μ and ν by changing the sign of k.

TABLES OF INTEGRALS.

Operator $(D+b)^{-1}$

Acting on	Integrals.
Υ	$\Upsilon\frac{1-\epsilon^{-bt}}{b}$
$\Upsilon\epsilon^{-mt}$	$\Upsilon\frac{\epsilon^{-mt}-\epsilon^{-bt}}{b-m}$
$\Upsilon\epsilon^{-bt}$	$\Upsilon t\epsilon^{-bt}$
$\Upsilon\epsilon^{-mt}\begin{matrix}\cos\\ \sin\end{matrix}(\omega t+\phi)$	$\Upsilon\frac{1}{\sqrt{\omega^2+(b-m)^2}}\left\{\epsilon^{-mt}\begin{matrix}\cos\\ \sin\end{matrix}(\omega t+\phi-\theta)-\epsilon^{-bt}\begin{matrix}\cos\\ \sin\end{matrix}(\phi-\theta)\right\}$ where $\tan\theta=\omega/(b-m)$

NOTE.—$D\,\epsilon^{-bt}\begin{matrix}\cos\\ \sin\end{matrix}(\chi t+\theta)$

$$=-\sqrt{p^2+b^2}\,.\,\epsilon^{-bt}\begin{matrix}\cos\\ \sin\end{matrix}(pt+\theta-\tan^{-1}p/m)$$

and $D^{-1}.\,\epsilon^{-bt}\begin{matrix}\cos\\ \sin\end{matrix}(pt+\theta)$

$$=-\frac{\epsilon^{-bt}}{\sqrt{p^2+b^2}}\begin{matrix}\cos\\ \sin\end{matrix}(pt+\theta+\tan^{-1}p/m)+\text{const.}$$

Operator $(\overline{D+b}|^2+p^2)^{-1}$

Acting on	Integrals.
Υ	$\frac{\Upsilon}{\theta_0}\left\{\frac{1}{\theta_0}-\frac{\epsilon^{-bt}}{p}\sin(pt+\theta)\right\}$ $\theta_0{}^2=b^2+p^2$, $\tan\theta=p/b$
$\Upsilon\epsilon^{-mt}$	$\frac{\Upsilon}{\theta_0}\left\{\frac{\epsilon^{-mt}}{\theta_0}-\frac{\epsilon^{-bt}}{p}\sin(pt+\theta)\right\}$ $\theta_0{}^2=(b-m)^2+p^2$, $\tan\theta=p/(b-m)$
$\epsilon^{mt}\cos(\omega t+\phi)$	$\frac{\Upsilon}{\theta_0}\left\{\epsilon^{mt}\cos(\omega t+\phi-\theta)-\frac{\sqrt{\omega^2+\theta_0\cos\phi\cos\overline{\phi-\theta}}}{p}\epsilon^{-bt}\cos(pt+\psi)\right\}$
$\Upsilon\epsilon^{-mt}\sin(\omega t+\phi)$	$\frac{\Upsilon}{\theta_0}\left\{\epsilon^{-mt}\sin(\omega t+\phi-\theta)-\frac{\sqrt{\omega^2+\theta_0\sin\phi\sin\overline{\phi-\theta}}}{p}\epsilon^{-bt}\cos(pt-\psi')\right\}$ $\theta_0{}^2=(\overline{b-m}^2+p^2-\omega^2)+4(b-m)^2\omega^2$ $\tan\theta=2(b-m)\omega/(\overline{b-m}^2+p^2-\omega^2)$ $\tan\psi=(\omega\tan\overline{\phi-\theta}-b+m)/p$ $\tan\psi'=(\omega\cot\overline{\phi-\theta}+b-m)/p$
$\Upsilon\epsilon^{-bt}\cos(pt+\phi)$	$\frac{\Upsilon}{2p^2}\epsilon^{-bt}\{pt\sin(pt+\phi)-\sin\phi\sin pt\}$
$\Upsilon\epsilon^{-bt}\sin(pt+\phi)$	$\frac{\Upsilon}{2p^2}\epsilon^{-bt}\{-pt\cos(pt+\phi)+\cos\phi\sin pt\}$

When the fractional operator reduces to third degree in the denominator we put

$$\frac{\alpha D^2+\beta D+\gamma}{(D+B')(D^2+2BD+G)}=\frac{A}{D+B'}+\frac{\mu D+\nu}{D^2+2BD+G},$$

where $$A=\frac{\alpha B'^2-\beta B'+\gamma}{B'^2-2BB'+G},$$

and $\mu=\alpha-A$, $\nu=(\gamma-GA)/B'$.

Applications.

As a first illustration take two circuits L_1, R_1 and L_2, R_2 without condensers, and suppose a cell switched into the primary at $t=0$. We have

$$i_1=(\Sigma\Omega_1)^{-1}\Upsilon,\quad i_2=-MD\Omega_2^{-1}\Upsilon,$$

per volt.

Then, putting $R_1/L_1=b_1$, $R_2/L_2=b_2$,

$$i_1=\frac{1}{L_1}\frac{D+b_2}{\sigma D^2+(b_1+b_2)D+b_1b_2}\cdot\Upsilon.$$

The quadratic denominator, if equated to zero, must have real roots, α, β say. Divide the operator into partial fractions:

$$i_1=\frac{1}{L_1\sigma(\alpha-\beta)}\left(\frac{\alpha-b_2}{D+\alpha}-\frac{\beta-b_2}{D+\beta}\right)\Upsilon.$$

This equation indicates, by the way, that the transformer is equivalent to two circuits in parallel. This may be at once integrated by the tables, and then reduces to

$$i_1=\frac{\Upsilon}{L_1b_1b_2}\left\{b_2+\frac{\beta(\alpha-b_2)\,\varepsilon^{-\alpha t}-\alpha(\beta-b_2)\epsilon^{-\beta t}}{\alpha-\beta}\right\}.$$

The Υ is retained in the solution merely to indicate that the solution applies to values of t after $t=0$, not to preceding time. The deduction of the secondary current will be left to the reader.

As a further illustration, the particular case of the high-frequency circuits of an experimental wireless telegraph station will now be worked out. The primary and secondary currents are to be produced by charging the primary condenser negatively till the spark-gap in the circuit breaks down. The measured constants of the circuits were kindly supplied to me by Mr. H. J. Round (by permission of the Marconi Company).

They were, for the primary $L_1=6{\cdot}0\times 10^{-6}$ henry, $C_1=10^{-8}$ farad, and $b_1=0{\cdot}98\times 10^5$; for the secondary $L_2=6{\cdot}0\times 10^{-5}$ henry, $C_2=10^{-9}$ farad and $b_2=3{\cdot}27\times 10^5$. Hence the angular velocity of both circuits was $p=4{\cdot}08\times 10^6$. The currents are given by

$$i_1=\frac{L_2}{L_1L_2-M^2}\,\frac{D^3+2b_2D^2+gD}{D^4+\frac{2}{\sigma}(b_1+b_2)D^3+\frac{2}{\sigma}(g+2b_1b_2)D^2+\frac{2g}{\sigma}(b_1+b_2)D+\frac{g^2}{\sigma}}(\Upsilon-1),$$

$$i_2=\frac{-M}{L_1L_2-M^2}\cdot\frac{D^3}{\text{same denominator}}(\Upsilon-1)$$

per volt at the spark-gap.

Let the operator of i_1 be resolved into

$$\frac{AD+F}{D^2+2BD+G}+\frac{A'D+F'}{D^2+2B'D+G'},$$

and that of i_2 into fractions with $aD+f$ and $a'D+f'$ as numerators.

Then

$$A=\frac{4(B-b_2)(BG'-B'G)+(G-g)(G-G')}{4(B-B')(BG'-B'G)+(G-G')^2},\qquad A'=1-A,$$

and
$$F=2\frac{A(B-B')+b_2-B}{1-G'/G},\qquad F'=-FG'/G.$$

The values of a, a', f, f', may be obtained by putting $b_2=g=0$ in the last equations.

Using the approximations

$$G=g/(1-k),\ G'=g/(1+k),\ B=\tfrac{1}{2}(b_1+b'_2)/(1-k)$$
$$B'=\tfrac{1}{2}(b_1+b_2)/(1+k),$$

we have approximately,

$$A=\tfrac{1}{2}(1+k),\ A'=\tfrac{1}{2}(1-k),\ a=(1+k)/2k,\ a'=-(1-k)/2k$$
$$F=\tfrac{1}{2}(b_2-b_1)(1+k)/k,\ F'=\tfrac{1}{2}(b_1-b_2)(1-k)/k,\ f=0,\ f'=0.$$

If the P.D. at the spark-gap when it breaks down is $-V_1$ then $e_1=-(1-\Upsilon)V_1$, or ΥV_1-V_1. The part due to ΥV_1 may be written down at once from the tables, and that due to $-V_1$ is got by putting $D=0$ in the operators.

There is no need to write these down here; nor need the voltages at the condenser terminals be dealt with further than to state that

$$v_1=(C_1D)^{-1}i_1,\quad v_2=(C_1D)^{-1}i_2.$$

The curves showing primary and secondary currents and voltages are given in Figs. 6 to 8. In Fig. 5 is seen the method of building up the secondary voltage curve of the 30 per cent. coupling case. The damped sine curves seen here are integrals arising from the quadratic partial fractions of the biquadratic operator in v_2.

As a final example we may take the problem of the currents produced in a magnetically coupled tuned receiving set, when an impulsive E.M.F., such as that due to a natural electric discharge or "stray," acts on the antenna. The stray may be represented by DΥ. Then

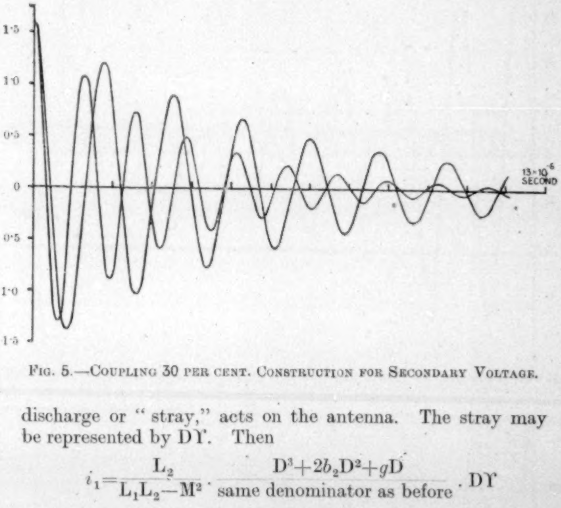


FIG. 5.—COUPLING 30 PER CENT. CONSTRUCTION FOR SECONDARY VOLTAGE.

$$i_1=\frac{\mathrm{L}_2}{\mathrm{L}_1\mathrm{L}_2-\mathrm{M}^2}\cdot\frac{\mathrm{D}^3+2b_2\mathrm{D}^2+g\mathrm{D}}{\text{same denominator as before}}\cdot\mathrm{D}\Upsilon$$

$$=\frac{\mathrm{L}_2}{\mathrm{L}_1\mathrm{L}_2-\mathrm{M}^2}\left[1+\left\{2\left(b_2-\frac{b_1+b_2}{\sigma}\right)\mathrm{D}^3+\left(g-\frac{2}{\sigma}\cdot\overline{g+2b_1b_2}\right)\mathrm{D}^2\right.\right.$$

$$\left.\left.-\frac{2g}{\sigma}(b_1+b_2)\mathrm{D}-\frac{g^2}{\sigma}\right\}\div(\text{same denominator})\right]\Upsilon.$$

The fraction splits into two partial fractions,

$$\frac{\mathrm{AD}+\mathrm{F}}{\mathrm{D}^2+2\mathrm{BD}+\mathrm{G}}+\frac{\mathrm{A}'\mathrm{D}+\mathrm{F}'}{\mathrm{D}^2+2\mathrm{B}'\mathrm{D}+\mathrm{G}'},$$

where, approximately,

$$A=\frac{1+k}{2k}\left\{2b_2-\frac{b_1+b_2}{1-k}\right\},$$

$$F=\tfrac{1}{2}\left(g+\frac{(b_1-b_2)^2}{k(1-k)}\right),$$

and A′, F′ are got from A and F by changing the sign of k.

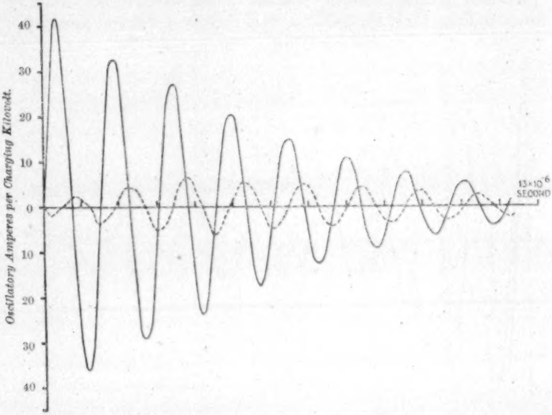


FIG. 6A.—PRIMARY AND SECONDARY (BROKEN LINE) CURRENTS, COUPLING 5 PER CENT. SECONDARY AMPERES MULTIPLIED BY 2·5.

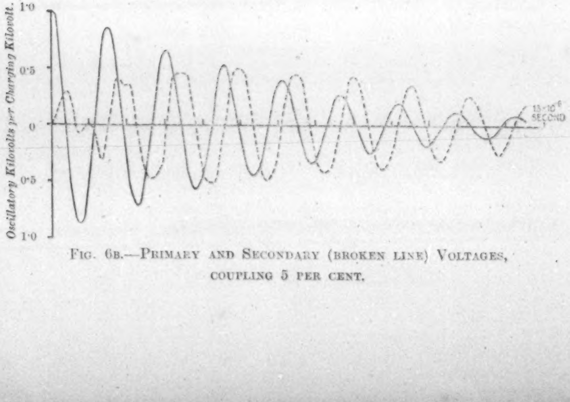


FIG. 6B.—PRIMARY AND SECONDARY (BROKEN LINE) VOLTAGES, COUPLING 5 PER CENT.

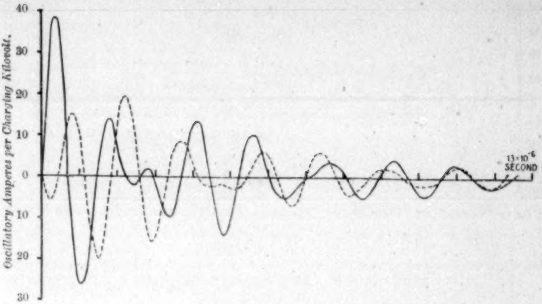


FIG. 7A.—PRIMARY AND SECONDARY (BROKEN LINE) CURRENTS, COUPLING 30 PER CENT. SECONDARY AMPERES MULTIPLIED BY 2·5.

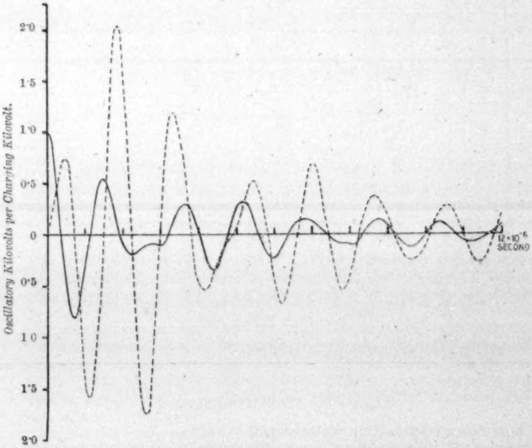


FIG. 7B.—PRIMARY AND SECONDARY (BROKEN LINE) VOLTAGES, COUPLING 30 PER CENT.

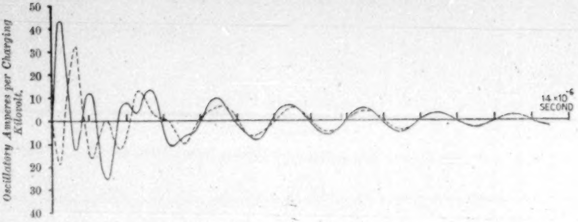


FIG. 8A.—PRIMARY AND SECONDARY (BROKEN LINE) CURRENTS, COUPLING 70 PER CENT

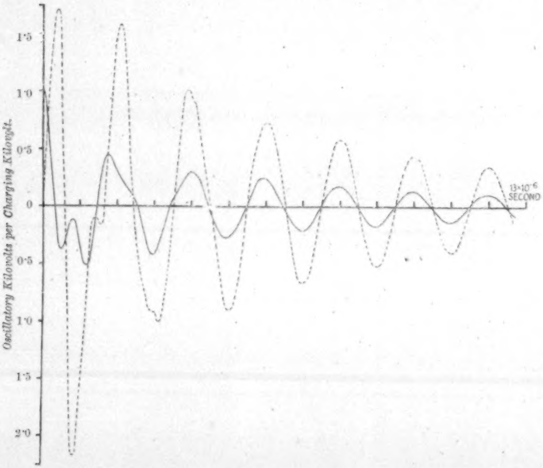


FIG. 8B.—PRIMARY AND SECONDARY (BROKEN LINE) VOLTAGES, COUPLING 70 PER CENT.

The secondary current is given by

$$i_2=\frac{-M}{L_1L_2-M^2}\;\frac{D^3}{\text{same denominator}}\,.\,D\Upsilon,$$

and its corresponding partial fractions are

$$\frac{aD+f}{D^2+2BD+G}+\frac{a'D+f'}{D^2+2B'D+G'},$$

where $$a=\frac{1+k}{2k(1-k)}(b_1+b_2),$$

$$b=\frac{1}{2k(1-k)}\cdot\{(1+k)g-(b_1-b_2)^2\},$$

and a', b' are obtained by changing the sign of k.

Thus the primary current is

$$i_1=\frac{L_2\Upsilon}{L_1L_2-M^2}\left[1+\frac{1}{\theta_0}\left\{\frac{F}{\theta_0}+\frac{\varepsilon^{-Bt}}{P}\cdot\varphi_0\sin(Pt+\theta-\varphi)\right\},\right.$$
$$\left.+\frac{1}{\theta'_0}\left\{\frac{F'}{\theta'_0}+\frac{\varepsilon^{B't}}{P'}\cdot\varphi'_0\sin(P't+\theta'-\varphi')\right\}\right],$$

where $$\theta_0{}^2=B^2+P^2=G,\quad \tan\theta=P/B,$$

$$\varphi_0{}^2=A^2G-2AB+F^2,\ \tan\varphi=\frac{AP}{AB-FP},$$

and similar definitions hold for θ'_0, etc. ; while

$$i_2=\frac{-M\Upsilon}{L_1L_2-M^2}\left[1-\frac{1}{\theta_0}\left\{\frac{f}{\theta_0}+\frac{\varepsilon^{-Bt}}{P}\cdot\psi_0\sin(Pt+\theta-\psi)\right\}\right.$$
$$\left.-\frac{1}{\theta'_0}\left\{\frac{f'}{\theta'_0}+\frac{\varepsilon^{-B't}}{P'}\cdot\psi'_0\sin(P't+\theta'-\psi')\right\}\right],$$

where $\psi_0{}^2=a^2G-2aB+f^2,\quad \tan\psi=\frac{aP}{aB-fP}$, etc.

The author desires to thank his colleague, Mr. J. Lister, for kind help in checking many of the formulæ given in this Paper.

ABSTRACT.

In circuits possessing constant inductance, resistance and capacity the differential equations for the currents and the voltages are linear with constant co-efficients, and may therefore be solved by aid of the known simple properties of symbolic operators. The symbolic operator method proves to be very compendious in problems concerning the determination of the primary and secondary currents and voltages of transformers whenever the applied E.M.F. can be expressed as an exponential function of the time, or when it consists of a sudden application of a constant or an exponential function, and also when it is impulsive.

In the Paper the method is first applied to a pair of "indirectly" coupled circuits—*i.e.*, circuits that are insulated from each other. The cases of pure magnetic coupling, pure electric coupling and of mixed magnetic and electric coupling are treated in turn and shown

to be operationally all of the same form. Then general operational equations are obtained for direct coupling and for mixed direct and indirect coupling—this latter, of course, includes all types of auto-transformers, both high and low frequency. Here again the operational equations are of the same form as in the case of magnetic coupling. The equations readily show that if the primary applied voltage be a pure sine or cosine alternating E.M.F. the well-known circle diagram for determining the instantaneous magnitudes and phases of the primary and secondary currents in an ordinary transformer is applicable to the most general type of transformer, and is capable of certain extensions. It is then shown that a similar circle diagram can be used for obtaining the primary and secondary currents in any general transformer under the application of the quasi-steady conditions of a slowly-damped alternating voltage. A further new theorem arises here, which affirms that a similar graphical method of computation can be used for deducing the voltages at the terminals of the primary and secondary condensers in a high-frequency transformer under steady or quasi-steady alternating current.

In the second section of the Paper it is shown how to calculate accurately the free periods and the damping factors of a coupled system with two degrees of freedom, and approximate formulæ are also deduced. The third section passes to the investigation of the currents and voltages produced in the circuits by sudden application or removal of a constant, a harmonic, or a damped harmonic E.M.F., and of those produced by the application of electromotive impulses. For this purpose formulæ expressing the effect of the various types of symbolic operator occuring in the Paper, on impulsive E.M.F.s and on "sudden applications," are deduced and tabulated. Using these tables of integrals it is very easy to write down quickly the solutions appropriate to any given transformer under the action of any of the types of E.M.F. specified. These tables include all the cases likely to arise in wireless telegraphy and in ordinary transformers.

As an example of the application of the methods the problem of a primary spark-circuit and its antenna is worked out under resonance conditions, and for various couplings and time graphs are made of the primary and secondary currents and voltages. A further example is the solution for the incidence of a natural electric wave (a "stray") on a receiving antenna and its resonant secondary.

DISCUSSION.

Dr. A. Russell said that it was not quite clear how much was Heaviside's and how much was Eccles' in the first part of the Paper. The latter part he considered an important piece of work.

Mr. R. Appleyard drew attention to a general method for the calculation of all transient phenomena, which had been worked out by Dr. H. Malcolm, described in "The Electrician" of May, 10, 1912. His method is there applied to the particular case of a submarine cable, but the same general method is directly applicable to the case treated by Dr. Eccles. In the case of the telegraph cable the constants are distributed. In the case of a transformer or of any network of conductors, such as the Wheatstone bridge, the constants are localised, and the problem is easier to solve. Briefly, Dr. Malcolm's method consists in converting by a formal transformation the periodic solution of the network—which is obtained in the usual manner by determinants or otherwise—into the solution for the sudden application of a constant E.M.F. Mr. Oliver Heaviside's theoretical conclusions are of enormous importance, but he himself realised how great is the difficulty of reducing them to arithmetical conclusions in specific cases. Dr. Malcolm's method is so direct and powerful in its application that it renders possible the discovery of new theoretical facts, and it so far simplifies the mechanical and mental labour of obtaining numerical results that it seems to justify the hope entertained by Mr. Oliver Heaviside that means would be found to combat the laboriousness of calculating the shapes of these wave-forms for transient currents.

XL. *Surface Leakage, with Alternating and Continuous Electric Pressures.* *By* G. L. ADDENBROOKE.

RECEIVED MAY 11, 1912. READ MAY 31, 1912.

IN the course of experiments on dielectrics at different temperatures, and using a wide range of periodicity, it was found that the results were in some cases complicated by surface leakage.

Further precautions were taken, and the results with leakage and without were compared.

From these it was manifest that the effects of surface leakage were sometimes considerable, and that the differences could not be accounted for by assuming that the surface leakage obeyed the ordinary laws of metallic conduction.

An attempt was, therefore, made to measure the leakage effects by themselves.

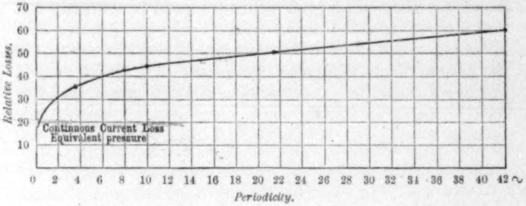
In the first place a sheet of crown glass was taken, the electrical properties of which had been carefully investigated beforehand.

On this sheet of glass two strips of tinfoil were pasted, each 25 cm. long and 2·5 cm. apart. The tinfoil strips were connected to the measuring arrangements which were electrostatic, and a current at 130 volts, first continuous and then alternating (R.M.S.), was turned on.

The resistance of the film of moisture at 15°C., and with 2° difference between the wet and dry bulb thermometers, worked out at 300 megohms, or the equivalent of 3,000 megohms across a surface of 1 sq. cm.; this is the mean of two final deflections with reversed poles, the initial deflections being larger, as described by Trouton and Searle (" Phil. Mag.," 1906).

The effects with continuous pressures were very similar to those described by the above authors, but as I experimented with only about one-tenth the potential slope used by them the variations found seemed to be within a smaller range. Taking the average continuous pressure loss as unity, after the deflection had become fairly steady, on turning on the alternating pressure at 42 periods the losses were found to be rather over three times as great. The deflection was quite steady if the film was protected from draughts.

The losses were next measured at several other periodicities, and the results are plotted in the curve below, where the ordinates represent relative loss and the abscissæ periodicities.



Next similar measurements were made to ascertain the effects of variations in the amount of moisture in the film. These are given below.

State of film on crown glass.	Continuous current resistance. Megohms.	Relative losses.	
		Continuous.	Alternating. 42 periods.
As above experiment	300	1	3
On a drier day......................	570	1	4
Film dried first in sun and cooled..............................	770	1	5

With another sheet of glass of much higher surface resistance I got a ratio of 1 : 15.

The power factor was very high in all cases.

The relative losses at 75 volts and at 150 volts were also measured and the square law was practically found to hold good.

Ebonite.—Grills made of thin strips of tinfoil connected together alternatively were used, pasted on a thin sheet, and also on a thick one. The film resistance per square centimetre was about 50 times that of the first glass tested. The ratios obtained were about 1 : 50 for the first specimen and 1 : 40 for the second.

Porcelain Insulators.—Two specimens were tested on account of the practical interest. Both gave ratios on a dry day of 1 : 60 approximately. As I was not sure how far this leakage might be through the body, I pasted two tinfoil rings round the outer shed of one of the insulators 15 mm. apart. On a rather damp day the relative leakages were as 1 : 14, but on a drier and warmer day the ratio rose to nearly the same as before. The continuous current resistance of these insulators was about the figures ordinarily quoted. If it is a fact that most of the resistance in ordinary porcelain insulators lies in the glaze, it is still possible that even in this last test part of the current went through the porcelain, when such a ratio in the losses might be expected, since through crown glass the relative losses under the above conditions of pressure and currents would give a ratio of about 1 : 150.

Effect of Thickness of Water Film.

A film of tap water about 1 mil thick between two glass plates showed no divergence from the ratio 1 : 1. Nor did a similar film of glycerine. Two pieces of plate glass were then taken and chemically cleaned. Two gold leaves were laid on one, 10 mm. apart, and a little distilled water put in the centre, and the top plate brought down. The two plates were then clamped together, with the water film between the gold leaves. Under these circumstances I seemed to get a difference of about 1 : 1·5, but I cannot put it forward with confidence. I have, through the good offices of Mr. C. W. S. Crawley, also been able to make some experiments on soap bubbles. Here, again, when the bubbles were thin I seemed to get evidence of some such a difference as in the last case, but again I cannot speak with confidence. For these latter tests it was necessary to reduce the pressure to 40 volts to avoid heating.

The thickness of even a fine soap bubble—viz., about $\frac{1}{20000}$ cm.—is, however, very great when compared with the thickness of the film of oil which can be got on water, and which is about $\frac{1}{20000000}$ cm. It is a film more of this class which I imagine must exist on glass surfaces under ordinary conditions.

Further Experiments.

The following also bear on this subject: A young lime tree leaf, which is very thin, showed no divergence from the ratio 1 : 1, but a last year's dry leaf showed a ratio of 1 : 2. The same ratio was also shown across a few inches of pine lath. An ordinary porous earthenware pot, tested as it was, showed no recognisable difference, but on drying this by heat and allowing it to cool a ratio of 1 : 2 was shown, the ordinary resistance having increased 900 times.

A set of experiments were next made by pasting two strips of tinfoil 12·5 cm. apart on a sheet of ordinary thin paper at room dryness. The ratio of losses for continuous current to alternating current 42 periods was as 1 : 2·7, and a curve over a long range of periodicities was the same as would have been got by testing through a number of sheets. Had this paper been artificially dried the losses would have been very much smaller, but the ratio of the continuous to the alternating loss would have been very much greater.

It may be interesting to note that measurements between the primaries and secondaries of a small induction coil and a small transformer gave ratios respectively of 1 : 4·5 and 1 : 8. In these cases part of the leakage was doubtless through the insulation.

Method of Measurement.

The losses, both continuous and alternating, were measured by a special form of electrostatic wattmeter. Under the conditions above it gives about 2 mm. deflection at 2 metres for 1 microwatt. I am in the habit of frequently testing its capabilities of measuring low power factors correctly by connecting it to a highly insulated air condenser, when no loss is shown beyond that due to half the resistance between the quadrants and in series with the circuit, the needle being connected directly across the circuit. Much care has also been used, in constructing the resistances, to reduce the capacity to a figure which will not influence the results. While, however, all these precautions have been taken, all the power factors met with in these experiments are high, so that no strain is thrown on the wattmeter, except in the point of sensitiveness. Expressed in the usual way, and with unity power factor, the instrument gives 250 mm. with $\frac{1}{10}$ volt between the quadrants and 200 volts on the needle.

Causes of the Effects Found.

That the losses in highly attenuated films of moisture with alternating currents should be higher than with continuous currents, which have been applied for any length of time, seems to follow directly from the observations on the latter of Trouton and Searle, and also that, to a large extent at any rate, the causes must be fundamentally the same. Now, in their Paper above quoted, Trouton and Searle give three suggestions as to the possible origin of the effects found.

Firstly, that they may be due to a highly insulating oxide of tin formed with time on one of the electrodes.

Secondly, that there may be an action tending to alter the thickness of the film near the electrodes, of somewhat the same character as occurs with the capillary electrometer.

Thirdly, that there may be a real departure from the simple form of Ohm's law.

From an interview I have had with Prof. Trouton I do not think he lays great stress now on the two former alternatives, and the probability of their being the true reasons seems less since in my experiments the potential slopes employed were in some cases not more than one-tenth, while the character of the effects seemed little altered.

I cannot avoid the impression that the third suggestion is the most likely, particularly when the effects as obtained above are compared with the effects found in making similar measurements through dielectrics over similar ranges of periodicity to that in the curve given above. These were described by me in outline in "The Electrician" for March 1st last. When testing through dielectrics the variations found in the losses are often of much greater magnitude than those given above, but the characteristics of the curves are strikingly the same. I have there pointed out the great differences made by very minute alterations in the amount of moisture present especially in the mineral oils, where it can be varied at will over a considerable range.

NOTE.—From further experiments since this Paper was read, I find that the above phenomena are connected with capacity effects, or effects which closely simulate capacity effects, and which cannot be accounted for by any calculations of the air capacity or capacity through the dielectric between the two conductors employed, as explained.

I hope to isolate this action more fully. It is not a constant per period, but is greater per period as the period is longer.

As a working hypothesis, it looks as if it might be due to comparatively free electrons moving backwards and forwards in the dielectric under the alternating field, as contrasted with the electrons intimately associated with the atoms, which latter, though far more resilient, are only very slightly displaced and without recognisable dissipation of energy.

DISCUSSION.

Prof. C. H. **Lees** pointed out that when alternating current was used there would be a capacity current flowing in the circuit as well as the surface leakage current.

Mr. A. **Campbell** thought the results were interesting. The study of surface losses was a very difficult subject and well deserved attention.

Mr. C. W. S. **Crawley** communicated the following remarks: Mr. Addenbrooke, I think, hardly emphasises the point sufficiently that the leakage curve he gets at different periodicities resembles so very closely the absorption curve in an ordinary condenser—so closely that I cannot help feeling that we are quite possibly dealing with one and the same phenomenon, the film in this case behaving as a condenser with negligible capacity but great absorption. I must congratulate him on being the first, as far as I know, to investigate these matters with very low frequencies. At ordinary commercial ones this present effect, like most of his results in absorption, would be masked, and they have, consequently, been overlooked.

XLI. *Demonstration of the Use of Specific Gravity Balls for determining very Small Differences of Density.* *By* T. H. Blakesley.

Exhibited June 14, 1912.

The present communication was made for the purpose of pointing out that the sensibility of specific gravity balls was, as a matter of fact, vastly under-rated by scientific men, and the author quoted passages from Prof. Balfour Stewart and Mr. Fownes which would tend to produce the belief that beyond the third decimal place specific gravity balls are not trustworthy. A variety of experiments was quoted which would indicate a sensibility 100 times as great as this; and, indeed, that the error which might be expected in a properly conducted experiment would be of the order 5 in the sixth decimal place. The author has been in the habit for the last quarter of a century of employing specific gravity balls for the purpose of discriminating between the qualities of potable waters in respect of density, and, therefore, of hardness, of testing the efficacy of softening processes, &c., for all of which the specific gravity ball is admirably suited. His process of observation differs from the ordinary rough-and-ready use in the fact that a thermometer of fairly open scale is employed to give the temperature at which a specific gravity ball is in equilibrium with a liquid which is being slowly warmed or cooled through that point of temperature. The error in such a temperature observation should not amount to more than three or four hundredths of 1°C.

If such a determination is made in distilled water at ordinary atmospheric temperatures it fixes the specific gravity of the ball at the temperature of equilibrium within four or five units in the *sixth* place of decimals. If a second observation with the same ball is made in a slightly heavier liquid, say common tap water, the temperature of equilibrium will be considerably higher, perhaps 2 deg. or more, than in distilled water. By applying the coefficient of cubical expansion the density of the ball at the higher temperature can be obtained, and this is the density of the second specimen of water at the second temperature. Reference to a table of densities of distilled water will furnish its density at the higher temperature, and the difference between the two numbers will give what the author calls the density excess of the second liquid over distilled water at the higher of the two temperatures. This density excess is best quoted in parts in one million. For example, the density excess for a specimen of London water was found to be 290 at 17°C.

For a few degrees on either side of this value this number is sensibly constant, a fact of some value, as it enables one to employ four or five balls at the same time in one experiment.

One noteworthy result of experiments made through a long course of years is that simple glass balls tend to expand, especially in the years immediately succeeding their manufacture. The rise of the zero point of thermometers has led to the belief that glass contracts with age.

Mr. Blakesley then proceeded to explain the sort of apparatus which he thought might make the extreme sensibility of the specific gravity ball available for quickly detecting the infusion of fresh water in the ocean due to the presence of icebergs in the neighbourhood. This problem is one which cannot be decided without definite experiments made upon water specimens taken from the sea at various noted distances from icebergs, observing direction of wind and current at the times of taking the specimens. These could be quietly submitted to examination, and the results would be of the first importance in settling some of the details of the apparatus for quick practical determination of fresh water infusion.

DISCUSSION.

Dr. Chree remarked that there was also a similar gradual change in the density of hydrometers with time. He questioned whether the density of sea water was uniform enough, especially near land, in order to apply Mr. Blakesley's test for icebergs.

Mr. A. Campbell asked how far dissolved gases affected the density of water. He asked whether the secular change in the balls might not be due to a change of weight on wearing.

Prof. C. H. Lees inquired whether the degree of accuracy obtained was greater than Dr. Warrington obtained with the same method seven or eight years ago.

Mr. F. E. Smith pointed out that the secular change in the balls might be due to the solubility of glass. If the balls were made of soft soda glass and the water were alkaline such a loss of weight could be seen in 10 minutes.

The President pointed out that the secular change in these bulbs was quite a different phenomenon from the secular change in thermometers. With respect to the application of the method to the detection of icebergs, he did not think it would be much help. What was wanted was not to detect the presence of a field of ice which was generally known, but some particular iceberg that was dangerously near.

The Author, in reply, remarked that a couple of balls which had irno wire inside them did not show the secular change of density. He had not tested the balls for loss of weight. A slight solubility of the glass was not detrimental, since the balls could be standardised in distilled water as often as desired. Unlike hydrometers, the balls eliminated all surface action.

XLII. *On Hysteresis Loss as affected by Previous Magnetic History at Liquid Air Temperature.* *By Prof.* ERNEST WILSON, B. C. CLAYTON *and* A. E. POWER.

RECEIVED MAY 14, 1912. READ JUNE 28, 1912.

ONE object of this Paper is to examine the effect of previous magnetic history upon the dissipation of energy by magnetic hysteresis at the temperature of liquid air. This effect at atmospheric temperature* and the effect of previous magnetic history upon permeability at liquid air temperature† have been already studied.

It is well known that if a piece of iron be subjected to a considerable magnetising (previous history) force, and then be tested for hysteresis loss and permeability, the results differ widely from what would have been obtained had the material been previously demagnetised. The hysteresis loss corresponding to a given maximum value of the magnetic induction **B** is increased, and the permeability corresponding to a given value of the magnetic force **H** is diminished, by the application of the previous history force. In Fig. 1, loop 1 is symmetrical about the origin and is obtained after the material has been carefully demagnetised. Loop 2 is obtained for the same value of the reversal force **H** after the application of a large previous history force, H_1, and after many reversals of the reversal force **H**. Loop 3 is in connection with a further effect. Suppose that the specimen is carefully demagnetised and that a magnetising force ($\mathsf{H}_2-\mathsf{H}$) supplied by an independent source is very gradually increased to its proper value, which is such that the change of magnetic induction corresponding to a reversal of the force **H** is exactly equal to the change observed with loop 2 for the same change of **H**. The change from loop 1 to loop 2 has been brought about by inter-molecular force, whereas the change from loop 1 to loop 3 has been brought about by the application of an externally applied constant force $\mathsf{H}_2-\mathsf{H}$. If the effect of inter-molecular force were capable of being exactly equivalent to the externally applied constant force ($\mathsf{H}_2-\mathsf{H}$), one would expect to find that the energy required to perform a complete cycle would be the same in each case—that is, the area of loop 2 would be equal to the area of loop 3.

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* *See* "Proc." Roy. Soc., A., Vol. 83, 1909, p. 1.

† *See* "Jour." Phys. Soc., Vol. XXIII., Part IV., June 15, 1911.

The ring experimented upon is built up of stampings of Stalloy and is the same as was used in the previous experiments upon permeability at liquid air temperature.* The method of test employs the ballistic galvanometer.

Referring to Fig. 2, the dissipation of energy is plotted in terms of the maximum value of the magnetic induction **B**, and curve A has been obtained by plotting the areas of a series of hysteresis loops (No. 1, Fig. 1) given by a carefully demagnetised specimen at atmospheric temperature. Curve B gives the corresponding areas at the temperature of liquid air.

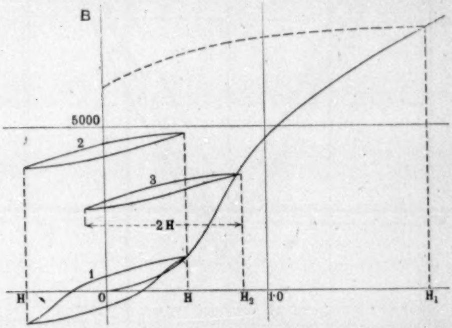

FIG. 1.

Curve C gives the areas of a series of loops (No. 2, Fig. 1) obtained at liquid air temperature after the application of a large previous history force (H_1) of about 26 C.G.S. units. Curve D gives the areas of a series of loops (No. 3, Fig. 1) obtained as above described with a carefully demagnetised specimen and during the application of a constant force H_2-H. Curves A and B show that the loss is increased by the application of liquid air. Curves B and C show that previous history at liquid air temperature gives rise to an increased value in the dissipation of energy for a given value of the maximum magnetic induction **B**, and this is the effect at atmospheric temperature also. Curves C and D show that the energy required to effect a reversal of the same value of the magnetic induction, the one after the application of a given previous history force of about 26 C.G.S. units, and the other under the application of a given constantly applied force (H_2-H) is greater in the former than in the latter case.

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* *See* "Jour." Phys. Soc., Vol. XXIII., Part IV., June 15, 1911.

In Fig. 3 the curve A is given again, and contrasted with it are the curves E and F as given respectively by loops 2 and 3,

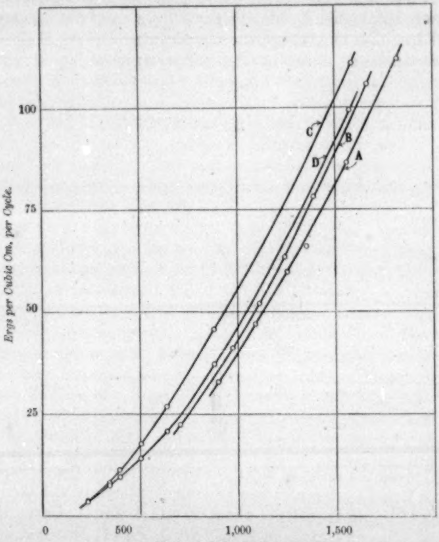

FIG. 2.

all at atmospheric temperature. The percentage increase in the dissipation of energy by magnetic hysteresis due to the application of a previous history force of about 26 C.G.S. units has been found by taking the difference between curves B and C and A and E respectively, and expressing it as a percentage of the corresponding ordinates of curves B and A.

As regards loop 3 (Fig. 1) it is interesting to plot the percentage increase of the area of loop 3 over that of loop 2, against reversal **H** for a given value of the previous history force. This is done in Fig. 4, the previous history force being 26 C.G.S. units. In Fig. 4, curve G gives the results obtained in the present experiments at atmospheric temperature, and curve H the results obtained at liquid air temperature. Curve I gives the results previously obtained at atmoshperic temperature with a different Stalloy ring.* As regards curves G and H, it cannot be said that there is a very great difference. The difference between curves H and I may be accounted for by the fact that they were obtained with different specimens of

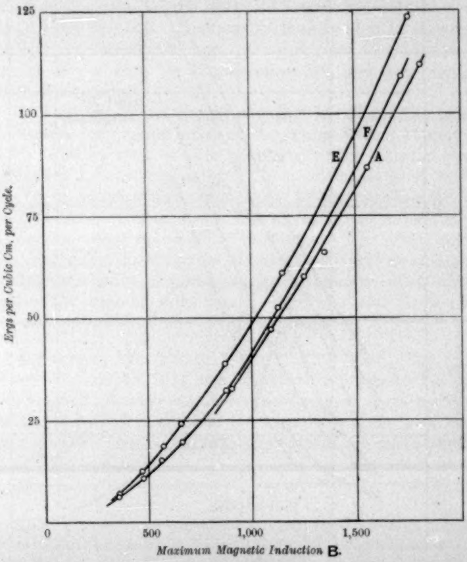

FIG. 3.

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* *See* "Proc." Roy. Soc., A., Vol. 83, 1909, p. 1.

Stalloy. In this material the effect mainly develops at a force **H** of about 0·5, and before the point of maximum permeability is reached.

Fig. 4 also shows the **BH** curves at atmospheric and liquid air temperatures, the lower ones after the application of a previous history force of 26 C.G.S. units, and the upper ones after careful demagnetisation.

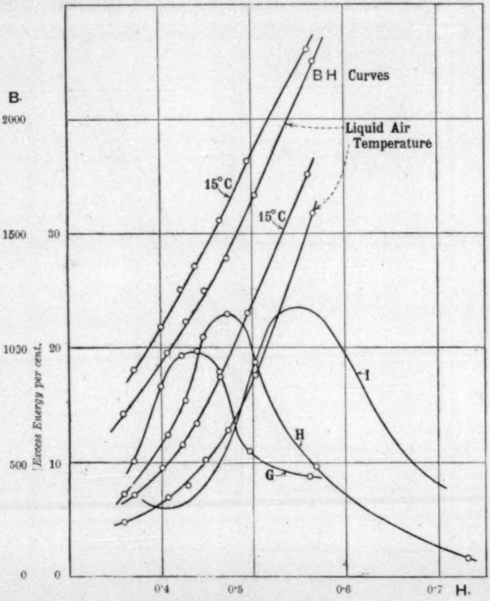

FIG. 4.

A general conclusion of the experiments is :—

1. That without previous history effect the loss due to magnetic hysteresis at the temperature of liquid air is larger than at atmospheric temperature for a given value of the

maximum magnetic induction; amounting to about 7·6 and 8·9 per cent. when **B** maximum has values of 1,000 and 1,500 respectively.

2. That the percentage increase in the dissipation of energy by magnetic hysteresis due to the application of a previous history force of 26 C.G.S. units varies from about 21 to 17 at atmospheric temperature and 27 to 15 at liquid air temperature, as the value of the maximum magnetic induction varies from 1,000 to 1,500. Previous tests show that at atmospheric temperature this percentage difference rises to a maximum of about 45 per cent. for **B** maximum 500, and then diminishes with **B** maximum.

3. That the excess energy required to reverse the magnetic induction for a given value of the reversal force **H** may be as high as 20 per cent. or over, according as the material has been polarised by the application of a previous force of 26 C.G.S. units, or continuously magnetised by the application of an independent external force, the total change in the magnetic induction being the same in each case. The effect occurs at its maximum in Stalloy at about $\mathbf{H}=0{\cdot}5$ and at a point preceding the point of maximum permeability. The effect has about the same magnitude and occurs at about the same force at atmospheric and liquid air temperatures.

NOTE.—In the Paper, "Jour." Phys. Soc., Vol. XXIII., Part IV., June 15, 1911, the temperatures at which Salloy loses its magnetic quality on heating and regains it on cooling were stated to be 850°C. and 830°C. respectively. Sir R. A. Hadfield has pointed out that these temperatures are more nearly 765 and 755 respectively.

ABSTRACT.

This Paper is concerned with hysteresis loss in iron at atmospheric and liquid air temperatures under three different conditions: (1) after the iron has been carefully demagnetised, (2) after it has been subjected to a large force (previous history) of about 26 C.G.S. units, and (3) whilst it is under the influence of an external constant magnetising force after demagnetisation. One object is to discover (*a*) how the dissipation of energy varies in the above three cases when the maximum magnetic induction **B** is the same in each, (*b*) how it varies in cases (2) and (3) when not only the maximum magnetic induction is the same, but the value of the reversal force **H** is the same in each. This matter should concern any molecular theory of magnetism.

It is shown both at atmospheric and liquid air temperatures that the loss for a given value of the magnetic induction **B** is greater in case (2) than in either (1) or (3) and that the loss in case (3) is greater

than in case (1). Moreover, the loss at liquid air temperature is for corresponding cases greater than at atmospheric temperature. For given values of the magnetic induction where the reversal force H and the temperature is the same for each the loss in case (2) is greater than in case (3), and the percentage difference between the losses rises to a maximum of about 20 when has H a value of about 0·5 C.G.S. units in stalloy. This percentage difference has about the same maximum at about the same value of reversal force H at atmospheric and liquid air temperatures.

As regards the importance of the subject to engineers, it may be mentioned that the watts dissipated by magnetic hysteresis may be increased by as much as 45 per cent., and the magnetising force by 85 per cent. on the values these quantities need only have when previous history effects have been wiped out. It would seem, therefore, that after severe short-circuits it might pay to demagnetise the cores of transformers, static-balancers, &c.

DISCUSSION.

Prof. C. H. **Lees** asked if the effect due to the previous history would not disappear after the alternating current had been applied for a little time.

Mr. G. L. **Addenbrooke** asked how Prof. Wilson distinguished between hysteresis and eddy current losses.

Mr. C. W. S. **Crawley** pointed out that the curve of percentage increase in energy loss showed a minimum for low values of H in the case of stalloy, as well as the maximum, and asked if the effect was a real one.

Mr. A. **Campbell** remarked that it is now being found that it is almost as important that the iron should be demagnetised before applying an alternating current to it as a direct one.

Prof. E. **Wilson**, in reply to Prof. Lees, stated that although the alternating current had been applied to the specimen for an hour or more during the tests it would not wipe out the effect of a previous history field that was stronger than itself. In order to demagnetise the specimen by an alternating current the current must be large enough to produce a magnetisation greater than the previous history effect it was desired to wipe out, and must then be gradually reduced to zero. The ballistic method had been employed, which would take no account of eddy currents. He thought the minimum shown by stalloy at low values of H was a real one, though the accurate measurements were very difficult to obtain at those low values. Lohys also showed the same minimum.

XLIII. *Dielectric Hysteresis at Low Frequencies.* *By* W. M. THORNTON, *D.Sc., D.Eng., Professor of Electrical Engineering in Armstrong College, Newcastle-upon-Tyne.*

COMMUNICATED BY SIR OLIVER LODGE, F.R.S.
RECEIVED JUNE 4, 1912. READ JUNE 28, 1912.

1. *Introduction.*—The polarisation of a solid insulator in a steady and uniform field of electric force differs from other polarisation, mechanical or magnetic, in the manner in which the final value is reached. It has been shown* that, after the initial displacement at the velocity of electromagnetic waves in a medium which occurs on starting the field, there is a slow increase of polarisation maintained at a uniform rate for several hours, and that the final value at which equilibrium is reached is independent of the intensity of the external field. The polarisation decays on stopping the latter at the same rate as that at which it was established, and in the end entirely disappears. The effect, which is common to all dielectrics, is an example of "elastic afterworking" unique in range and duration. There is, however, in every case a well-marked period of transition between the first polarisation and the complete establishment of the second stage, and it is in this that any electric hysteresis loss must arise, for in the first stage the displacement is elastic, and in the second is unaffected by the external field. In this paper an attempt is made, by the examination of wave forms of alternating current in condensers built up of flat sheets of dielectric, to find the range and magnitude of polarisation under simple harmonic electric stress, at a frequency low enough to retain the features of hysteresis observed in steady fields and high enough to be examined by the oscillograph in the usual way. A summary of the considerable body of work upon this subject is given in Winkelmann's "Handbuch der Physik." 2 Aufl., Band IV., p. 159, and a full bibliography in a recent paper by Mr. E. H. Rayner in the Journal of the Institution of Electrical Engineers. The effect is observed in practice by the accumulation of charge on the conductors of direct-current cables left insulated after use, giving rise to voltages sometimes greatly in excess of the working pressure. It is observed in high-

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* "On the Polarisation of Dielectrics in a Steady Field of Force." Proc. Phys. Soc., Lond., Vol. XXII.; also Phil. Mag. Vol. XIX., March, 1910.

voltage alternating-current systems by heating of the insulation of cables through the conductor of which the only current passing is that required to supply the dielectric polarisation.

2. *Dissipation of Energy in, and Power Factor of, Condensers.* The general case of the loss of energy per cycle in condensers may be stated briefly as follows. Consider a condenser of capacity C, having a polarisable dielectric of mean constant K, to which a sine wave voltage V, is applied at a frequency n. The charge on the conductors at any instant is CV, and the current $C\frac{dV}{dt}+V\frac{dC}{dt}$. The apparent power Vi is then $CV\frac{dV}{dt}+V^2\frac{dC}{dt}$, and the mean activity, *i.e.*, the energy absorbed per second, is

$$\int Vidt=n\int CVdV+n\int V^2dC.$$

The integration being taken round a cycle the first term on the right is zero, and the energy absorbed is $n\int V^2dC$. This is the dielectric hysteresis loss, and it depends upon the change of the capacity, that is, of the electric constant, during the cycle. Since the observed index of the loss never exceeds 2 and appears to be in some cases less, the variation δC depends upon a term of the form aV^{-x}, where x is less then unity and is generally small.

The power factor of a condenser, *i.e.*, the ratio of the watts to the volt-amperes, may be expressed in terms of the variation of capacity during the cycle. Let v and i be the root mean square voltage and current, and φ the angle between their vectors. We have then

$$vi\cos\varphi=n\int v^2dC.$$

The energy component of the current is usually so small that without sensible difference we may write $i=2\pi nCv$. The integral may also be regarded as the product of the maximum range of v^2 and the mean range δC of the capacity. Since the hysteresis loop is symmetrical about zero the loss is the same as that produced by a double frequency change of amplitude v_0^2, and since in a sine wave $v_0^2=2v^2$ we have

$$2\pi nCv^2\cos\varphi=4nv^2\delta C,$$

and $\cos\varphi=\frac{2}{\pi}\,\frac{\delta C}{C}$, that is, the power factor of a condenser is proportional to the mean relative change of the dielectric constant during the cycle.

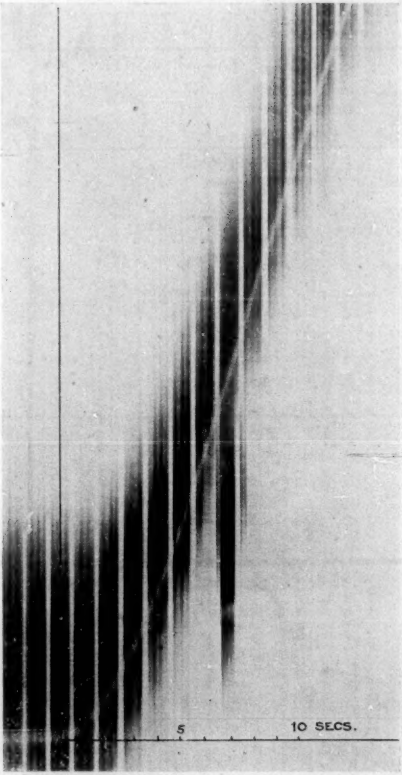

FIG. 1.—LINEAR RATE OF ORIENTATION OF INDIARUBBER CYLINDER IN STEADY FIELD OF FORCE.

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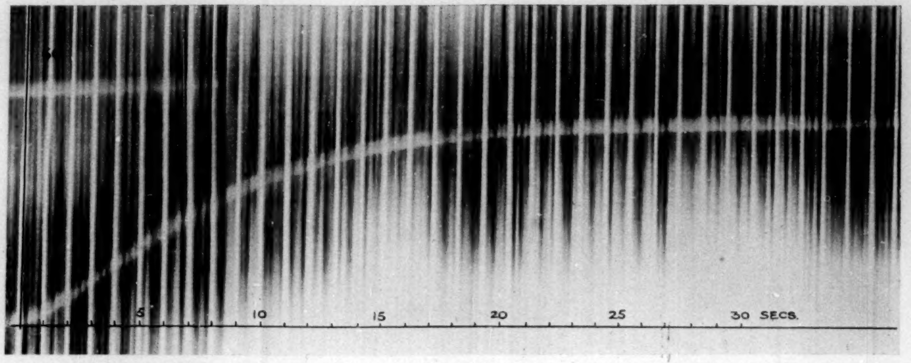

FIG. 2.—END OF FIRST STAGE OF SLOW POLARISATION IN STEADY FIELDS, AND COMMENCEMENT OF SECOND IN INDIARUBBER.

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3. *Early Stages of Polarisation in Steady Fields.*—The currents in a cable charged for many hours at a steady voltage and then short-circuited have been observed by Ashton.* On integration of the curves of charge and discharge of an okonite cable, the total quantity found to be absorbed during charge at a uniform voltage, taking the sum of several operations, was $29{\cdot}71 \,.\, 10^{-5}$ coulombs; that discharged on short-circuit $61{\cdot}35 \,.\, 10^{-5}$ coulombs. The quantity discharged was, therefore, within the limits of experimental error, twice that received. In either case the quantity passing is $CdV+VdC$; the quantity Q_c absorbed on charge when V is constant is VdC; at discharge the circuit voltage is that which is given to the conductors by the charge leaking out of the dielectric. This starts at the value V and falls to zero, while at the same time the coefficient C returns to its previous value. On discharge, therefore, if there is no permanent polarisation, $Q_d=CdV+VdC$, and since by symmetry these terms are equal, going through the same range of values in each case, $Q\ =2Q_c$. The above experimental results show that a polarised dielectric parts with the whole of its charge if allowed to do so at its own rate, and that there is nothing of the nature of permanent polarisation, as in the magnetisation of iron. The same conclusion was reached by the experiments in steady fields previously referred to, using suspended ellipsoids through which conduction currents could not pass from the pole plates, and is well authenticated by many experimental records.

The curves of change of dielectric constant with time recorded in the previous paper† on slow polarisation do not show the nature of the change in the first few seconds after the field is applied. This part of the rise of polarisation was, therefore, examined in the present case by suspending the cylinder of indiarubber which had been used in the previous work under alternating stress so as to lie at a small angle to a steady electric field. On starting the field the cylinder began to polarise and moved slowly into line with the field, the torsional control being such that the first polarisation was only sufficient to overcome a small part of the angle of set. The motion was made dead beat by attaching to the underside of the cylinder a fine glass tube dipping into oil, the viscosity of which to give correct damping was determined by trial. The photographs of Fig. 1 and Fig. 2 give the rate of change of angular displacement of the cylinder, one showing the change in the first few seconds from starting the field, the other up to the time of establishment of the linear change given in Fig. 2 of the previous paper, at which it proceeds for several hours to complete equilibrium in the field.

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* A. W. Ashton, "On the Resistance of Dielectrics." Phil. Mag., 1901, p. 517. † *Loc. cit.*, p. 401.

The initial polarisation of indiarubber from the above-mentioned figure, having a value for the dielectric constant of 3·08, and the polarisation after 30 seconds being 4·5, the rate of change on the straight part of Fig. 1 of the present paper in the first five seconds after the start of the field is at the rate of 0·6 per second. To explain the facts of dielectric hysteresis, the rate of change of K must be much greater than this. It became necessary, therefore, to examine in some other way the change which takes place in the small fraction of a second from the start of the field, and which is disguised by the inertia of the above arrangement. For this purpose several large condensers were made of such a size that the wave form of the current taken by them, with a suitable voltage gradient in the dielectric, could be observed in an oscillograph and photographically recorded. Wave forms of capacity currents in cable networks have been previously obtained, but the voltage wave forms of the generators employed have been so irregular that no trustworthy measurements of cable power factors from oscillograms have been possible. There is an additional difficulty in the interpretation of measurements of energy loss in cables that the voltage gradient in the dielectric is not the same everywhere. The influence of this on cable power factors is examined in § 8.

4. *Experimental Details.*—Before constructing the condensers it was first necessary to obtain a pure sine wave at a frequency of 20 to 80 a second, and at a maximum pressure of 1,000 to 2,000 volts. A low-pressure wave was eventually found from a large machine, which, when analysed, showed no signs of distortion. The next step was to transform the 60 volts so obtained to the required maximum to be used on the condensers without introducing harmonics in the sine wave. In the end a Westinghouse voltmeter transformer was used with air core choking coils in series with its primary circuit. Its normal step was from 100 to 5,500 volts, but by choking the primary current the secondary voltage was lowered to the desired values. This had the effect of lowering the core flux of the transformer and was necessary for the following reason. The capacity current flowing in the secondary high-tension circuit being 90 deg. in front of the secondary voltage is in phase with

the primary magnetising current. The result is that unless the primary terminal volts were already lowered the core flux might be strengthened to such an extent that hysteresis would distort the secondary wave form. On open circuit or on a resistance load the voltage might have a true sine wave, and on a capacity load show a third harmonic of iron hysteresis. So long as the core flux follows a sine law the generated primary and secondary volts must also be sinuous, but the currents will not be so if there is either dielectric or magnetic hysteresis in the system. In a condenser supplied by a transformer the greatest precautions have to be taken in order that the capacity current wave form should not show a spurious harmonic derived from the magnetisation of the iron. The nearness of the core to hysteretic distortion cannot be seen by overloading the secondary circuit on a non-inductive resistance, for that only changes the primary ampere-turns, the core flux remaining nearly constant. If, however, the ratio of the secondary to the primary ampere-turns is found when the transformer is on capacity load, a resistance then substituted for this load, and the primary voltage increased in the above ratio, the secondary current being kept at its normal full-load value, the distortion of the primary current wave will only occur when the maximum induction in the core is above the straight part of the curve of magnetisation of the iron. This final test did not in the present case reveal the least trace of magnetic hysteresis in the primary current wave. If, therefore, the capacity current shows a third harmonic with a true sine wave primary voltage well reduced, the distortion can be said with certainty to arise from the dielectric alone.

5. *Observations and Results.*—The condensers were made up with flat sheets of dielectric overlapping intermediate sheets of tin-foil by an inch at each edge. The substances used were glass, ebonite, indiarubber, gutta-percha, paraffined paper, Empire cloth, Empire paper, presspahn, and a paper impregnated with a resin oil compound used in the manufacture of high-tension paper-insulated cables. The frequency was constant throughout at 36·5 complete periods a second, unless otherwise mentioned.

The oscillograph was of the Duddell Cambridge Instrument Co.'s double high-frequency permanent magnet type. One strip was connected through a suitable non-inductive resistance across the terminals of the condenser under test, the other strip being in series with the condenser without added resistance.

The wave forms were recorded on a falling photographic plate, and for the purpose of measurement were enlarged to give the waves an amplitude of 10 cm. on a half-wave base of about 19 cm. The ordinates for typical substances are recorded in Table I., in centimetres in each case. The scale for each curve

TABLE I.—*Temperature 18°C., Frequency 36·5 p.p.s.*

Phase angle.	Glass.			Gutta-percha.			Indiarubber.			Ebonite.			Paraffined Paper.		
	V.	I.	V.I.	V.	I.	V.I.	V.	I.	V.I.	V.	I.	V.I.	V.	I.	V.I.
5	1·15	7·71	8·87	0·85	5·14	4·36	1·0	5·55	5·55	0·85	8·16	6·94	0·96	9·38	9·00
15	3·15	7·25	22·82	2·53	4·97	12·56	2·13	5·23	11·13	2·45	7·75	19·00	2·96	9·03	26·72
25	4·80	6·72	32·22	4·09	4·53	18·52	4·14	4·82	19·92	4·05	7·12	28·81	4·87	8·46	41·20
35	6·30	6·13	38·60	5·51	4·08	22·50	5·62	4·30	24·30	5·54	6·36	35·20	6·60	7·16	47·26
45	7·51	5·50	41·30	6·86	3·60	24·70	6·80	3·73	25·36	6·80	5·50	37·40	8·13	6·75	54·85
55	8·75	4·70	41·10	7·94	2·96	23·50	7·90	3·04	24·00	7·80	4·50	35·08	9·43	5·01	47·32
65	9·70	3·63	35·2	8·85	2·05	18·12	8·75	2·22	19·40	8·62	3·28	28·25	10·52	4·07	42·82
75	10·4	2·30	23·9	9·45	1·04	9·84	9·26	1·32	12·22	9·12	2·07	18·86	11·22	2·42	27·15
85	10·73	0·90	9·66	9·70	0·15	1·45	9·55	0·45	4·30	9·45	0·60	5·67	11·60	0·92	10·66
95	10·73	0·72	7·72	9·68	0·65	6·29	9·55	0·50	4·77	9·45	0·65	6·14	11·56	0·90	10·40
105	10·3	2·26	23·28	9·40	1·55	14·56	9·25	1·40	12·95	9·18	2·00	18·36	11·22	2·43	27·25
115	9·5	3·70	35·12	8·80	2·31	20·32	8·70	2·30	20·00	8·62	3·42	29·48	10·50	3·34	35·05
125	8·57	4·84	41·46	7·98	3·09	24·64	7·90	3·18	25·13	7·88	4·65	36·62	9·56	5·15	49·20
135	7·35	5·85	43·00	6·90	3·81	26·30	6·76	3·95	26·70	6·75	5·80	39·18	8·25	6·33	51·16
145	5·83	6·73	39·20	5·56	4·48	24·90	5·54	4·50	24·92	5·50	6·75	37·12	6·77	7·42	50·2
155	4·32	7·38	31·85	4·12	4·95	20·39	4·02	5·00	20·10	4·08	7·51	30·65	5·0	8·25	41·25
165	2·36	7·77	18·32	2·56	5·24	13·41	2·40	5·40	12·95	2·48	8·05	19·95	3·13	8·94	27·95
175	0·74	7·94	5·87	0·86	5·36	4·61	0·93	5·60	5·21	0·90	8·35	7·51	1·03	9·30	9·58
R.M.S.	7·57	5·61		6·88	3·717		6·79	3·85		6·74	5·72		8·25	6·49	
Mean product }	42·48		0·87	25·60		2·20	26·2		0·72	38·58		1·09	53·6		0·55
cos ϕ	0·0205			0·086			0·0275			0·283			0·0103		

may be found by reference to the particulars of voltage and current given later. The power factors determined from these numbers could be repeated to the second significant figure.

Where there is no distortion of either wave form it is satisfactory to measure the angle of phase displacement of the curves on the axis, and to calculate the power factor from it; but where there is distortion this is quite unreliable on account of the phase difference introduced by the harmonic terms. These may easily give rise to an apparent displacement of a degree along the axis and change the power factor in proportion. For this reason, since the displacement is rarely more than a few degrees, all the substances whose power factors it was desired to obtain with fair accuracy were dealt with by measuring the ordinates of the voltage and current wave forms at equal intervals of time, and finding from them the root-mean-square values of current and voltage. The sum of products of corresponding ordinates in the two waves was also found for the complete period. The power factor of the condenser is then the ratio of this algebraical sum to the product of the root-mean-square values of voltage and current. The method will be clear from Table I. The condenser measurements are given below. Their special feature is the large volume of dielectric under stress.

CONDENSERS.

Glass.—This condenser had 66 sheets of photographic negative plates separated by tin-foil 13·5 cm. by 19·8 cm. The mean thickness of the plates was 0·135 cm., and the volume under stress 2,380 cubic cm. A current of 47·5 milliamperes was taken by the condenser on a voltage of 3,060. The wave forms of the voltage and current are given in Fig. 3, and the magnitude of the hysteresis effect can be seen by the third harmonic prominent in the current wave. There is evidence in the voltage wave—in this case only of all those examined—of slight distortion, not sufficient, however, to account for that of the current wave. Power factor 0·0205, dielectric constant 5·85.

Gutta-percha.—Two condensers each having 58 sheets of a very pure Silvertown gutta-percha, 0·028 cm. thick; tin-foil sheets 16·5 cm. by 17·5 cm. Volts 600, current 35 milliamperes with two condensers in parallel. The wave forms in Fig. 4 show very marked distortion. Power factor 0·086, mean dielectric constant from equivalent sine wave 2·48. Special care was taken in preparing and rolling the sheets to free them from moisture; the low value found for the dielectric constant is probably due to this cause. Several days were spent in testing this condenser under varying conditions, but at the same frequency, 36·5. Rayner obtained a power factor of 0·04 at a frequency of 50 from a short piece of cable.

Indiarubber.—59 sheets of pure Silvertown rubber 0·054 cm. thick, tin-foil 16·7 cm. by 17·8 cm. Volts 1,060, current with two condensers in parallel 31 milliamperes. Wave forms in Fig. 5, showing marked distortion in the current wave. Power factor 0·0275, mean dielectric constant 2·21.

Ebonite.—70 sheets of second-grade ebonite specially rolled, 0·092 cm. thick, separated by tin-foil sheets 25·4 cm. square. Volts 1,840, current 61 milliamperes.

The wave forms are given in Fig. 6, the current wave showing marked distortion, though less than glass. The power factor from the curves is 0·0283 and the mean dielectric constant 3·25.

Empire Paper.—96 sheets of paper impregnated with linseed oil and specially treated, 0·0065 cm. thick; tin-foil 25·4 cm. square. Volts 120, current 65 milliamperes. Wave forms in Fig. 7. Rather more distortion than in Empire cloth. Voltage wave here reversed. There is in this case also a high-power factor, 0·15. Dielectric constant 2·84.

Presspahn.—100 sheets, 0·02 cm. thick, kindly supplied by Messrs. C. A. Parsons & Co.; tin-foil 25·4 cm. square. Volts 400, current 126 milliamperes. Wave forms in Fig. 8. Dielectric constant 4·9, power factor 0·29. This high value may possibly be owing to traces of moisture. The material, though lightly varnished, is hygroscopic, and even after drying it is difficult to avoid contact with the atmosphere whilst assembling the condenser. Values as high as 0·447 have been found by Rayner for a similar material with about the same voltage gradient. On the same unidirectional voltage the current was 3·5 milliamperes.

Paper with Resin Oil Compound.—70 sheets of paper, 0·01cm. thick, heavily dressed with a resin oil compound; tin-foil 25·4 cm. square. Volts 200, current 38 milliamperes. Wave forms in Fig. 9. Current shows very slight distortion. Voltage wave reversed. The dielectric constant under light mechanical pressure is 2·1. Power factor 0·0087. This power factor is much lower than that of finished paper-insulated cables, which is

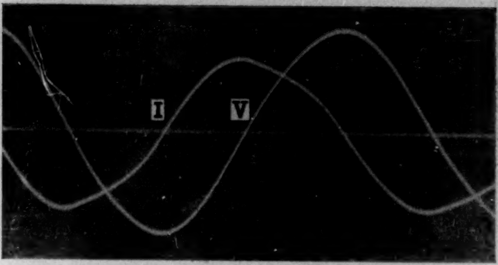

FIG. 3.—GLASS.

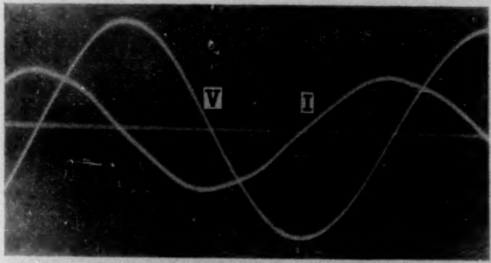

FIG. 4.—GUTTA-PERCHA.

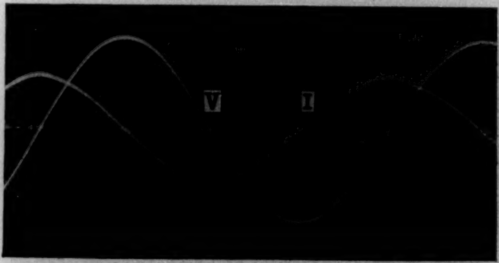

FIG. 5.—INDIARUBBER.

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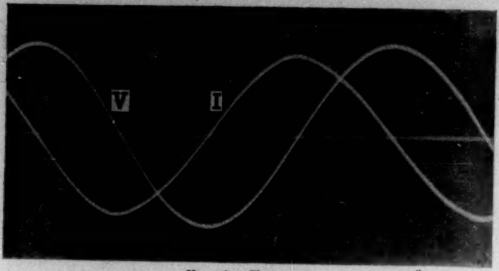


FIG. 6.—EBONITE.

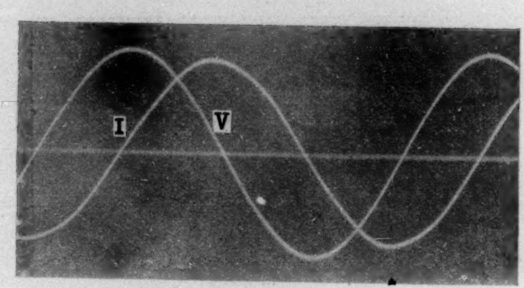


FIG. 7.—EMPIRE PAPER.

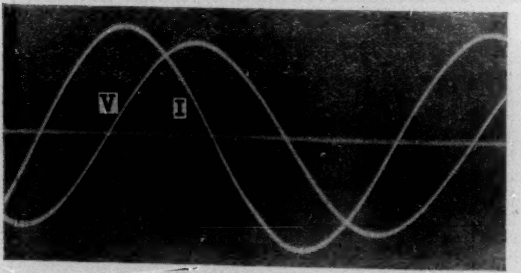


FIG. 8.—PRESSPAHN.

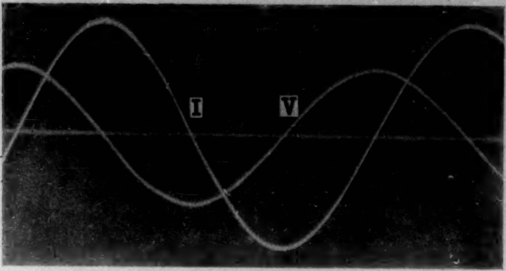


Fig. 9.—Resin-oiled Paper.

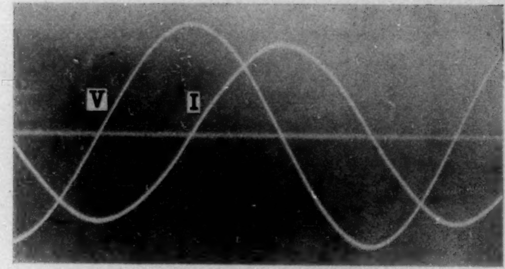


Fig. 10.—Paraffined Paper.

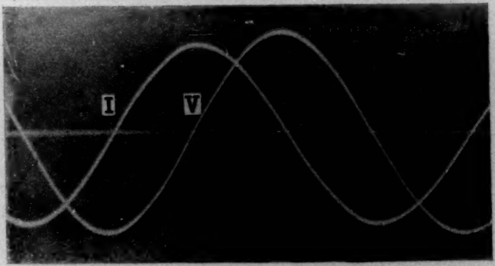


Fig. 11.—Empire Cloth.

usually 0·028. How far this result is owing to excess of insulating compound, or to inequality of voltage gradient in the cable case, or to difference in mechanical conditions, could not be determined. The present result is under light pressure, but sufficient to make good contact. It was, however, difficult, even by warming, to remove every trace of air when attaching the tin-foil. The dielectric constant is, if anything, high.

Paraffined Paper.—190 sheets of thin paper, 0·0063 cm. thick, impregnated with paraffin wax; tin-foil sheets 20·5 cm. by 25·4 cm. Volts 126, current 91·2 milliamperes. Wave forms in Fig. 10. There is slight distortion in the current wave. The voltage wave is here accidentally reversed. These condensers were made by the Stanley Mfg. Co., Pittsburg, U.S.A., and have a very low power factor. The mean dielectric constant is 2·28, power factor 0·01 (paraffin wax 2·32, paper 1·8 to 2·1).

Empire Cloth.—96 sheets of cloth, 0·015 cm. thick; tin-foil 25·4 cm. square. Volts 300, current 140 milliamperes. Wave forms in Fig. 11. No distortion to be seen. Power factor 0·110. Dielectric constant 2·77.

The substances are arranged in order of visible distortion of the current curves in Table II., in which are also given the voltage gradients, the relative change $\delta C/C$, calculated from the relation $\cos\varphi = \frac{2}{\pi} \cdot \frac{\delta C}{C}$, the volume under stress, and the specific loss of energy per cycle. To find the power factor of distorted waves it is always necessary to find the equivalent sine wave.

TABLE II.

Material.	Mean lielectric constant.	Power factor.	$\frac{\delta c}{c}$.	Volts per cm. thickness	Ergs per c.c. per volt per cycle.	Active volume of dielectric, c.c.
Glass	5·85	0·0205	0·032	22,700	0·0136	2,380
Gutta-percha ...	2·48	0·0860	0·135	22,000	0·0230	955
Indiarubber	2·21	0·0275	0·043	20,000	0·0065	1,890
Ebonite	3·25	0·0283	0·044	20,000	0·0104	4,150
Empire paper ...	2·84	0·153	0·240	18,500	0·0440	402
Presspahn.........	4·90	0·290	0·455	20,000	0·1550	1,290
Resin oiled paper	2·10	0·0087	0·0136	20,000	0·0020	450
Paraffined paper	2·28	0·0103	0·016	20,000	0·0025	625
Empire cloth ...	2·77	0·110	0·172	20,000	0·0660	950

Influence of Mechanical Pressure.—The condensers were all made up in dried hard-wood boxes, with a plate of thick dry board on top pressed down by two broad distance pieces on the cover, which were so adjusted that when the lid was screwed down the pressure was equivalent to that of a 15 lb. weight. To examine the influence of pressure in improving contact between tin-foil and dielectric the lid of one of the gutta-percha condensers was removed. The current then taken at 585 volts was 15 milliamperes. With a 28 lb. weight uniformly distributed over the surface the current was 17·5 milliamperes, and no further rise was obtained on increasing the weight to 84 lb.

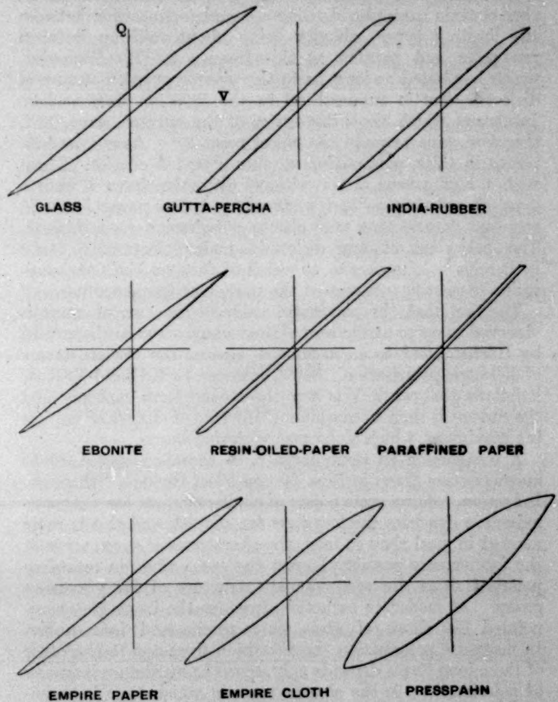
The current in the presspahn condenser treated in the same way rose from 53·5 milliamperes with the cover removed to 60·5 with 33 lb., 63·2 with 61 lb., and 65·2 with 89 lb. The current from which the dielectric constant was calculated corresponds to a total pressure of 30 lb. distributed over an active surface of 1 square ft.

The amplitude δC of the change of capacity varies in the materials showing marked distortion, from 3 to 13 per cent. In this group the great amplitude of gutta-percha may be compared with its very rapid polarisation in steady fields recorded in Fig. 2 of the previous paper. It is noticeable that the substances in common use for cables—indiarubber and resin oiled paper—have very small loss in comparison with the others, and that presspahn, universally employed in the insulation of electrical machinery, absorbs a large part of the energy of polarisation transmitted through it.

6. *Hysteresis Loops derived from Pressure and Current Wave Forms.*—Electric hysteresis loops may be drawn in the same way that the magnetic hysteresis loop of iron is obtained from magnetising current and flux waves. Defining the capacity at any instant as the ratio Q/V, where Q is the quantity in the condenser and V the terminal voltage, there are two methods of finding it from the curves: (1) by integrating the current wave up to the point at which the voltage is measured and plotting the derived quantity against this voltage; (2) by differentiating the voltage wave and plotting current against rate of change of volts. Where the wave forms are much distorted the former is preferable, but for a sine wave voltage the latter is quicker; both have been used in the present case.

The disadvantage of the falling-plate method is that the time base of the photographs has to be corrected for the slight acceleration during exposure. This can be done with accuracy from the known frequency and height of fall.

Hysteresis loops (Fig. 12) have been drawn for all the materials tested. Their most characteristic feature is that their sides are straight and parallel for the simpler substances. More composite materials, like presspahn, Empire cloth and paper, show curvature, both on the rising and falling sides. Lens-shaped loops of this kind can be produced by simple electrical viscosity, causing retardation proportional to the rate of change of



GLASS GUTTA-PERCHA INDIA-RUBBER

EBONITE RESIN-OILED-PAPER PARAFFINED PAPER

EMPIRE PAPER EMPIRE CLOTH PRESSPAHN

Fig. 12.

voltage. In the straight-sided type the retardation occurs at the commencement of the fall of voltage and is therefore more truly defined as hysteresis. The limit of this type, the closing to a straight line, is approached in paraffined paper even at the low frequency of 36·5. The dielectrics tested thus contain examples showing a complete transition between the limiting types, ebonite being about midway between presspahn and paraffin at this frequency. The discussion, which has lasted so long, as to the viscous or other nature of dielectric loss is summed up in the light of these curves. Insulators which show distortion of the current wave, and, therefore, give loops of the chisel point type, have true hysteresis in their polarisation at these low frequencies. Those with a high power factor without distortion have a viscous type of polarisation and hysteresis. A low power factor in any case means that true elastic polarisation predominates. The behaviour of any dielectric under alternating stress is therefore a question as to which of the two kinds of polarisation is most in evidence at the particular frequency used.

The fact that the maximum value of the current wave is observed to occur at the zero of the voltage wave has been used by Höchstädter[*] as an argument against the viscous nature of dielectric polarisation. Since, however, $i = \mathrm{K}d\mathrm{V}/dt + \mathrm{V}d\mathrm{K}\ dt$, it follows that when V is zero the second term vanishes, and the current is then a maximum if the rate of change of voltage is a maximum, which is the case with sine waves.

A comparison of these loops with those of graded cyclic magnetisation given in Figs. 50 and 52 of Ewing's "Magnetic Induction," shows some points of similarity. At low magnetic induction densities the loops are len-shaped, and this is more marked in steel than in iron, the characteristic segregation in the former case possibly having the same effect in retarding polarisation as the very complex structure of the presspahn group. At moderate induction densities the loops are chisel-pointed like those of glass, gutta-percha and indiarubber. In magnetic polarisation the transition from one to the other of these loop forms depends only upon the maximum intensity of polarisation; in the electric case, in which time is so important a factor, it should be found to depend chiefly upon frequency.

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[*] "Electrotechnische Zeitschrift," May-June, 1910. "Science Abs.," 558, 1910.

7. *Influence of Frequency.*—It has been shown by Threlfall[*] that at frequencies of a million a second there are no hysteresis effects in dielectrics and, since it has been previously shown that all substances have hysteresis in prolonged unidirectional fields, it follows that the hysteresis loop of each substance *must pass through every phase of type* as the frequency is raised from zero. On account of the difficulty of obtaining a sine wave over a wide range of frequency, and of recording the curves, only a part of this could be in the present case directly observed.

The particular phase, elastic or viscous, characteristic of any dielectric will probably be revealed most fully by investigation of the current wave forms at very low frequencies, say, up to 5 a second. The range of frequency used in heavy electrical engineering is from 15 to 80, and in the following examples attention has been confined to these. It has been shown that paraffined paper and ebonite may be regarded as approaching the elastic type at a frequency of 36, whilst presspahn is typically viscous. It might then be expected that the latter would be more sensitive to changes of frequency than the former, and this is found to be the case.

PRESSPAHN.

Frequency	15	22	30	33	36·5	78·5
Current ...m.a.	34	44·75	56	60·9	63·2	122·5
K	6·3	5·7	5·2	5·1	4·9	4·3

The increase of K begins to be marked below a frequency of about 40, and is very rapid at the lower frequencies. The dielectric constant of paraffined paper increased 0·4 per cent. as the frequency was reduced from 80 to 20, and that of ebonite 0·66 per cent. over the same range. In the latter cases no great rate of change might be expected until the frequency is reduced to about five a *minute*, as shown by their initial rate of polarisation in steady fields.

It would be interesting to observe the loop of presspahn at telephonic frequencies. The behaviour of paper would lead one to expect that the loop would close to a straight line.

8. *Influence of Voltage Gradient on Capacity.*—To observe the possible influence of voltage gradient on the mean dielectric constant the paraffined paper and ebonite condensers were tested, at constant frequency, from gradients of 3,500 volts per

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[*] "Physical Review," 189.

cm. to 39,600 in the former and 35,400 in the latter. In both cases the constant had the same value at the higher as at the lower limit, but in each a slight increase, 4 per cent. above the mean, was found at an intermediate gradient. This is possibly owing to a combination of instrumental errors, and within this range it may be said that for these typical substances the voltage gradient has no measurable influence on the mean dielectric constant.

9. *Influence of Voltage Gradient and of Frequency on the Power Factor of Cables.*—Many of the measurements of energy loss in dielectrics have been made on cables, the voltage gradient in which is not uniform over the section. It would appear to be generally agreed that the dielectric loss from all causes varies nearly as the square of the gradient. The rate of heating by reason of dielectric hysteresis must, therefore, be very unequal. Apart from differences in material the power factors of cables will not be the same unless the distribution and intensity of the stress is also identical. Fleming and Dyke have recently found a wide difference in the power factors of gutta-percha cables of dissimilar section. It would be interesting to compare the distribution in these cases by the stream line method. The distribution of stress in a three-core cable during a cycle has been worked out in this way and numerical values given from which the loss might be determined from point to point.*

In the case of a concentric cable the voltage gradient can be readily calculated at all points in the dielectric, and is greatest at the inner conductor. The rate of heating can thus be estimated over the section on the assumption that it varies as the square of the gradient. It is found that whether the average gradient be taken as the potential difference divided by the thickness or as the mean of the extremes at the conductors, the result is that the power factor of a parallel plate condenser having the same mean gradient and the same volume of dielectric is the same as that of the concentric cable and equal to $2c/Kn$, where c is the loss per unit volume at unit gradient, K the dielectric constant and n the frequency. It has been previously shown that the rate of loss—in so far as it is indicated by the form of the loop—must depend upon the frequency ; c must, therefore, vary with n in some manner to be determined experimentally. At telephonic periods Fleming and Dyke have shown that the "conductivity" of a condenser is a linear function of the frequency. The rate of change of the conductivity and c is then directly proportional to n. At high frequencies, therefore, the power factor should increase only as the dielectric constant K diminishes. From their observations the increase of cos φ per period between 920 and 2,760 periods per second is $1{\cdot}23 . 10^{-7}$ for pure rubber, $1{\cdot}25 . 10^{-7}$ for paraffin wax and $1{\cdot}2 . 10^{-6}$ for gutta-percha. The increase though small is perceptible, and the higher value for gutta- percha agrees with the previously observed fact that the rate of change of slow polarisation is greater for this than for any other dielectric examined.

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* W. M. Thornton and O. J. Williams, "The Electrician," Sept. 3 and 10, 1909.

ABSTRACT.

The Paper is an attempt to determine from dielectric hysteresis loops the nature of the change of polarisation which gives rise to the absorption of energy. Current and voltage wave-forms in large condensers were oscillographically recorded at a frequency of 36 a second. Certain substances, like glass or gutta-percha, are found to show a marked triple frequency harmonic in the current wave, out of phase with the fundamental. Most of these have a low power factor; others, like presspahn, show little distortion, but have a high power factor. The loops for the former have straight parallel sides and a true hysteresis retardation as the voltage begins to fall; the latter have lens-shaped loops corresponding to a purely viscous retardation. Every variety of transition is observed between these extreme types.

The cause of the former would appear to be inter-attraction between the induced molecular charges, which reaches a maximum when these are greatest—*i.e.*, at the highest voltage; that of the latter is a resistance to their movement through the substance during the establishment of the usual polarisation, the retardation reaching a maximum when the rate of change of the voltage is greatest—*i.e.*, in passing through zero.

The dielectric constants of substances with lenticular loops are more affected by change of frequency than the straight-sided type, that of presspahn changing from 4·3 at 78 to 6·3 at 15 per second. Ebonite and paraffined paper, on the contrary, change by less than 1 per cent. during the same range. For the same substances change of voltage gradient, short of breakdown voltage, has little or no influence on dielectric constant at this frequency. Power factor appears to increase with frequency within the range determined.

DISCUSSION.

Prof. E. **Wilson** asked what degree of accuracy the author had reached. The accuracy of results obtained with an oscillograph were marred by the thickness of the lines.

Prof. C. H. **Lees** asked if the different loops could not be accounted for by assuming the dielectric to be made up of two parts, in one part of which the polarisation was proportional to the electric field and independent of of the time, and in the other part in which it rose with the time to its final value following an exponential law.

Mr. G. L. **Addenbrook** thought there were two effects here, one depending on the periodicity, and the other an absorption effect.

Mr. D. **Owen** asked if Prof. Thornton had ascertained there was no C^2R loss or leakage current in the condensers.

Mr. A. **Campbell** thought the different classes of hysteresis loops obtained were interesting. The fact that the author had dealt with dielectrics in practical use increased the importance of his Paper.

Prof. W. M. **Thornton**, in reply, stated that he thought the accuracy of the oscillograms he had obtained was sufficient for the purpose. In reply to Prof. Lees, he did not think the loops could be accounted for without assuming more than one exponential term. Leakage current was tested for by applying a continuous-current pressure of 480 volts. In no case was any measurable current observed except with presspahn, which showed a current of $3{\cdot}5 \times 10^{-6}$ amperes

XLIV. *I. The Efficiency of Generation of High-frequency Oscillations by Means of an Induction Coil and Ordinary Spark-gap. II. An Oscillographic Study of Some Induction Coil Phenomena. By Prof.* G. W. O. Howe *and Mr.* J. D. Peattie.

Received May 22, 1912. Read June 28, 1912.

I. The Efficiency of Generation of High-frequency Oscillations by Means of an Induction Coil and Ordinary Spark-gap.

The primary object of the experiments described in this communication was to determine the efficiency with which high-frequency currents are generated in small radio-telegraph stations, using an ordinary spark-gap and a coil supplied with continuous current through a mercury interrupter.

In order to explain the peculiarities observed it proved necessary to investigate the action of the spark coil itself, and an account of this investigation forms the second part of the Paper.

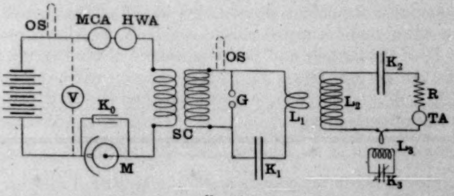


Fig. 1.

The apparatus employed and the connections are shown diagrammatically in Fig. 1. SC is a 10 in. Marconi spark coil supplied from accumulators through the motor-driven mercury break M. This break works in coal-gas; it is shunted with a condenser, K_0. MCA and HWA are two ammeters connected in series, the former, of the moving coil type, reading the mean value of the intermittent current, the latter, of the hot-wire type, the root-mean-square value. K_1 represents three glass-plate condensers immersed in oil, the combined capacity at high frequencies being 0·00485 mfd. The coil

L_1 is 10 in. square, and consists of four turns of copper tape. The spark-gap G consists of massive brass electrodes of large radius (about 2 in.), with micrometer adjustment. The natural frequency of the primary oscillatory circuit corresponded to a wave-length of 360 metres.

The coil L_2 consists of 15 turns, 9 in. square, of 3 mm. copper wire; K_2 is an oil condenser of 0·0018 mfd. The circuit K_2L_2 takes the place of the aerial, and a resistance, R, is introduced to take the place of the total damping due to radiation and C^2R losses. The resistance R consists of fine Eureka wire having the same resistance to high-frequency and continuous currents. The energy dissipated in it could be determined from the temperature rise of the oil in which it is immersed, or more simply from the reading of the thermal ammeter TA. This energy constitutes the high-frequency output of the plant, and the overall efficiency is obtained by dividing it by the continuous-current input to the induction coil. The coupling between L_1 and L_2 can be varied by sliding L_2 along a graduated bench. Loosely coupled with the circuit K_2L_2 is the wavemeter K_3L_3, by means of which the frequency and the nature of the oscillations can be determined.

Measurement of Input to the Induction Coil.—The unidirectional intermittent current was supplied from a battery of accumulators at some distance. If E be the constant E.M.F. of the battery and R the resistance of the battery and leads, the average power supplied to the coil will be $(E \,.\, C_{\text{av.}}) - (C^2_{\text{R.M.S.}}R)$, and this can be calculated from E and R and the two ammeter readings.

Determination of the Coupling.—If the coefficient of mutual induction between the coils L_1 and L_2 be M, then if $k = \frac{M}{\sqrt{L_1L_2}}$, k is called the coefficient of coupling. This was determined for various positions of the secondary coil in the following manner. If the two coils are connected in series in such a way that their inductive effects are additive, the combined inductance $L' = L_1 + L_2 + 2M$, whereas, if the connections of one coil are reversed, the combined inductance $L'' = L_1 + L_2 - 2M$. Now L_1, L_2, L' and L'' can be determined by putting a spark-gap and a known capacity in the circuit and determining the frequency with a wave-meter. Since the coefficient k is merely a numerical ratio, it is not necessary to know the value of the capacity employed, if it be unchanged throughout. If the coil of the wave-meter be also unchanged, the inductance under test is proportional to the capacity of the wave-meter condenser when adjusted to resonance. The leads used to connect L_1 and L_2 may have a considerable inductance, which we may call L_a, but their mutual inductance with the coils L_1 and L_2 can be made practically negligible, so that the combined inductance of L_1 and L_a, or of L_2 and L_a, is unaffected by reversing the connections to the coil. We have then

$$L_1+L_2+L_a+2M=aK'_1$$

and

$$L_1+L_2+L_a-2M=aK'_2,$$

where a is a constant and K'_1 and K'_2 are the capacities of the wave-meter condenser for resonance.

Hence,

$$M=\frac{a(K'_1-K'_2)}{4},$$

and

$$L_1+L_2+L_a=\frac{a(K'_1+K'_2)}{2}.$$

With coil L_1 alone we have $L_1=aK'_3$, and with coil L_2 alone, $L_2=aK'_4$, and

$$k=\frac{M}{\sqrt{L_1L_2}}=\frac{(K'_1-K'_2)}{4\sqrt{K'_3K'_4}}.$$

Hence, all that is required to determine the coupling is an accurately calibrated variable condenser. Some of the results of the calibration are given in the following table :—

$$K'_3=1\cdot87\ :\ K'_4=4\cdot98\ :\ \sqrt{K'_3K'_4}=3\cdot05.$$

Reading on bench.	Approx. distance between mean planes of coils.	K'_1.	K'_2.	$\frac{K'_1+K'_2}{2}$	$\frac{K'_1-K'_2}{4}$	k.
70	4 cm.	10·24	4·40	7·32	1·46	0·48
65	9 ,,	9·48	5·21	7·34	1·07	0·35
60	14 ,,	8·58	6·06	7·32	0·63	0·21
50	24 ,,	7·81	6·90	7·36	0·23	0·075
40	34 ,,	7·51	7·17	7·34	0·085	0·028
30	44 ,,	7·41	7·21	7·31	0·05	0·016

By plotting these values of k, the coupling can be found for any position of the secondary coil.

The constants of the primary and secondary oscillatory circuits were as follows : $K_1=0\cdot00485$ and $K_2=0\cdot00183$ microfarads, $L_1=7\cdot5$ and $L_2=20$ microhenries. $\lambda=360$ metres.

Results of Tests.—On plotting either the aerial current or the current taken by the coil, against the spark-length, peculiar results were obtained. After the electrodes had been polished readings were taken for different lengths of gap between 0·5 mm. and 5·0 mm., first gradually increasing the length and then gradually decreasing it. As shown in Fig. 2, the two sets of readings do not agree. This is due to the fact that with freshly polished electrodes the spark takes the shortest path, whereas when the central part becomes burnt and blackened the spark prefers the longer path between either the unburnt part of the electrodes or the edge of the burnt portion. This can be seen on observing the gap, and is also confirmed by the horizontal displacement of the return curve in Fig. 2. It will be noticed that with a gap of 1 mm. the average current taken by the coil gradually decreases, whereas with a gap of 1 mm. it gradually increases. The uncertainty thus introduced was avoided by polishing the electrodes before each experiment and then taking readings at regular intervals after switching on. By plotting the results and producing the curves backward, the probable initial values were obtained. Except at the first moment, the effective length of the gap is greater than the measured length.

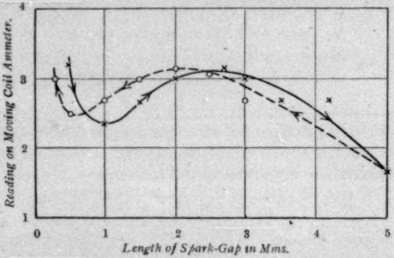


FIG. 2.

The first series of tests were made with a resistance of 6·4 ohms in the dummy aerial circuit, corresponding to a logarithmic decrement $R/2\sim L$ of 0·19 per period. The induction coil was supplied from a battery of six accumulators, and the mercury break gave 23·8 interruptions per second. In all the experiments the spark-gap was subjected to a moderate air-blast. Six values of the coupling were tried, each with about 10 different lengths of gap. The results are shown in Figs. 3 and 6, where the high-frequency output and the over-

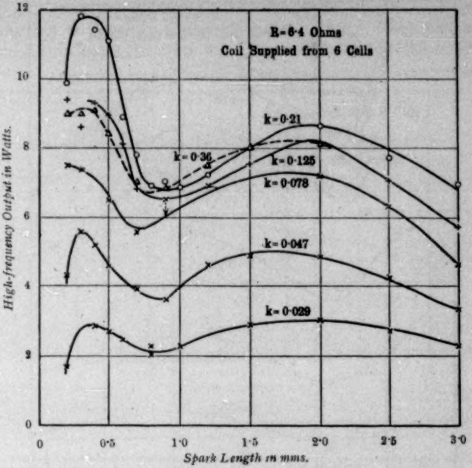

R=6·4 Ohms
Coil Supplied from 6 Cells
k=0·21
k=0·36
k=0·125
k=0·078
k=0·047
k=0·029
High-frequency Output in Watts.
Spark Length in mms.
0
2
4
6
8
10
12
0
0·5
1·0
1·5
2·0
2·5
3·0

FIG. 3.

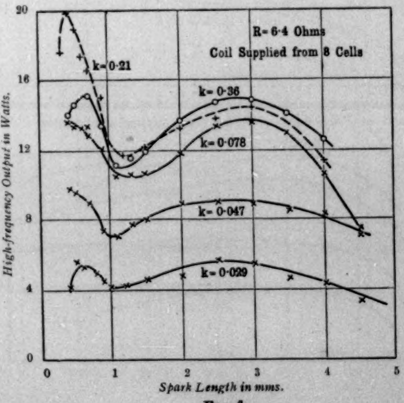

R=6·4 Ohms
Coil Supplied from 8 Cells
k=0·21
k=0·36
k=0·078
k=0·047
k=0·029
High-frequency Output in Watts.
Spark Length in mms.
0
4
8
12
16
20
0
1
2
3
4
5

FIG. 4.

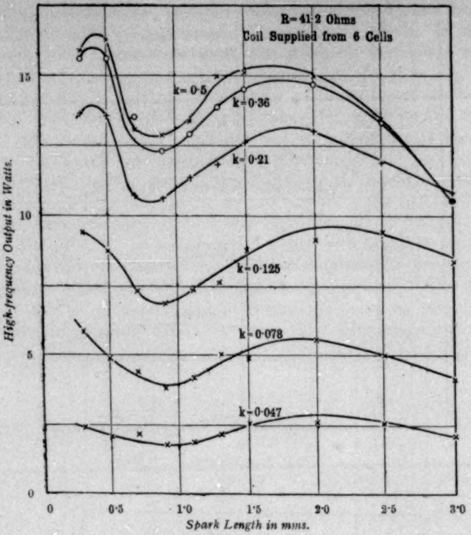

R=41·2 Ohms
Coil Supplied from 6 Cells
k=0·5
k=0·36
k=0·21
k=0·125
k=0·078
k=0·047
High-frequency Output in Watts.
15
10
5
0
0
0·5
1·0
1·5
2·0
2·5
3·0
Spark Length in mms.

FIG. 5.

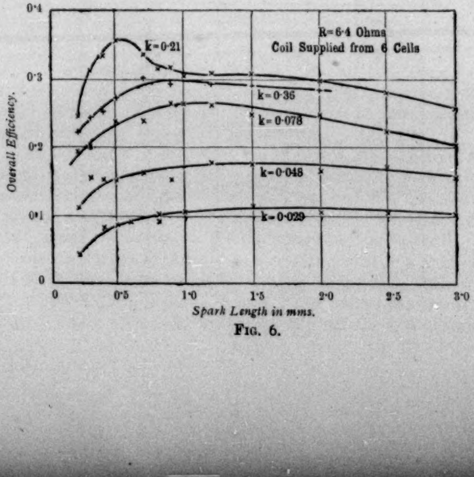

R=6·4 Ohms
Coil Supplied from 6 Cells
k=0·21
k=0·36
k=0·078
k=0·048
k=0·029
Overall Efficiency.
0·4
0·3
0·2
0·1
0
0
0·5
1·0
1·5
2·0
2·5
3·0
Spark Length in mms.

FIG. 6.

all efficiency are plotted against the spark-length, each curve referring to a given value of the coupling. With the same aerial resistance the tests were repeated with the coil supplied

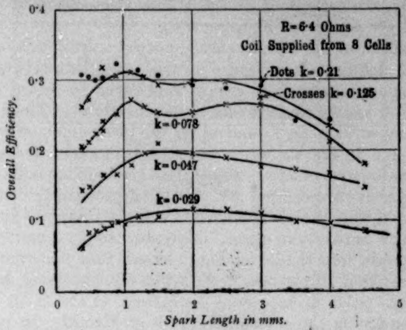


FIG. 7.

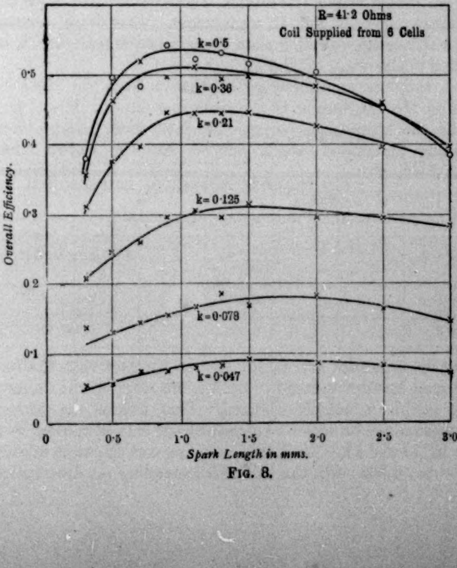


FIG. 8.

from 8 and 10 cells respectively; the results when using 8 cells are plotted in Figs. 4 and 7. Using 6 cells the tests were repeated with "aerial" resistances of 21·5 and 41·2 ohms respectively, corresponding to logarithmic decrements of 0·65 and 1·24 per period (*see* Figs. 5 and 8).

In addition to the high-frequency output and the efficiency one must determine the nature of the oscillations produced. For this purpose resonance curves were taken by means of a wave-meter loosely coupled with the aerial circuit. These are reproduced in Figs. 12, 13 and 14 for the three different aerial resistances. In Fig. 12, which corresponds more nearly to the conditions found in practice, it is seen that the coupling cannot be increased above about 8 per cent. without decreasing the power transmitted from the aerial circuit to a slightly damped tuned circuit such as the wave-meter. If it were simply a matter of getting power into the aerial Fig. 3 shows that the coupling should be about 20 per cent. With this tight coupling, however, beats occur in the aerial oscillations as shown by the double humps in the resonance curves in Fig. 12. In practice, the decrement of the aerial is often less than the value, 0·19 per period, which the dummy aerial circuit had when the resonance curves of Fig. 12 were taken. This would necessitate a still looser coupling than 8 per cent. if beats are to be avoided in the aerial oscillation.

The coupling is sometimes calculated from the distance apart of the two humps of the resonance curve. If $\sim_1$ and $\sim_2$ be the frequencies corresponding to these humps, then assuming undamped oscillations, it can be shown that $k=\frac{\sim_1^2-\sim_2^2}{\sim_1^2+\sim_2^2}$. The following table shows, however, that the coupling, as determined in this way, may be far from correct:—

Fig. 4.	$\frac{\sim_1^2-\sim_2^2}{\sim_1^2+\sim_2^2}$.	Actual value of k.
Curve D	0·059	0·125
Curve E	0·194	0·214
Curve F	0·348	0·363

The discrepancies are mainly due to the fact that the formula used assumes undamped oscillations whereas the dummy aerial circuit is heavily damped. That this is the correct explanation can be seen by comparing the curves marked F in Figs. 12, 13 and 14. The actual coupling was the same in each case—viz., 0·363—but the calculated coupling, as determined

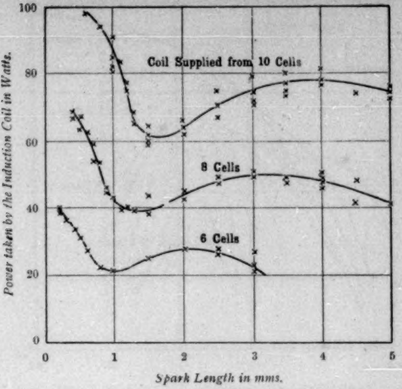

Power taken by the Induction Coil in Watts.
100
80
60
40
20
0
Coil Supplied from 10 Cells
8 Cells
6 Cells
0
1
2
3
4
5
Spark Length in mms.

FIG. 9.

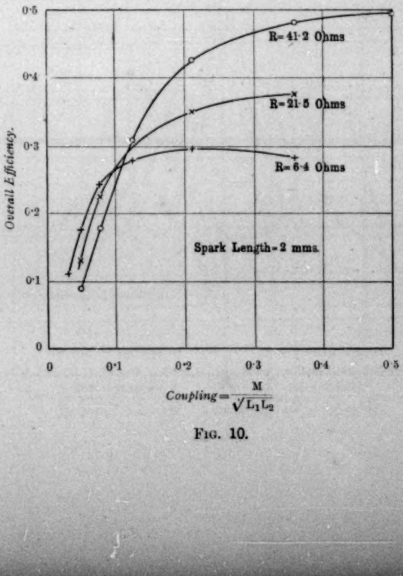

Overall Efficiency.
0·5
0·4
0·3
0·2
0·1
0
R=41·2 Ohms
R=21·5 Ohms
R=6·4 Ohms
Spark Length=2 mms.
0
0·1
0·2
0·3
0·4
0·5
Coupling = $\frac{M}{\sqrt{L_1 L_2}}$

FIG. 10.

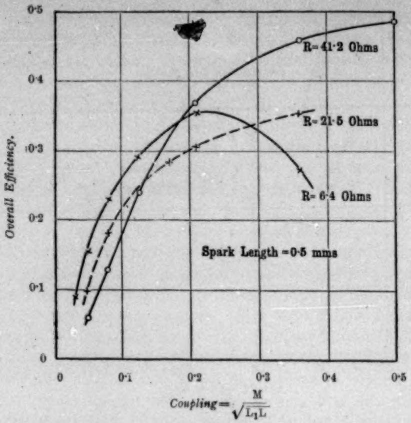
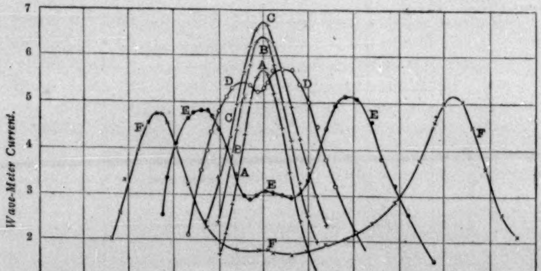


FIG. 11.



Using 10 in. Spark Coil, with Mercury Break. Length of Spark Gap 1·2 mm. with Air-Blast.

A Coupling = 2·9 %
B " " 4·75 %
C " " 7·8 %
D Coupling = 12·55 %
E " " 21·4 %
F " " 36·3 %

FIG. 12.

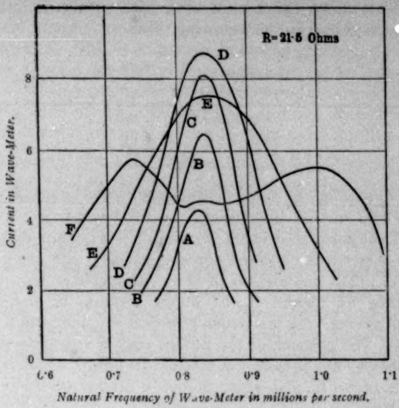

R=21·5 Ohms
A
B
C
D
E
F
0
2
4
6
8
0·6
0·7
0·8
0·9
1·0
1·1
Current in Wave-Meter.
Natural Frequency of Wave-Meter in millions per second.

FIG. 13.

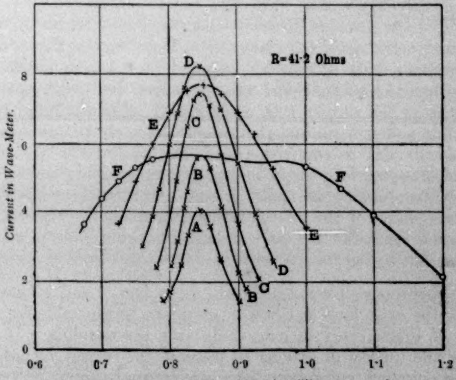

R=41·2 Ohms
A
B
C
D
E
F
0
2
4
6
8
0·6
0·7
0·8
0·9
1·0
1·1
1·2
Current in Wave-Meter.
Natural Frequency of Wave-Meter in millions per second.

FIG. 14.

by the position of the humps, decreases as the damping is increased, as shown in the following table :—

Curve F.	Resistance in aerial circuit.	$\frac{\sim_1^2 - \sim_2^2}{\sim_1^2 + \sim_2^2}$.	Actual k.
Fig. 4	6·4 ohms	0·348	0·363
Fig. 5	21·5 „	0·298	0·363
Fig. 6	41·2 „	0·157	0·363

If the figures in the third column are plotted against those in the second column, and the curve is produced backwards to find what the calculated value of k would be when the damping was zero, a result is obtained agreeing exactly with the actual value of k. The calculation of the coupling from the resonance curves by means of the above formula is, therefore, capable of giving very erroneous results if the damping is considerable. The position of the humps may also be affected to some extent by the presence of the middle hump, which is seen in Fig. 12, curve E, and which is due to premature quenching of the spark.

The absence of any pronounced double hump in Fig. 14 is due to the increased resistance in the aerial circuit, the dissipation of energy being too rapid to allow of its storage in the secondary circuit and subsequent return to the primary circuit.

Power taken by the Induction Coil.—These curves (Fig. 9) are of the same shape whether the supply be taken from 6, 8 or 10 cells, but the maxima and minima occur for longer spark-lengths as the supply P.D. is increased. The causes of the maxima and minima and the general shape of the curves are discussed later in connection with the oscillograph records. For short spark-lengths the increased input is shown to be due partly to sparking at make as well as at break, while the general shape of the curves is dependent on the time that elapses between the last spark and the re-making of the primary circuit.

High-frequency Output.—The curves in Figs. 3 and 4 follow, much the same lines as those for the input to the coil in Fig. 9, but, of course, vary in magnitude with the coupling. They show the futility of seeking to increase the output by using closer coupling, quite apart from questions of tuning, as very little is gained by increasing the coupling beyond a certain point. The curves also show the tendency for the output to increase abnormally with short sparks and a coupling of about 20 per cent. This is undoubtedly due to premature quenching of the spark. In Fig. 5 the secondary circuit has a much greater decrement than an ordinary radio-telegraph aerial; there is here no sign of quenching and the output is largely increased by tightening the coupling up to about 50 per cent.

The Overall Efficiency.—From Figs. 6, 7 and 8 it will be seen that the efficiency is very low at short spark-lengths, due probably to the arcing that occurs between the electrodes. The efficiency increases with the spark-length up to a certain point, and then gradually decreases; it varies very little, however, over a wide range. The effect of premature quenching is seen in Figs. 6 and 7 for a spark-length of 0·5 mm. and 21 per cent. coupling. The effect of varying the coupling and the decrement of the aerial can best be seen from Figs. 10 and 11, which are plotted for fixed spark-lengths of 0·5 mm. and 2·0 mm. respectively. If the coupling be loose and the secondary damping large, we shall get a feeble forced oscillation, whereas if the secondary damping be small, a fairly large oscillation may be built up in the secondary before the gap in the primary opens, and the energy so stored will be dissipated entirely in the secondary circuit. With tight coupling, however, matters are very different. If the secondary damping is small, the energy surges backwards and forwards between the two circuits, and is largely lost in the spark-gap. As the secondary resistance is increased a greater proportion of the energy is dissipated in the secondary circuit. This explains the crossing of the curves in Figs. 10 and 11.

With an aerial resistance of 6·4 ohms and a coupling of 8 per cent. the overall efficiency has a maximum value of about 26 per cent. This represents approximately the conditions of radio-telegraphy when using sending apparatus similar to that employed in these experiments.

The logarithmic decrement due to the radiation from an earthed vertical wire can be shown from theoretical considerations to be about 0·2.*

The total decrement due to radiation and energy dissipation in the aerial and earth will depend largely on the nature of the earth connection. It may be much greater than that due to radiation alone.

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* *See* M. Abraham, "Theorie der Elektricität," Vol. II., p. 307.

T-shaped antennæ and other types having a large capacity relatively to the height have a smaller radiation decrement, and their total decrement may be much smaller than the smallest value employed in these experiments—viz., 0·19.

II. OSCILLOGRAPHIC STUDY OF SOME INDUCTION COIL PHENOMENA.

In order to explain the peculiarities of the curves showing the input to the coil for various spark-lengths, a number of oscillograms were taken. The oscillograph strip, suitably shunted, was placed in one of the two positions shown in Fig. 1. The frequency and regularity of the sparks was determined by means of a revolving mirror from which the sparks were reflected and focussed on a falling photographic plate.*

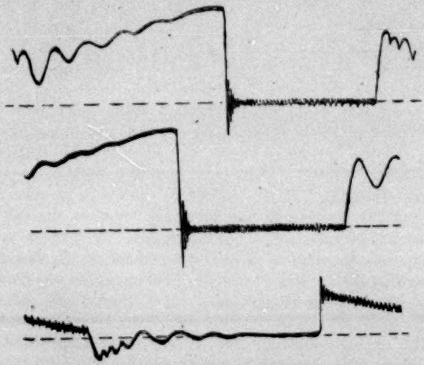

FIG. 15.—0·5 MM. PRIMARY. SPARKING ON MAKE.
FIG. 16.— „ „ NOT SPARKING ON MAKE.
FIG. 17.— „ SECONDARY. SPARKING ON MAKE.

With a gap of 0·5 mm. (Figs. 15, 16 and 17) the coil usually sparks on "make," causing a rapid rise of the primary current. The fact that the core is already magnetised by the secondary current which has not yet died away will also cause the primary current to rise very suddenly, and Fig. 16, in which there is no spark at make, shows that this is the principal cause of the sudden rise of the primary current. It appears from Fig. 17 that the "make" causes three or four sparks, whereas a torrent of sparks is maintained during the whole duration of the "break." Immediately after the break there are signs of one or two distinct sparks, followed by a period of great regularity in the current oscillations, from which it would appear that the phenomenon is similar to the musical arc, the frequency of the sparks being controlled by that of the oscillations due to the inductance of the coil and the capacity of the condenser across the mercury break. After a time this regularity ceases, and the sparks occur at increasing intervals until finally the primary circuit is again closed. In such oscillograms one is always rather sceptical—and rightly so—as to the instrument being properly damped. This was always checked by making and breaking the current through a non-inductive resistance; a typical result is shown in Fig. 18.

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* *See* Dr. Fleming, "Journal" of the Inst. of Elec. Eng., Vol. XLIV., p. 363.

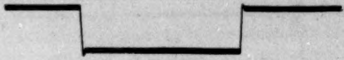
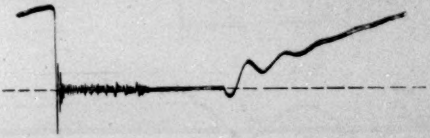


Fig. 18.



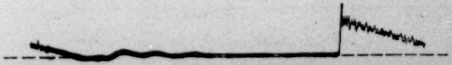
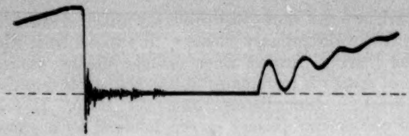
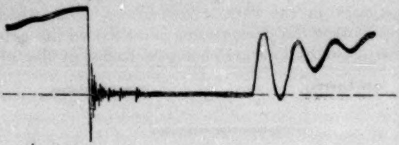


Fig. 19.—1 mm. Primary. Fig. 20.—1 mm. Secondary.

Oscillograms are given of primary and secondary currents as the spark-gap is increased in length from 0·5 mm. to 6·0 mm. and it is interesting to trace the gradual changes. It is only with the shortest gap that sparking occurs at make. In all cases the exact number of sparks occurring at break can be counted in both primary and secondary oscillograms. This was confirmed by spark photographs. Fig. 24 was taken with a 2 mm. gap, and should be compared with Figs. 22 and 23.





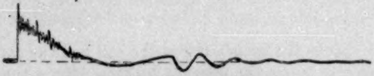


FIG. 21.—1·5 MM. PRIMARY.
FIG. 22. - 2·0 MM. PRIMARY.
FIG. 23.—2·0 MM. SECONDARY.

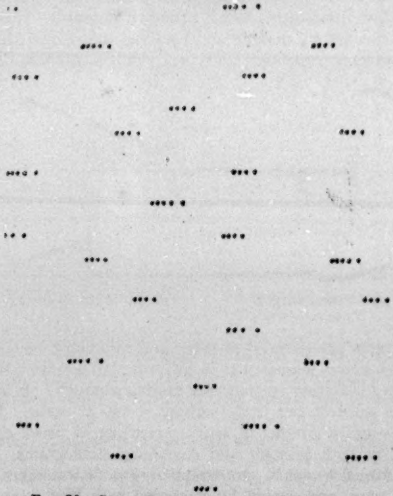


FIG. 24.—SPARK PHOTOGRAPH WITH 2 MM. GAP.

With a gap of 1·2 mm. the number of sparks varies between 8 and 10 per break, with 2·0 mm. between 4 and 5, and with a gap of 2·5 mm. between 2 and 3. It will be noticed that the successive sparks in each train die away in brilliancy, the first one being always especially bright, also the intervals between successive sparks gradually increase.

In all the oscillograms it will be noticed that there are oscillations of three distinct frequencies, occurring at various times. At make, the exponential rise of the primary current has superposed upon it an oscillating current which sometimes (Figs. 22 and 26) reaches a very large value. This oscillation ($\sim$=266) is equally prominent in the secondary circuit. It is due to the condenser K_1, which is connected across the secondary of the coil. The inductance, which must be taken into account when calculating the frequency of this oscillation, is not the full inductance of the secondary winding, but only its leakage inductance, as the primary coil is practically short-circuited through the mercury jet and the battery.

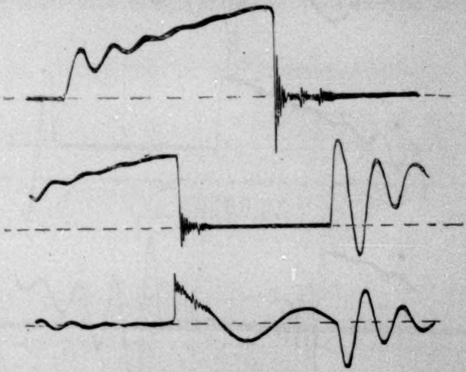
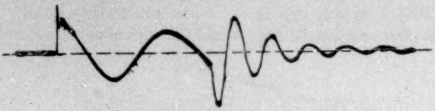
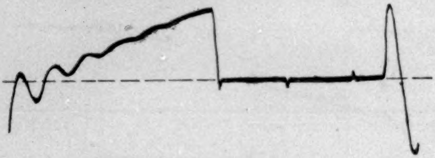


Fig. 25.—2·5 mm. Primary.
Fig. 26.—3·5 mm. Primary.
Fig. 27.—3·5 mm. Secondary.

There are other oscillations occurring only in the secondary circuit and of a considerably lower frequency—viz., 86 $\sim$ per second. They occur only when the primary circuit is open and, therefore, when the effective inductance of the secondary winding is not diminished by the latter being tightly coupled with the closed primary winding. It is interesting to note the immediate effect on the frequency of this oscillation of closing the primary circuit in Figs. 23, 27 and 28. It is not quite correct, however, to say that the primary circuit is open during the intervals of break, for the mercury interrupter has a condenser of about 0·6 mfd. across its terminals. This condenser is connected, through the battery, across the primary winding. The natural frequency of this circuit is so high, however, that it appears very "stiff" to the low-frequency oscillations which we are now considering, with the result that they do not appreciably affect it. To show this, the condenser across the interrupter was increased by an additional 9·1 mfd. The gap was very long, so that no sparking took





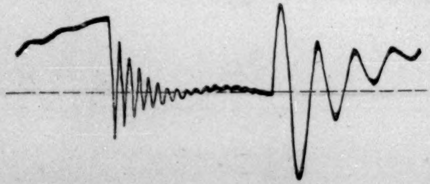


Fig. 28.—Secondary. Long Gap, no Sparking.

Fig. 29.—Primary. „

Fig. 30.—Primary. With increased Capacity across the Mercury Break.

place. The primary current was previously as shown in Fig. 29; it is now modified as shown in Fig. 30, where the presence of the low-frequency oscillation is quite evident. We have simply decreased the stiffness of the primary circuit.

More striking in this last oscillogram is the change in the high-frequency oscillation to which we have not yet referred. It occurs on break and in Fig. 30 is only just damped out when the make occurs. This is the natural oscillation of the primary winding with the condenser across the interrupter. By increasing the capacity of the condenser to 16 times its former value we have lowered the frequency to a quarter and have greatly increased the amplitude. In all the preceding oscillograms the frequency of this oscillation has been so high that one might have thought it due to under-damping of the oscillograph. The damping was, however, almost critical. In calculating the frequency of this comparatively rapid oscillation (under normal conditions $\sim = 4{,}250$) it must be remembered that the secondary winding is practically short-circuited by the glass-plate condenser K_1. The inductance involved is only that due to the leakage flux of the primary coil. The free period of an oscillatory circuit is approximately $2\pi\sqrt{KL}$, but, if the coil be coupled with another short-circuited coil, it can be shown that the free period is reduced to $2\pi\sqrt{KL}\sqrt{1-k^2}$, where k is the coefficient of coupling $\sqrt{\frac{M^2}{L_1L_2}}$. The periods of the various oscillations as determined from the

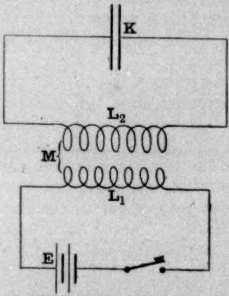


FIG. 31.

oscillograms agree closely with the values calculated by means of this formula from the measured inductances and capacities.

An interesting point in connection with the oscillograms is the gradual change which occurs in the oscillations of the primary current at make, as the length of the spark-gap is increased. As there is no spark at make (except at 0·5 mm.) this effect is rather unexpected. At 1 mm. the primary current sets off in a negative direction. At 1·5 mm. the start is positive, and the magnitude of the oscillation gradually increases with the length of the gap as far as 3·5 mm. The reason for these differences can be seen from a consideration of the secondary current curves. The amount of energy left in the secondary circuit when the spark quenches depends on the length of the gap. This energy oscillates in the secondary circuit with a frequency of 86 cycles per second, the primary circuit being open, except for the condenser across the interrupter. When the primary circuit is closed again, the frequency is changed to 266 cycles per second, and an oscillation is set up in the primary circuit with this frequency. This oscillation is superposed upon the exponential growth of the primary current. The direction of the initial current at make may be negative, as in Fig. 19 ; this depends on the time which has elapsed since the last spark and on the energy left in the coil when the last spark was quenched.

An appendix has been added to the Paper in which this is shown mathematically. Curves have been plotted from the calculated results to the same scale of current as applies to the oscillograms, and the results will be seen to agree very well. In Figs. 19 and 20 the make occurs at the moment when the secondary current is zero ; in this case the primary oscillations are superposed on an exponential curve rising from the base line.

In Fig. 23 the secondary current at the moment of make is a maximum ; as shown in Fig. 22, the primary current under these conditions consists of oscillations superposed upon an exponential curve, which starts from a point above the base line. In Figs. 28 and 29 the secondary current at the moment of make is approaching a maximum, but in the opposite direction, and the primary current curve is here displaced downwards. The result of this is that the final value reached by the primary current at the moment of break, and still more so the average value of the current, is much smaller in Fig. 29 than in Fig. 22. This explains the shape of the curves in Fig. 9, where the input to the coil is plotted for various spark-lengths ; in the oscillograph experiments six cells were used, and under these conditions the input is a maximum for a spark-length of 2 mm. The position of the maximum in Fig. 9 depends, therefore, on the time which elapses between the last spark and the subsequent make ; it will obviously vary with the number of cells employed, for with more cells sparking will continue longer, unless the spark-gap is lengthened. This explains the curves for 8 and 10 cells in Fig. 9 being displaced to the right.

It is obvious that the spark-length for which the input is a maximum will vary with the design of the mercury interrupter and with the speed at which it is run.

What has been said applies also to the high-frequency output curves, which follow generally the same shape as the input curves.

The effect of the oscillations in the primary current was evident in the relative values of the readings of the moving coil and hot-wire ammeters. With a gap of 0·5 mm. the ratio was 1·33, while with a gap of 3·3 mm. it had increased to 1·64.

Although in many cases the initial oscillation of the primary current exceeds in amplitude the final value of the current, it would be useless breaking the circuit at this moment, for the large primary current is almost neutralised by the secondary ampere-turns and the resultant magnetic flux is small.

APPENDIX.

It has been seen that the growth of the current in the primary circuit of the induction coil when the mercury interrupter closes the circuit depends on the magnitude and phase of the remanent oscillation in the secondary circuit of the coil at that moment.

Let L_1 and L_2 be the inductances of the primary and secondary windings of the coil and M their mutual inductance. Let E be the E.M.F. of the battery supplying the coil. The secondary circuit has an oscillatory current with an amplitude I, and the primary circuit is suddenly closed. Since we are mainly concerned with what happens at the moment of closing the switch we can assume that both R_1 and R_2 are zero. The equations for the two circuits may be written :—

$$L_2\frac{d^2q_2}{dt^2}+M\frac{d^2q_1}{dt^2}+\frac{q_2}{K}=0, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$L_1\frac{d^2q_1}{dt^2}+M\frac{d^2q_2}{dt^2} \qquad =E. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Eliminating q_1 we have

$$\frac{d^2q_2}{dt^2}\left(1-\frac{M^2}{L_1L_2}\right)+\frac{q_2}{KL_2}+\frac{ME}{L_1L_2}=0.$$

Putting $\frac{M^2}{L_1L_2}=k^2$, $\frac{1}{KL_2(1-k^2)}=\omega^2$ and $\frac{ME}{L_1L_2(1-k^2)}=a$,

we have
$$\frac{d^2q_2}{dt^2}+\omega^2q_2+a=0$$

and
$$q_2=A\sin(\omega t+\varphi)-\frac{a}{\omega^2},$$

$$i_2=\frac{dq_2}{dt}=A\omega\cos(\omega t+\varphi). \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From (2)
$$\frac{d^2q_1}{dt^2}=\frac{E-M\frac{d^2q_2}{dt^2}}{L_1}$$

$$=\frac{E+MA\omega^2\sin(\omega t+\varphi)}{L_1}.$$

$$\therefore \quad i_1=\frac{dq_1}{dt}=\frac{E}{L_1}t-\frac{\omega AM}{L_1}\cos(\omega t+\varphi)+B, \quad . \quad . \quad . \quad (4)$$

where A and B are constants to be determined from the initial conditions.

If, in the first place, the "make" be assumed to occur at the moment of maximum current I in the oscillating secondary circuit, then we have for $t=0$, $q_2=0$ and $i_2=I$.

$$\therefore \quad A\sin\varphi=\frac{a}{\omega^2} \text{ and } A\omega\cos\varphi=I.$$

$$\therefore \quad A=\frac{\sqrt{I^2+\frac{a^2}{\omega^2}}}{\omega} \text{ and } \tan\varphi=\frac{a}{\omega I}.$$

$$\therefore \quad i_2=\sqrt{I^2+\frac{a^2}{\omega^2}}\cos\left(\omega t+\tan^{-1}\frac{a}{\omega I}\right). \quad . \quad . \quad . \quad (5)$$

Hence, the oscillatory current is increased in amplitude whichever way the battery is switched in, but the change of phase depends on whether E is positive or negative.

$$i_1=\frac{E}{L_1}t-\frac{M}{L_1}i_2+B,$$

where B is a constant, such that when $t=0$, $i_1=0$.

$$\therefore \quad i_1=\frac{E}{L_1}t-\frac{M}{L_1}i_2+\frac{M}{L_1}I. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Hence, the primary current consists of three components—the first, directly proportional to the time (neglecting resistance); the second, a sine wave; and the third, a constant current. The first term may be positive or negative, and on this depends the phase displacement of the second term.

If, in the second place, the primary circuit be closed when the secondary current is passing through zero, the constants in (3) and (4) are such that for $t=0$, $i_2=0$; $q_2=\frac{I}{\omega}$; $i_1=0$.

$$\therefore \quad A=\frac{I}{\omega}+\frac{a}{\omega^2}, \ \varphi=\frac{\pi}{2} \text{ and } B=0.$$

$$\therefore \quad i_2=-\left(I+\frac{a}{\omega}\right)\sin \omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and

$$i_1=\frac{E}{L_1}t-\frac{M}{L_1}i_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

In this case the oscillatory current is unchanged in phase, but is increased or decreased in amplitude, depending on the direction of E as compared with the direction of the initial oscillation induced in the primary circuit. The amplitude changes from I to $I+\frac{a}{\omega}$, where a can be + or —, and

$$\frac{a}{\omega}=E\sqrt{\frac{K}{L_1}}\sqrt{\frac{k^2}{1-k^2}}.$$

Hence if the coefficient of coupling k be nearly unity, as it is in the case of the induction coil, the increase of amplitude of the secondary current and the oscillation set up in the primary circuit may be very considerable, as is shown by the oscillograms.

In Fig. 32 the four cases have been plotted. The values used in the calculation are as follows :—

$L_1=0{\cdot}02$; $L_2=570$; $M=3{\cdot}2$ henries ; $K=0{\cdot}005$ mfd. ; $E=12$ volts ; $I=0{\cdot}025$ ampere ; $k=0{\cdot}946$.

Although the only difference in the assumptions in the four cases is in regard to the phase of the secondary current at the moment of make, the differences in the nature of the oscillations produced are very great. In the actual case the oscillations are gradually damped out by the resistance and the term $\frac{E}{L_1}t$ becomes an exponential term, rising finally to $\frac{E}{R_1}$.

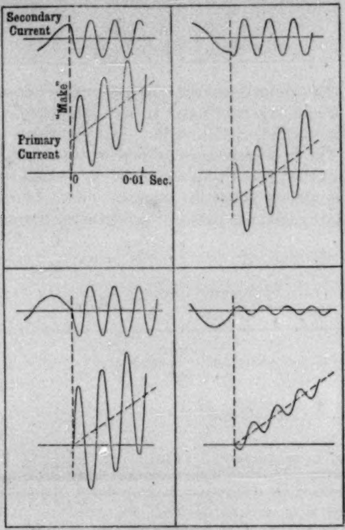


Fig. 32.

ABSTRACT.

The apparatus used was similar to that employed in small radiotelegraph stations. A 10 in. induction coil, operated from cells through a mercury interrupter, supplied power to an oscillatory

circuit containing a spark-gap between spherical electrodes. Coupled to this circuit was another oscillatory circuit representing the aerial, and containing a variable resistance which constituted the high-frequency load. The input, output and efficiency were determined for various degrees of coupling, various aerial decrements, different lengths of spark-gap and with various primary voltages, the object being to determine the effect of these various factors on the working of a small radio-telegraph station.

A second part of the Paper dealt with some oscillographic records of the primary and secondary currents of the induction coil. These showed many points of interest, and explained several peculiarities in the working of the coil, especially the variation of input with spark length. This was shown to be due to the particular phase at the moment of "make" of the remnant current in the primary induced in it by the oscillations in the secondary produced by the last spark; this depended on the time which had elapsed since the last spark.

XLV. *The Self-Demagnetisation of Annealed Steel Rods.* By S. W. J. Smith, *M.A., D.Sc., and* J. Guild, *A.R.C.S., of the Imperial College of Science.*

Read June 28, 1912.

Almost 50 years ago von Waltenhofen observed that short rods of soft iron sometimes exhibit residual magnetism which is anomalous in the sense that the feeble residual polarity is of opposite sign to that existing before the magnetising field is withdrawn. In the course of experiments upon the thermomagnetic properties of iron-carbon alloys we have observed another kind of anomalous residual magnetism which seems to have escaped notice hitherto. It is more marked than von Waltenhofen's phenomenon, being easily perceptible in comparatively slender rods, but its existence is not detected until the temperature is raised some 200°C. above that of the air. It does not occur in soft iron, but in annealed steel containing about 0·3 per cent., or more, of carbon.

All the rods we used were 7 cm. long and 0·55 cm. in diameter; for convenience in the measurement of temperature each of them had a cylindrical hole, 0·3 cm. in diameter, drilled through it axially from end to end. One of the rods was constructed out of steel containing 0·85 per cent. of carbon described in a previous Paper ("Proc." Phys. Soc., XXIV., p. 62, 1911). The others were made from materials, numbered 1 to 6 below, for which we are indebted to Mr. E. A. Wraight, of the Royal School of Mines, who also supplied the analyses which follow :—

No. of Rod.	Percentage of C.	Percentage of Mn.	Percentage of Si.
1	0·15	0·20	0·09
2	0·36	0·20	0·11
3	0·60	0·21	0·17
4	0·71	0·22	0·17
5	1·10	0·23	0·17
6	1·53	0·235	0·165

There were traces of sulphur (0·01 per cent. and under) and of phosphorus (0·03 per cent. and under) in each of the steels.

The phenomenon to which we wish to draw attention was shown by the rods when in the condition following slow cooling from about 850°C. It occurred to a greater or less extent in all except that containing the lowest percentage of carbon. The properties of the rods when hardened by quenching have also been observed but will be omitted from this Paper, which is not intended to be more than a preliminary account of the work.

After being annealed, each of the rods was magnetised as nearly as possible to saturation by means of a powerful electromagnet, the field current being reduced by steps of similar magnitude in each case before the magnetised rod was removed. In one case the magnetising current was reduced much more gradually than in the others without producing any important difference in the subsequent behaviour of the rod. Unlike the Waltenhofen phenomenon, therefore, the present effect occurs when the removal of the field is gradual.

The residual magnetism was measured magnetometrically, each rod being placed at right angles to the meridian within a copper tube which could be heated from below by means of a row of small gas jets. The temperature was determined by means of a calibrated platinum-rhodium thermo-couple placed within the rod.

Fig. 1 gives a general idea of the effects observed in the different cases, at the air temperature and during subsequent heating, the ordinates being the approximate residual intensities in absolute units at the temperatures represented by the abscissæ.

In every case, except that of the 0·15 per cent. C steel, the residual magnetism changes sign when the rod is heated sufficiently. It diminishes rapidly from the first and continues to fall until, at a temperature depending upon the percentage of carbon, and lying between 180°C. and 200°C., the residual moment becomes zero. Further heating produces a negative moment which reaches a maximum value in the neighbourhood of 220°C. and then slowly falls, becoming zero again in the neighbourhood of 750°C.

It is with the possible significance of this negative magnetism that we wish to deal.

It will be noticed at once that it is dependent upon the presence of carbon. A rod of the purest iron obtainable commercially showed no trace of the phenomenon, its residual magnetism diminishing slowly and continuously as the temperature rose up to and above 260°C. In the rod containing 0·15 per cent. C the magnetism never becomes negative, but the effect of the presence of carbon is shown by the very distinct change in slope of the magnetism-temperature curve which occurs near 200°C., as in the steels containing higher percentages of the added element.

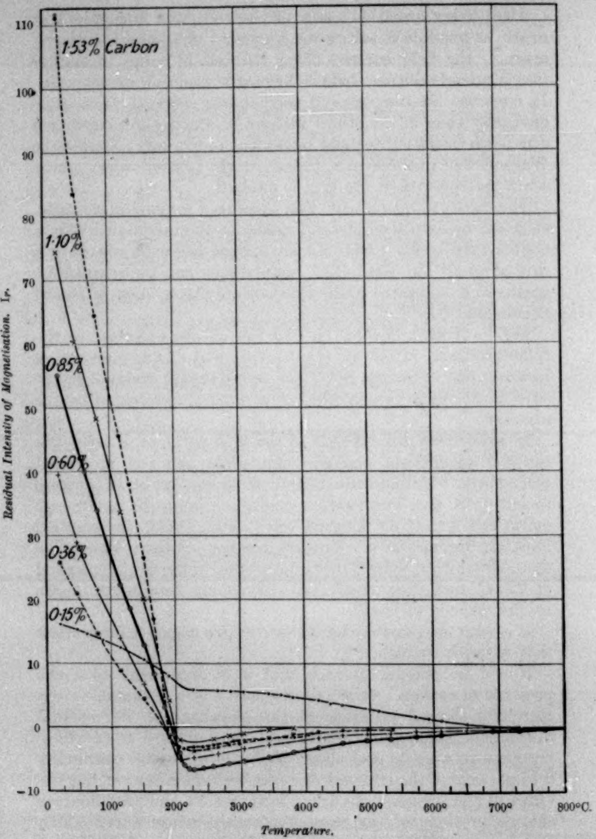

1·53% Carbon
1·10%
0·85%
0·60%
0·36%
0·15%
Residual Intensity of Magnetisation. Ir.
110
100
90
80
70
60
50
40
30
20
10
0
−10
0
100°
200°
300°
400°
500°
600°
700°
800°C.
Temperature.

FIG. 1.

It will also be seen from Fig. 1 that the amount of negative magnetism exhibited does not increase continuously as the percentage of carbon in the steel rises. The alloy containing 0·85 per cent. C shows more than any of the others.

According to present views, annealed carbon steels consist almost entirely of iron and iron carbide. When the percentage of carbon is about 0·89 the steel is an intimate " eutectoid " mixture of the two components. With lower percentages there is " free " iron, with higher percentages " free " carbide, present along with the eutectoid mixture.

It appears, therefore, that the effect is most marked in the so-called " saturated " steel, in which the development of the " ferrite-cementite " eutectoid is a maximum.

While the effect does not occur in the absence of carbon, it nevertheless seems certain that the negative magnetism is due entirely to the iron.

It has already been shown by Messrs. White and Barker, and one of us (*loc. cit.*, p. 64), that the ferromagnetism of iron carbide becomes relatively very small above 230°C. and it will be noticed that the negative magnetism reaches its maximum value in each steel at a temperature not far short of this value. Below 230°C., any positive polarity that the carbide may possess will be diminishing continuously ; above it, the polarity will be very small and any variation of residual magnetism that is observed will be due, mainly at least, to the iron. It is unlikely that the negative polarity exhibited by the iron increases much with rise of temperature. Consequently it would seem that the positive polarity shown by the rods at the lower temperatures must be due to the positive magnetism of the carbide.

The facts, therefore, suggest the following view of the course of events within each rod. Withdrawal of the magnetising field leaves the rod subject to the influence of its own self-demagnetising force which, at first, is considerable. This force, acting upon the iron and upon the carbide, tends to reverse the polarity of both. The alignment of the iron is more easily disturbed than that of the carbide and a greater amount of reversal is produced in the former. The reversing field diminishes continuously and, when it becomes incapable of producing further change, the feeble residual polarity is made up of a small positive polarity due to the carbide and of a still smaller negative polarity due to the iron.

When the rod is heated the polarity of the carbide diminishes more rapidly than the opposite polarity of the iron for several reasons. One of the most important is that the carbide is approaching its transition temperature more rapidly than the iron, and another is that, until the zero moment in the neighbourhood of 200°C. is passed, the demagnetising field tends to maintain the polarity of the negatively magnetised constituent.

The initial parts of some of the curves of Fig. 1 are comparatively steep, and suggest that the material is at first in a relatively unstable condition. This suggestion is confirmed by the fact that, when a series of observations is made upon the same specimen, the values of the initial residual magnetism vary over a considerable range while corresponding values at higher temperatures are much more nearly equal. The data, so far as we have examined them, suggest that this initial instability is due mainly to the carbide and that a small disturbance is sufficient to produce a relatively considerable change in the polarity of this constituent.

We have endeavoured to represent in a single diagram (Fig. 2) a number of observations we have made with the object of throwing further light upon the changes occurring within the material when subjected to various disturbances. The experiments were made upon the steel containing 0·85 per cent. of carbon. The thick-lined curve ABC represents the mean behaviour of the material during several successive heatings from 20°C. to 800°C., the different series of observations being represented by distinctive marks. One of the first of the additional experiments consisted in heating the material to 135°C. (the point marked A in the figure), and then allowing it to cool. The dotted curve A*a* represents the return path. In another experiment the material was heated to 350°C. The return path in this case is represented by B*b*. The specimen was next heated (after remagnetisation at the air temperature as before) to 590°C., and behaved during cooling as represented by the curve C*c*. When the heating was continued to 800C°. there was no appreciable return of magnetism on cooling.

In another experiment, after remagnetisation, we heated the specimen to 245°C. We now destroyed the residual negative magnetism by means of a solenoid wrapped round the tube containing the rod and remagnetised the latter by means of this solenoid so that its residual polarity was positive. The value is represented by the point D in the figure. The temperature was now raised to 380°C., the residual positive magnetism falling as represented by the curve D*d*′. The rod was then allowed to cool to the temperature of the air, with the result shown by the curve *d′d*. It will be seen that there was no perceptible change until about 240°C. The magnetism then began to rise rapidly as shown. Between 240°C. and 20°C. the curve *d′d* is practically identical in form with the corresponding part of the curve C*c*. This suggests that the magnetism which

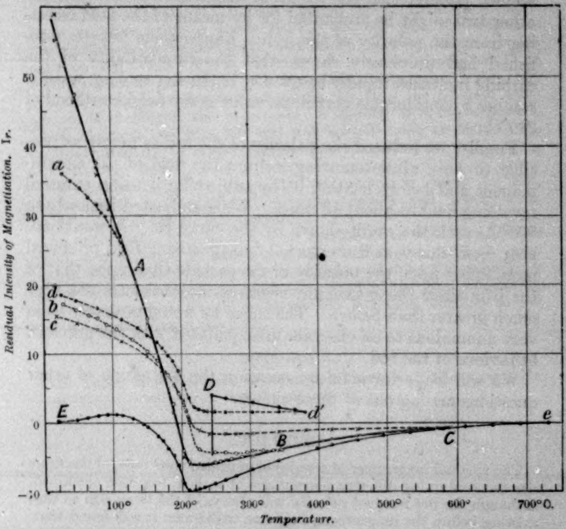


FIG. 2.

returns below 240°C. in each of these cases is mainly due to the carbide. Experiments with the soft iron rod showed that the increase in the magnetism of iron, over this range, is relatively small.

It is noteworthy that below 210°C., which is approximately the " critical " temperature of the carbide, the curves beginning at B, C and *d′*, respectively, become nearly the same in form if the ordinates are reckoned from base lines passing through the readings at 240°C. They are also very similar to the curve given by Weiss as characteristic of the ferro-magnetic state when the temperatures are expressed as fractions of the critical temperature. Spontaneous return of magnetisation in the carbide, after it has been heated much above its critical temperature, is suggested by the curves C*c* and *d'd* considered together. Until we had obtained the latter curve it seemed possible that the return of magnetisation shown in the other two might be accounted for by means of the field resulting from the polarity of the iron. Experiments not yet published had previously shown that the susceptibility of the carbide increases rapidly below 230°C., during cooling, until it reaches a considerable maximum value in the neighbourhood of 210°C.

Finally, we reduced the polarity of the rod as nearly as possible to zero, after removing it from the field of the electromagnet and before heating it, by subjecting it to an external reversing field of about 15 units. We then heated it slowly to 800°C., with the result shown by the curve E*e*. It would appear from this that the external demagnetising field produced more effect upon the polarity of the carbide than upon that of the iron since the maximum negative magnetism is not very much greater than before. The curve E*e* would probably seem very anomalous to an observer unacquainted with the previous behaviour of the rod.

We will leave for a future occasion the discussion of other questions arising out of these curves.

ABSTRACT.

The residual magnetism of a series of annealed steel rods, of the same size but containing different percentages of carbon, has been examined. Although the net residual polarity was always of the same sign as that existing before the magnetising field was withdrawn it was found that, except when the percentage of carbon was small, the residual polarity of the *iron* was apparently of opposite sign. The observed positive polarity owed its sign to the carbide. This was shown by heating each rod until its temperature was above that at which the ferro-magnetism of the carbide practically disappears. The residual polarity could then be due to the iron only and was in every case negative. Moreover, the rate of growth of the negative magnetism as the temperature rose corresponded with the rate of decay of the magnetism of the carbide which has been examined in an earlier paper. The effect appears to be most marked in "saturated" or "eutectoid" steel, and is an example of the superposition of magnetic properties presenting several points of interest. Some of these are described.

The constituents, iron and iron carbide, are easily traceable in annealed steel, owing to the differences between their magnetic pro-

perties. The ferro-magnetic transition point of the carbide is about 500°C. lower than that of the iron. The carbide is also magnetically harder at ordinary temperatures and possesses greater coercive force, although, like iron, it is magnetically very soft at temperatures near the transition point. In consequence of these facts, the effect of heat upon the residual magnetism of an annealed steel rod is peculiar and at first sight mysterious. As the temperature rises the residual magnetism falls continuously until it becomes zero in the neighbourhood of 200°C. It then changes sign and reaches a maximum negative value at about 220°C. Beyond this, the negative magnetisation decreases slowly, and finally becomes imperceptible between 700°C. and 800°C. If the rod is cooled from 800°C. it remains without perceptible polarity as the temperature falls; but if the heating is interrupted before the whole of the residual magnetism is destroyed the behaviour on cooling is quite different. Thus, to quote one case, the rod was heated until, at 600°C., the residual intensity of magnetisation was about —0·5. On cooling the intensity increased to a maximum negative value of about −1·6 at about 245°C. Then the magnetisation began to fall, reached zero at about 210°C., became positive and, finally, was about +15·5 at the air temperature.

An explanation of these and other results which were described is given in the Paper, in which it is shown that the residual magnetism of short annealed steel rods is determined by the retentivity of the carbide, and that the residual polarity of the iron is negative. The iron may thus be said to contribute less than nothing to the residual magnetism of the rods.

DISCUSSION.

The Paper was discussed by Profs. Mather and Howe and Mr. Campbell, to whom the Secretary replied.

XLVI. *The Resistance to the Flow of Water along a Capillary Soda-Glass Tube at Low Rates of Shear.* *By* A. Griffiths, D.Sc., *and* Miss C. H. Knowles.

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§ 1. Introduction and experiments on the relative flow along two tubes in parallel along which the water is flowing at different rates of shear.
§ 2. Absolute measurement of the viscosity of water at low rates of shear.
§ 3. General remarks and results.

§ 1. In the course of some experiments on the viscosity of liquids at low rates of shear, Prof. W. Duff (" Phil. Mag." 1905, Vol. IX., p. 685) observed that the resistance to the flow of water in a capillary glass tube at low rates of shear increased with the time that the water had been stationary in the tube, and he concluded that " very minute quantities of the constituents of glass dissolved out by the water have very large effects on the viscosity at low rates of shear."

Some years later L. E. Gurney (" Phys. Rev." 26, pp. 98–122, Jan., 1908) allowed water to remain in contact with powdered glass for a week, and then, by a method involving the employment of a rotating cylinder found, that the value of the coefficient of viscosity was equal to that of pure water within the limits of experimental error.

Early in 1911 one of the authors of this Paper concluded from some unpublished work that the apparent value of the coefficient of ordinary distilled water in a capillary glass tube might be anything from the normal value to infinity ; but that the value of the coefficient in the case of air-free distilled water always remained constant.

The objects of the conjoint research were :—(*a*) To discover why ordinary distilled water in soda glass tubes developed in the course of time apparently anomalous values of the coefficient of viscosity, and, if possible, to eliminate the cause ; (*b*) to determine the absolute value of the viscosity at extremely low rates of shear.

The flow at low rates of shear was measured by a modification of the method described in the "Proc." Lond. Phys. Soc., Vol. XXIII., Part III., April, 1911, the coloured index in the experiments of the present Paper consisting of a solution of one quarter of a gram of *uranine* to one litre of water.

To the authors, Duff's explanation of the remarkable variation in the viscosity of water seemed reasonable. He mentions in his Paper that glass dissolves in water; and he thinks that "the large effect produced may be due to the tendency of silicic acid to separate out from the solution in a gelatinous form." It was thought by the authors that, if two tubes of unequal bore were arranged in parallel it was very probable that no obstruction would form in the tube along which the flow was relatively rapid; and it was hoped that the wide tube would act as a standard of reference for the narrow one.

Two carefully calibrated tubes, one of about 2 mm. diameter, the other of about $\frac{1}{3}$ mm. diameter, each about 1 metre in length, were connected in parallel. The flow along the fine capillary was measured by means of a coloured index, and the combined flow along the two tubes was determined by a method of weighing. The ratio of the total flow of ordinary distilled water along the two tubes to that through the narrow one was less than that calculated from the dimensions; and the ratio diminished in the course of a fortnight to one-third of the calculated value. Careful inspection showed the presence of fungoid growths in the wider tube, which doubtless flourished there, on account of the greater supply of food and air provided by the relatively rapid stream.

To sterilise the apparatus it was washed with a solution of corrosive sublimate. But there was still a deposit of some sort on the walls of the wider tube. This was removed by washing first with strong sulphuric acid which charred the deposit and then by passing a solution of potassium bichromate in strong sulphuric acid until the carbonaceous residue was removed.

That fungoid growths are liable to develop in distilled water is well known; but it was not known to the authors that they grow so easily and are practically invisible inside a glass tube.

To prevent their growth one drop of a 20 per cent. aqueous solution of copper nitrate was added to a litre of air-free distilled water. The experiments with the tubes in parallel were repeated. These experiments showed no signs of any decrease in the ratio of the total flow to the flow in the capillary with increase of time. A careful experiment performed in a thermostat-room after water had been in the tubes for a week gave a ratio 1·1 per cent. less than the calculated value. This result, as the tubes were not tested for circularity, was quite as near as could be expected.

To the authors it seems very probable that Duff's remarkable results were due to organic growths.

§ 2. *Absolute Measurements.*—Although it was probable from the experiments described in the preceding section that the viscosity of sterilised water is normal at low rates of shear, even when the water has been in contact with the glass for a long time, still it was just possible that the water in each tube might have been affected to the same extent. It was therefore decided to make absolute measurements by a modification of Duff's method.

The apparatus consists of two large cylindrical vessels of approximately the same cross-section joined by a capillary tube of length l, and of radius r. Let h represent the initial difference in level between the surfaces of the water in the tanks, t the time, η the coefficient of viscosity of the liquid, and d the density; then it can easily be proved that η equals hgr^2t/ld to an accuracy of one-tenth per cent. in the case of the experiments described in this Paper.

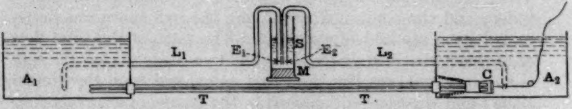

FIG. 1.

The two large tanks are shown at A_1 and A_2 (Fig. 1.). T is the tube through which the flow of water under the test takes place. T enters the tanks as close to the bottom as possible, as in this case the head will be scarcely affected by temperature changes.

The tube T can be closed by a copper cap C (Fig. 2), which is fitted with a rubber pad. The cap is held pressed against the end of the tube by means of three spiral springs. To open the tube the cap can be pulled back by means of a cord of fish-line, which passes under a horizontal bar, H. In order to keep the tube open a small ring, R, attached to the cord is slipped over a pin, P, on the horizontal bar H.

In order to bring the two surfaces of the water to approximately the same level the tanks are connected by means of a levelling arrangement, which consists of two tubes, L_1, L_2 (Fig. 1)—really parts of the same tube cut in the middle—the ends, E_1, E_2, of which dip into a vessel, S, containing a layer of mercury, M, covered with water. The levelling arrangement is placed in or out of action by adjusting the height of the mercury. The average diameters of the two tanks are 33·717 cm. and 33·773 cm. respectively; the length of the tube T is 130·175 cm.; the value of r^2 is 0·0004376. The value of r^2 was found by nearly filling the tube with mercury, and obtaining the weight of a column of measured length. The cross-section was assumed to be circular. The tube T was carefully calibrated by means of a thread of mercury when in position on the black scale of the apparatus. Mr. J. M. Scholer, B.Sc., assisted in the design of the apparatus and constructed it.

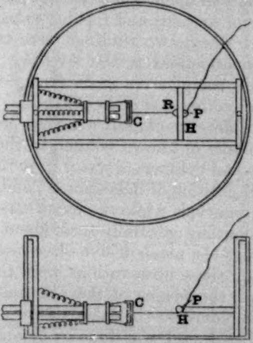

FIG. 2.

The series of operations requisite for the determination of the coefficient of viscosity usually takes two days to complete, the first day being occupied with cleaning the tanks and getting the index into a suitable position for the viscosity experiment proper. It is unwise to hasten the proceedings, for if the index be moved too quickly its ends become diffuse. The following gives the plan of work when the viscosity is determined as early as convenient after the water is introduced into the capillary tube. A very slight modification is involved when the viscosity is determined after the water has stood a long time in the tube.

I. Tanks cleaned with caustic soda, dilute nitric acid, and water, chamois leather being used at the finish to clean the walls of the tanks.

II. Index introduced and apparatus usually left all night.

III. Water removed from tanks and glass cleaned, as in I.

IV. Water put in tanks to about 10 cm. depth; leveller opened; stirrers added; water stirred every half-hour for the next three-and-a-half hours.

V. Levelling device closed; a reading taken of the position of the index; and seal opened at carefully noted time. Index allowed to move for an hour and reading taken of the index at end of an hour. These two readings were taken in order to allow for any imperfections in the levelling. The movement was always small, being never more than 0·8 mm. per hour In general the correction was quite negligible

VI. A known difference of head was next produced either by adding copper blocks of known volume to the left tank or by withdrawing unequal volumes of water from the tanks.

VII. Time and position of index noted; and further readings taken at regular intervals while the index was travelling along the tube. The journey generally took about 5 hours. Each reading was always the mean of five observations. When the index was moving these observations were taken at carefully noted times; and the means of the observations of position and time were employed in the calculations.

The stirring of each tank was done by means of a small plate of copper, 1 cm. square, attached to the end of a copper wire, which was fastened to a glass rod supported horizontally on the edge of the tank. When the stirring was over the wire always dipped to the same extent in the water, and there was therefore no disturbance of the head due to any change in the quantity immersed. It was proved experimentally that the stirring itself had no appreciable effect on the resultant flow. The water in all the experiments contained approximately one drop of a 20 per cent. solution of copper nitrate to a litre of water. The results obtained are given in the table opposite :—

§ 3. *General Remarks and Conclusions.*—The way in which the calculated value of the coefficient of viscosity varies in an individual experiment proves that there is a disturbing

	Length of time water had been in the tube.	t in seconds.	d in centimetres.	Times of stirring.	Method of producing head.	Head in centimetres.	Temp. C°.	η.
1	1 day ...	3,026 6,072 8,668 13,760	10·58 21·60 30·87 49·13	Stirred just before first reading in operation V.	By adding copper blocks to Tank A_1.	0·0954375	15·54	0·01126 0·01106 0·01105 0·01102
2	12 days ...	3,558 6,910 15,472 18,945	12·43 24·34 54·89 67·30	Stirred just before first reading in operation V.	By adding copper blocks to Tank A_1.	0·09472	15·39	0·01118 0·01109 0·01100 0·01099
3	1 day ...	3,484 8,381 14,825 19,014	12·41 29·51 52·14 66·88	Stirred after first reading of V. and after blocks had been introduced.	By adding copper blocks to Tank A_1.	0·09472	15·44	0·01096 0·01109 0·01111 0·01110
4	5 days...	3,694 9,034 14,711 18,008	13·04 31·62 51·74 63·41	Stirred just after tube was first opened in V. and just after introduction of blocks.	By adding copper blocks to Tank A_1.	0·0955	16·03	0·01115 0·01125 0·01119 0·01118
5	1 day ...	3,408 9,006 15,079 20,750	12·04 31·73 52·91 73·00	Stirred every 20 minutes throughout experiment.	By adding copper blocks to Tank A_1.	0·09472	15·70	0·01105 0·01109 0·01113 0·01110
6	5 days...	3,629 7,818 14,275 18,896	12·80 27·52 50·30 66·69	Stirred every 20 minutes throughout experiment.	By adding copper blocks to tank A_1.	0·09472	15·85	0·01110 0·01109 0·01109 0·01107
7*	1 day ...	3,539 7,321 14,434 18,447	12·42 25·89 50·81 65·37	Hourly	By adding copper blocks to Tank A_2.	0·09504	15·82	0·01116 0·01109 0·01113 0·01106
8*	5 days...	5,447 8,953 18,109	19·82 32·42 65·72	Hourly	By adding copper blocks to tank A_2.	0·09444	16·37	0·01070 0·01075 0·01073
9†	1 day ...	6,140 13,269 18,171	26·98 58·49 80·21	Hourly	By withdrawing 100 cc. water from A_1, 200 cc. water from A_2.	0·1157	16·89	0·01085 0·01082 0·01080
10†	1 day ...	8,437 15,069 16,867	36·21 64·67 72·07	Hourly	By withdrawing 100 cc. water from A_1. 200 cc. water from A_2.	0·1139	16·43	0·01095 0·01094 0·01099
11†	4 days...	4,689 12,123 18,000	20·25 52·30 77·81	Hourly	By withdrawing 100 cc. water from A_1. 200 cc. water from A_2.	0·1124	16·72	0·01087 0·01089 0·01087
12‡	10½ hours before first reading of VII.	4,219 7,424 20,170	15·33 26·75 72·96	Hourly	By adding copper blocks to Tank A_1.	0·09403	16·80	0·01067 0·01076 0·01072

* Index moved from Tank A_2 to Tank A_1; that is in the opposite direction to its motion in the previous experiments.

† The water was removed with a pipette and run into a flask. Its volume was then found by weighing. In experiments (10) and (11), 100 cc. of water were removed from each tank before the first reading of V.

‡ In this experiment the whole of the work, usually extendihg over two days, was done continuously in one, so as to reduce to a minimum the time that the water was in the tube.

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influence at work which sometimes accelerates the flow and sometimes retards it.

The disturbances were much greater in some preliminary experiments. In one of the earliest experiments with tanks of 18·9 cm. diameter, and with a capillary tube of about 1 mm. diameter, the initial head was 0·004 cm., and the water was covered with a layer of oil. The index moved one-third the distance expected, slowly returned and nearly came to rest, and then shot out of the tube in a direction opposite to that in which it entered. In another experiment the initial head was the same, no oil was used, and the surfaces were not stirred; the index moved one-half the calculated distance at a reasonable speed and then slowly crept forward. There is no doubt that, as found by Duff, the stirring is very important. But the experiments of this Paper suggest that, however carefully the stirring be done, surface effects must always be reckoned with. To fix the ideas experiments (9), (10) and (11) may be considered. Here the initial head is 0·114 cm. The head which the surface tension is capable of exerting, if the angle of contact of the water surface with the glass is zero, is given by the formula $2T/gr$, where T is the surface tension and r is the radius. Taking T as equal to 81 dynes per centimetre, the effective head is 0·0098 cm. Thus the head which the surface is capable of supporting is about 8·5 per cent of the actual head employed, and it can easily be proved that a change in the head equivalent to a drop in the value of T of 9·5 dynes per centimetre, not at all a large amount, would cause an apparent change in the viscosity of 1 per cent. These figures are suggestive rather then directly applicable; more important would be the calculation of the instantaneous variation in the flow due to a change in the surface tension.

Since the percentage error due to a mistake in the reading of an index diminishes with increase in the distance (d) travelled, the last calculated value of the coefficient in each experiment is considered to be the most reliable. On account of the fact that the experiments were generally performed in pairs (1 and 2, 3 and 4, 5 and 6, 7 and 8, 10 and 11), in which all the conditions and operations were approximately the same, with the exception of the time that the water had been in the tube, it is possible to test the effect of prolonged contact with the glass. The mean of the calculated values of the viscosity after prolonged standing of the water is only one-tenth per cent. less than the mean of the values obtained with water that had stood in the tube for one day only. In experiment (12) the time of standing was reduced to a minimum, and the value of the viscosity is substantially the same as that given by the other experiments.

Of the results given in the tables the greatest weight must be given to experiments (9), (10) and (11), in which the stirring is most thorough, and in which the difference of level is caused by lowering the level of the water in each tank, thus increasing the chance of the glass being covered with a layer of water. The maximum rate of shear in these experiments is one radian per second, *i.e.*, the rate of shear is about one-fifth of that at which Duff worked. The mean apparent value of the coefficient of viscosity at an average temperature of 16·68°C. is 0·01089; whilst the value of the coefficient at the same temperature derived from Hosking's formula ("Phil. Mag." Vol. XVIII., p. 262, 1909) is 0·01093. It may be mentioned that some careful determinations of the coefficient when the rate of shear is 1,300 radians per second give a value of the coefficient 0·6 per cent. greater than that given by Hosking's formula. This result proves that the cross-section of the tube is nearly circular.

The authors conclude (*a*) that the apparent increase in the viscosity of ordinary distilled water in soda-glass tubes developed in course of time is due to organic growths. (*b*) That consistent values of the viscosity may be produced by sterilising the water by the addition of a trace of copper nitrate. (*c*) That with the apparatus described surface effects prevent the attainment of a high degree of accuracy in the determination of the absolute value of the coefficient of viscosity; but that, nevertheless, the experiments prove that the viscosity of water when the rate of shear is 1 radian per second is approximately the same as at high rates.

ABSTRACT.

In 1905 Prof. W. Duff concluded that the viscosity of water at low rates of shear increases when the water is allowed to stand in a capillary glass tube. He employed Poiseuille's method. Later, L. E. Gurney, by a method involving the use of a rotating cylinder, found that there is no increase in the viscosity of water which has stood in contact with powdered glass for a week.

The authors, working at Birkbeck College, in the City of London, find that almost invisible fungoid growths develop in ordinary distilled water in glass tubes, and they suggest that organic growths were probably the cause of Duff's results. They detect no increase

in the viscosity of water that has stood for days in a glass tube when the water is sterilised by the addition of a trace of copper nitrate. Their apparatus is a modification of Duff's, and consists of a calibrated capillary tube 130 cm. long of ⅓ mm. bore, joining two cylindrical tanks of ⅓ metre diameter. The difference of head is about 1 mm. The flow is measured by means of a coloured column, consisting of a feeble aqueous solution of uranine.

The following Papers were read at the Joint Meeting of the Society *with the* Optical Convention *on June* 21, 1912. *They are here printed in lengthy abstract only, but will be found in full in the* Proceedings of the Optical Convention, 1912.

XLVII. *The Trend of Geometrical Optics.* *By* Silvanus P. Thompson, *D.Sc.*, *F.R.S.*

The purpose of this communication is to explain certain points in the methods of geometrical optics, to describe certain models based thereon, and to raise some questions as to notation and nomenclature.

The method of Gauss for treating the properties of an optical system of central lenses by means of his conception of points and planes is now all but universally accepted. It presents a scheme for demonstrating the performance of an instrument on the assumptions, not only that the aberrations of definition of the image are absent, but also that the well-defined image is devoid of curvature and distortion—in other words, that the optical system is both stigmatic and collinear. It furnishes the means of finding the image-point which is conjugate to any object-point, without any consideration of the particular refracting surfaces or thicknesses of the lenses, for it reduces all centred optical systems, whether the refracting surfaces be spherical or aspherical, to a species of equivalent lens, of which it is needful merely to know the two principal foci and the two principal points (Haupt-punkte), together with the respective focal planes and principal planes drawn orthogonally through them. The width between the principal planes (constituting, so to speak, the reduced thickness of the equivalent lens) being known, and the focal lengths as measured from the principal planes to the respective principal foci being also known, the whole performance of the optical system can be stated in the simplest way in terms of these data. For the width between the principal points and planes simply does not count in the calculations ; and if the distances of object-plane and image-plane measured from the respective principal planes are reckoned, as in ordinary geometry, as positive if measured to the right and negative to the left, the following relations hold good :—

$$\frac{1}{u}+\frac{1}{f}=\frac{1}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$-pq=f^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$M=v/u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where f is the principal focal length (assuming the first and last media to be alike, both air) ; u the distance of the object-plane from the first principal plane ; v the distance of the image-plane from the second principal plane ; p the distance of the object-plane from the first principal focal plane ; q the distance of the image-plane from the second principal focal plane ; M the linear magnification. In either form the equations (1) and (2) are simply an expression of collinear relationship.

To these, the original features of Gauss's scheme, Listing added the two nodal points which coincide with the two principal points only if the first and last media are alike (both air), but otherwise are displaced toward the side on which the end medium is the denser. To the system of points thus conceived Casorati assigned the name " cardinal " points. To them Toepler added another pair, which he termed " negative principal points," but which, for reasons presently to be discussed, I prefer to call the " symmetric " points. These points are situated at distances respectively equal to one focal length beyond the principal foci on the two sides. They are conjugate to one another. Planes drawn orthogonally through them are such that they are planes of unit magnification, an object in one symmetric plane having its image in the other, of equal size, but inverted. Toepler also showed that if, on the image side of the lens, planes are drawn at successive distances apart equal to the focal length, they constitute a family of planes in which the magnifications of the images formed therein differ by unity successively from one another. Mr. Blakesley has further developed this plan, and, as he attributes negative signs to the magnifications of inverted images, he would write the successive magnifications, beginning with the plane through the principal focus, as 0, −1, −2, −3, &c. The two principal planes of Gauss are also conjugate to one another, and are planes of unit magnification without inversion. The principal focal planes are, of course, respectively conjugates to planes at $+\infty$ and $-\infty$. The name symmetric planes is apt, not only because these two planes lie symmetrically right and left of the system, but because of the symmetry which there exists between object and image. Any point in the first symmetric plane, having co-ordinates *yz* in that plane, will have as its image a point having co-ordinates $-y$, $-z$ in the second symmetric plane. This circumstance lends itself to the graphic solution of the problem how to find the position, after refraction, of any given oblique ray, and is applicable even in cases where the given ray is askew in direction and never meets the axis of the system. Let Fig. 1 depict the system, having principal points E_1, E_2, principal foci F_1, F_2, and symmetric points S_1, S_2. And let six planes be drawn orthogonally through these six points. Then let it be required to find the course of a ray of which the part PP′ is given. Produce PP′ both ways, intersecting the first symmetric plane in *a* and meeting the first principal plane at *j*. The ordinate of *a* being *y*, set off the conjugate ordinate $-y$ in the second symmetric plane, thus fixing the conjugate point *b*, through which the ray must pass. Transferring the ray across the equivalent thickness *jk* between the principal planes, join *k* to *b* by the line *kb*, which is the refracted ray sought. A model based on this property of the symmetric planes is shown in Fig. 2. It has

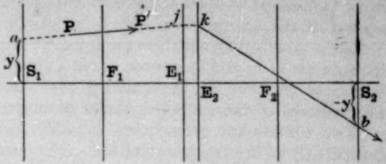

Fig. 1.

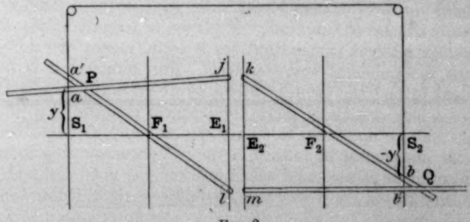

Fig. 2.

done duty since 1886. Two pairs of wooden laths are articulated at *jk* and *lm* respectively. By a cord passing over pulleys, the distance that *b* is let down in the second symmetric plane is made equal to the distance that *a* is raised in the first symmetric plane. By a second cord, the same thing is done for the points *a'b'* for the other pair of laths. If the two laths are caused to cross one another at the point P, the other portions at once adjust themselves to cross at the conjugate point Q.

The property of the symmetric planes being at the same distances from the principal foci as these are from the principal planes was made use of in the focometer which I described to the Royal Society in 1891 for determining the true focal lengths and positions of the principal planes of microscopic objectives. Two transparent micrometers suitably mounted are set independently at the two principal foci. They are then clamped to carriers on a long right and left-handed screw which is parallel to the axis of the lens. By turning this screw the two micrometers are moved away from one another through equal distances until they arrive at the symmetric planes, as ascertained by their then being the image one of the other. The distance through which they have thus been displaced is the true focal length, and the positions of the principal planes are found by measuring back the same distance from the focal points. The instrument is adapted also to determine the cardinal points of camera lenses.

The existence of the family of planes of magnifications of successive integral values furnishes a useful means of readily solving a number of lens problems. For instance, if it be required to find the position where an object must be placed before a lens system of given focal length in order that the image may be magnified *m* times, one has only to remember that the image must be formed in the m^{th} plane counting from the focal plane as zero, and will therefore be at $m+1$ times the focal length from the second principal plane; and dividing this distance by the magnification *m* will give $\frac{m+1}{m}$ times the focal length as the distance of the object from the first principal plane.

The construction which enables one to draw directly the continuation of any incident ray suggests naturally that in a certain sense the emergent ray, taken as a whole, is the image of the incident ray, seeing that it consists of a range of points, each one of which is the image of a corresponding one in the range of which the incident ray may be considered to consist. Similarly, the part of the axis itself, which lies on one side of the lens system, may be considered in the same sense an image of the part which lies on the other side. If we confine our attention to a linear object placed in the axis, its image will be also linear, lying in the axis at some other part, and, in general, of a different length. Thus, in Fig. 3, the object PP′, an arrow, will have its image at QQ′. Whatever the distance of the arrow from the lens it will point always in the same sense, and will never be inverted, that is, pointing backward. The image may be real, as in Fig. 3, or virtual, as in Fig. 4, or partly real and partly virtual, passing through ± ∞, as in Fig. 5.

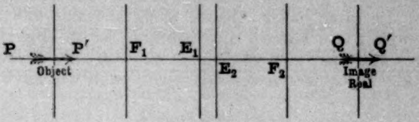

FIG. 3

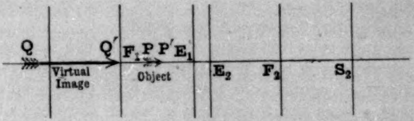

FIG. 4.

It is, of course, well known that the longitudinal magnification or "elongation," as Maxwell called it, does not follow the same rule as the transverse magnification, but is, for a small object, placed at any position, proportional to the square of the transverse magnification for the same position. There are, for example, only two positions in which the image of a small cube

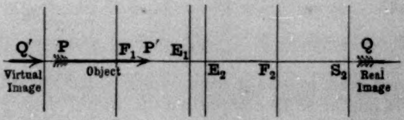

FIG. 5.

can be also a cube—namely, where object and image are in places of unit magnification—namely, either at the principal points or at the symmetric points. In all other positions the image of the cube will be distorted, becoming elongated axially if the image is enlarged and compressed axially if the image is diminished. We may consider the case of a series of objects all of the same height, placed along the axis like a row of telegraph poles, spaced equidistantly, as in Fig. 6. Their images will constitute a harmonic range of lines of diminishing length. The object at the symmetric point will have an equal inverted image, and all the others to infinity will have their images crowded together, their heads lying on the sloping line running through the second focal point.

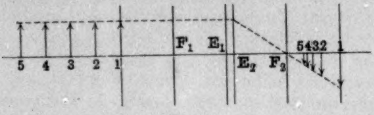

FIG. 6.

This unequable crowding together of the images of more distant objects is responsible for the curious effect noticeable in modern binocular field glasses of high power, but seen also in opera glasses and telescopes, in which the successive planes of landscapes seem exaggerated and flattened, almost like the flat scenery of the theatre. Their trees and hedges, for example, seem to occupy definite planes, and the more distant objects appear to be compressed up toward those in front of them. This phenomenon, to which Dr. von Rohr has given the name of *porrhallaxia*, is not due, as some suppose, to any binocular hyperstereoscopic effect, but is the consequence of the law of longitudinal magnification.

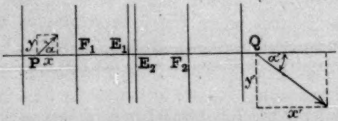

FIG. 7.

It was stated above that the system of Gauss is now all but universally accepted for the treatment of lens systems. The exceptions are two. In the case of the formation of images upon the retina of the eye the collinear relation ought not to be fulfilled, since the images are not required to be thrown upon a plane, but on the interior of a globe; and as Listing and, more recently, Gullstrand have pointed out, the fact that the centre of rotation of the eyeball does not coincide with the second nodal point of the eye, introduces into the problem of free vision considerations outside those of instruments for which the eyeball may be regarded as fixed in direction.

The other exception is the system proposed by Biot (1844) and Bossoha (1876), and adopted by Cornu (1877), and more recently by Sissingh (1900), in which no reference is made to principal planes. In this system the distances of object and image are measured, as are also the focal lengths, from the vertices of the first and last refracting surfaces of the instrument, and the performance is expressed in terms of four constants: the angular magnification, the focal power, the angle subtended at the first vertex by unit length placed at the last vertex, and the "dilatancy" of the system, this last term meaning the ratio of the diameter of the emerging conical pencil as it traverses the last surface to the diameter of an incident cylindrical pencil where it traverses the first surface. It is claimed for this method of treatment that the distances of object and image are more readily measured than in the system of Gauss; but, on the other hand, the mathematical relations are less direct, and do not lend themselves with equal facility to extended use when proceeding to consider the various aberrations.

In connection with the relation of the magnification of lenses as dependent on the relative distances of the conjugate points, mention may be permitted of a simple process communicated by the author to the Optical Society in 1899, for deducing the power of a (positive) lens from its magnification. From equation (2), preceding, we have $\frac{M}{v}=\frac{1}{u}$. If we substitute this value in equation (1), and remember that for the production of a real image u and v must be of opposite signs, we obtain for the power P of the lens

$$P=\frac{1}{f}=\frac{M+1}{v}.$$

If, therefore, we so adjust the distance of the object from the lens that $v=1$, we have the relation

$$P=M+1.$$

The model proposed by the author to carry out the method consisted in a frame wherein the lens whose power was to be measured was set up at a distance of 1 metre from a vertical scale graduated in centimetres. As object was used an aperture 1 cm. square placed in front of the flat flame of a paraffin lamp. The object was moved to such a distance from the lens that its image was sharply focused on the scale, enabling the size of the magnified image to be read off directly; when the addition of 1 to the linear size of the image at once gave the power of the lens in *dioptries*.

It is germane to the subject to suggest that some improvement is needed in defining the different uses of the term magnification. Most commonly the term is applied to the *transverse* (or linear) *magnification*, the ratio of a vertical ordinate in the image to that of the object, as depicted in the ordinary diagrams of the text-books. But there is also a horizontal transverse magnification, identical indeed with the vertical magnification if the optical system consists of spherical surfaces only. But the optical system may be an astigmatic one of special design to give different magnifications in azimuth and altitude. Secondly, we have to consider the *longitudinal magnification* or *elongation* already alluded to; and, thirdly, there is the so-called *angular magnification*, the ratio of the magnitude of an angle in the image to that of the corresponding angle of the object. Let there be an optical system such that PQ are two of its conjugate points. At P, an object, let there be placed in the plane of the diagram a small rectangle of height y and length x; its image at Q will have height y' and length x'. The transverse magnification $M = y'/y$; the longitudinal magnification $L = x'/x$. The diagonal of the object rectangle makes an angle $\alpha = \text{arc} \tan y/x$, and the image of that angle is the angle $\alpha' = \text{arc} \tan y'/x'$. The ordinary statement of the angular magnification is to define it as the ratio α'/α. It would be better to adopt the plan, certainly in use by more than one Continental writer, of defining the term *angular magnification* as $\tan \alpha'/\tan \alpha$. For then, since $L = M^2$, we have the relation

$$\text{Angular magnification} = \frac{\tan \alpha'}{\tan \alpha} = \frac{y'}{y} \cdot \frac{x}{x'} = \frac{M}{L} = \frac{1}{M}.$$

In other words, the angular magnification thus defined is the reciprocal of the transverse magnification.

But there are yet other senses in which the word *magnifica-*

tion is used—namely, in term the *apparent magnification* produced by a simple magnifying glass, which is the ratio of the angle subtended by the visual image (transverse)—formed presumably at the *punctum proximum* of the observer's eye—to the angle subtended by the object itself at the place where it is. There is, similarly, an apparent longitudinal magnification and an apparent angular magnification. Further, there are the extrusions of these terms to the cases of telescopic and microscopic vision.

It is much to be desired that some general agreement should be arrived at amongst opticians and physicists, not only respecting the use of these terms and their proper definition, but also respecting the right use of certain other terms and conventions. At present there are wide differences in usage as to the terms *positive* and *negative*. In lens problems some authorities reckon axial lengths positive when measured, as mathematicians always do, to the right of the origin; others reckon lengths positive when measured in the sense opposed to the light. Some take as the origin the surface of the lens, others always reckon from the "principal" points. Some writers call a divergent lens positive, while in the optical trade universally a positive lens is understood as a convergent one.

Further, suitable English terms are much needed for several matters for which at present none are recognised. The central point on the face of a lens is sometimes called its "pole," or even its "apex"; the German term is *Scheitel*. A name is wanted to denote the distance between the pole of a lens and its principal focus, the German term being *Schnitt-weite*. We possess no equivalents for the three most useful German terms, *Durchbiegung*, *Durchrechnung* and *Abbildung*. An English term is also needed for the phenomenon called *porrhalaxie*, introduced by Dr. Moritz von Rohr for the phenomenon due to longitudinal magnification, mentioned above.

XLVIII. *Demonstration of Laboratory Apparatus and Experiments.* *By* G. F. C. SEARLE, *Sc.D., F.R.S., University Lecturer in Experimental Physics, Cambridge.*

[Dr. Searle explained a number of methods of determining optical constants and exhibited the apparatus used by the students in his Practical Class at the Cavendish Laboratory, Cambridge, in applying those methods to optical systems.]

I. Revolving Table Method of Determining the Curvature of Spherical Surfaces.[*]

In the revolving table method of determining the radius of a spherical surface designed by the author, Mr. A. C. W. Aldis and Mr. G. M. B. Dobson, the radius is found directly from two readings on a straight uniformly divided scale.

A table turning about a vertical axis is required. The plane of the top of the table is normal to the axis of revolution, and the top carries a straight scale against which slides a carriage bearing the spherical surface (*see* Figs. 1 and 3). We shall assume that the scale is so adjusted on the table that the straight line described by the centre of curvature of the spherical surface when the carriage slides along the scale, *intersects* the axis of revolution of the table. The position of the carriage relative to the table-top when the centre lies on the axis of revolution will be called the *first position.* If the table be turned through any angle about the vertical axis when the carriage is in the first position the only effect of the motion is to substitute one part of the spherical surface for another. Hence, if rays from an object fall upon the surface, the reflected rays will be unaffected by the motion.

The carriage is now moved along the scale into a *second position* in which the axis of the table is a tangent line to the spherical surface. If, now, the table be turned about its axis, a grain of lycopodium placed at the point of contact of the vertical tangent line and the surface will remain stationary. By using a microscope for observing the image of the object in the first case and the lycopodium in the second, the settings can be made with great accuracy.

The radius of curvature of the surface is now given at once by the difference of the two scale-readings of the carriage in the first and second positions.

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[*] A full account of this method will be found in the "Philosophical Magazine," February, 1911, pp. 218-224.

The revolving table rests upon a tripod stand, and the table-top turns about a vertical rod carried by the tripod. The top of the table carries a scale graduated in millimetres, which can be clamped in any position on the table by the screw seen in Fig. 1. The adjustments of the surface to be tested are facilitated by the use of a small lathe-head to form the "carriage" mentioned above.

The lathe-head is attached to a board which rests upon the top of the table with its straight edge in contact with the scale. The spindle of the lathe-head is screwed at one end to fit a face-plate with three screws which serve to adjust a second plate to which the lens or mirror to be tested is attached. Each end of the spindle is turned to a conical point, and the vertex of each cone lies on the axis of the spindle. One arm of a rod bent at right angles can be secured by a set-screw in a socket in the casting supporting the table-top, and a small clip carrying a pin can be fixed in any position on the other arm of the rod by a set-screw. The bent rod also serves as a means of clamping the board to the table in the manner shown in Fig. 2.

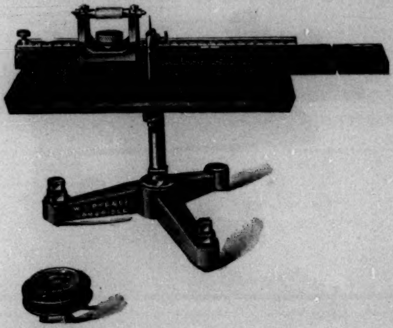

FIG. 1.

The first step is to make the axis of the spindle of the lathe-head parallel to the edge of the board. The clamping screw enables the lathe-head to be properly adjusted on the board.

After this adjustment of the lathe-head on the board the scale is adjusted on the table-top so that the axis of the lathe spindle *intersects* the axis of the table.

The face-plate is now attached to the spindle and the lens or mirror to be tested is fixed to the adjustable plate. In the case of a lens, the back surface should be blackened to stop reflexion at that surface. The spindle is then rotated and some object is observed by reflexion at the spherical surface. If the rotation cause a motion of the image, the three adjusting screws are manipulated until the image remains stationary when the spindle is revolved. The centre of curvature then lies on the axis of the spindle.

The preliminary adjustments are now complete. If they are not quite perfect, the error which they cause in the radius of curvature as found by this method will be very small, since the error of radius depends on the *second* powers of the errors in the preliminary adjustments.

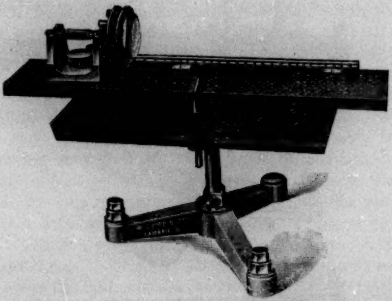

Fig. 2.

The board carrying the lathe-head is now set into the "first position" in which the centre of curvature of the surface lies on the axis of revolution of the table. The image of a vertical line ruled upon a piece of ground glass is observed in the microscope. If the image move when the table-top is turned about its axis, the board carrying the lathe-head is moved along the scale until the image remains at rest. The centre of curvature then lies on the axis of the table. The scale-reading of an index mark on the board is then taken.

The board is next adjusted to the "second position" in which the axis of the table is a tangent line to the reflecting surface. A patch of lycopodium is placed on the surface and then a pointed piece of wood is held against the surface while the lathe-spindle is rotated, so as to remove all the lycopodium but a circular patch about a millimetre in diameter. The board is then adjusted along the scale so that the central grains of the patch remain stationary when the table-top is turned about a mean position in which the axis of the lathe-spindle is *parallel* to that of the observing microscope. The scale-reading of the index mark is then taken.

The difference between the two scale-readings gives the radius of curvature of the surface. Each of the two settings is easily made to $\frac{1}{10}$ mm. with the apparatus described.

II. Goniometer Method of Measuring Focal Lengths of Optical Systems.

The goniometer (Fig. 3) designed by the author and Messrs. W. G. Pye & Co., of Cambridge, England, provides a means of measuring angles up to about $\frac{1}{4}$ radian (about 14°). The base is formed of a strip of wood furnished at one end with a spherical pivot and at the other with a cross-bar carrying a scale. Angles are measured by means of a movable arm which turns at one end about the pivot, while the other end moves over the scale on the cross-bar. The optical system consists of a lens fixed to the arm near the pivot and of a fine vertical wire attached to the other end of the arm and adjusted to be in the focal plane of the lens.

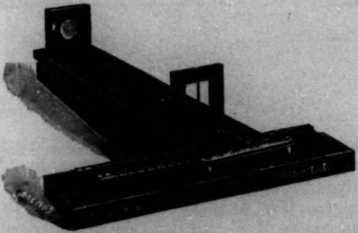

FIG. 3 (Registered Design, No. 526,858).

The spherical pivot enters a conical hole turned out of a block of brass attached to the arm. The arm also carries two brass feet which rest upon the cross-bar.

The scale on the cross-bar is divided into millimetres, and the ball is adjusted so that its centre is 40 cm. from the edge of the scale. The readings are taken by means of a fine wire passing across an opening in the arm. The scale is provided with an anti-parallax mirror.

The lens attached to the movable arm is achromatic and has a focal length of about 35 cm. The vertical wire is held in an adjustable frame attached to the arm, and the image of a distant point will fall upon the wire, if the arm be properly directed. If a plane mirror be placed so that the lens lies between it and the wire, the image of the wire formed by two refractions through the lens and one reflection at the mirror may be made to coincide with the wire itself.

The instrument may be used to measure the angle between two beams of parallel light. The arm is moved so as to bring each beam in turn to a focus on the wire; the angle turned through by the arm is then equal to that between the two beams.

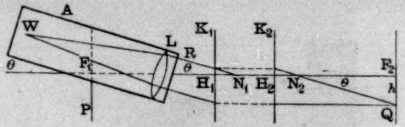

Fig. 4.

One of the uses of the goniometer is its application to measure the focal length of any coaxal system of lenses by a method depending upon the properties of the nodal points.

In the diagram (Fig. 4) let F_1P and F_2Q be the two focal planes of the system, H_1K_1 and H_2K_2 the two principal planes, and N_1 and N_2 the two nodal points. The lenses forming the optical system under test are *not* shown in the diagram. Let $F_1H_1=f_1$, and $F_2H_2=f_2$; then f_1 and f_2 are the two focal lengths of the system. The nodal points are defined by the property that any incident ray QN_2, whose direction passes through N_2, gives rise to an emergent ray N_1R whose direction passes through N_1 and is parallel to QN_2. It can be shown that $N_2F_2=f_1$ and that $N_1F_1=f_2$. When the medium is the same at both ends of the system, the nodal points N_1, N_2 coincide with the principal points H_1, H_2 and then $f_1=f_2=f$.

The point Q in the focal plane through F_2 will emit a pencil of rays which will give rise to an emergent beam of parallel rays, as indicated in the diagram. If the goniometer arm A be properly adjusted, this parallel beam is brought to a focus on the wire W by the lens L. Similarly, a point at F_2 gives rise to a parallel beam which may be brought to a focus at W by turning the arm through θ radians, the angle between the two beams. But, by the property of the nodal points, the angle QN_2F_2 is also equal to θ. But, if $F_2Q=h$, we have, for small angles, $\theta=h/N_2F_2=h/f_1$, and thus $f_1=h/\theta$. Since we know θ, f_1 can be calculated as soon as h is known.

If a plane mirror be mounted on the goniometer arm, the image of Q may be made to coincide with Q itself by properly adjusting the arm.

The distance h or F_2Q is measured by a glass scale placed with its divided face in the focal plane. The adjustment is easily made if the scale be mounted on a sliding carriage similar to that shown in Fig. 5.

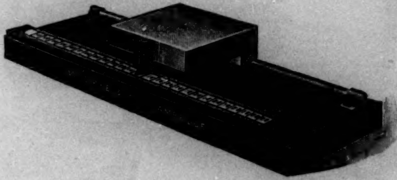

FIG. 5 (Registered Design, No. 526,859).

The goniometer method may be used for finding the focal length of a microscope objective if a micrometer scale, divided to $\frac{1}{10}$ mm. and provided with an eye-piece, be substituted for the glass scale. When the micrometer is in adjustment, the image of the goniometer wire will be sharply focused on the micrometer scale. Two readings are taken of the position of the image of the goniometer wire on the micrometer scale for two positions of the goniometer arm, and the focal length is calculated as before.

The sliding carriage (Fig. 5) rests on a steel rod and a steel scale 30 cm. in length divided to millimetres, the rod and the scale being mounted parallel to each other on a base-board. The carriage has three projections on its lower side. In two of these a V groove is cut and these two V's rest upon the rod, the remaining projection resting upon the scale. For many purposes it is convenient to fit a vertical rod to the carriage, as in Figs. 6 and 7, and to attach objects to the rod by the aid of suitable clamps.

III. Revolving Table Method of Measuring Focal Lengths of Optical Systems.

The revolving table shown in Figs. 6 and 7 has been designed by the author and Messrs. W. G. Pye & Co. for use in this method. The table-top turns, without side-shake, about a vertical steel rod carried by a tripod. The table carries a bar graduated in millimetres, which can be clamped in any position on the table by the screw seen in Figs. 6 and 7. A cranked rod can be secured by a set screw in a socket in the casting which supports the table-top. This rod, as shown in Fig. 7, is fitted with an adjustable clip, in which a scale can be held.

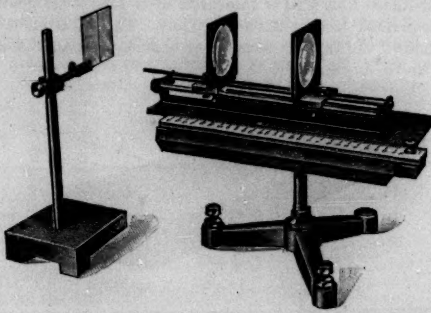

FIG. 6

A system of lenses may be arranged for use with the revolving table by attaching the system to a base-board with the optical axis parallel to one edge of the board; this edge, which must be straight, is kept in contact with the graduated bar and is provided with an index mark by which its position relative to the bar may be observed.

In finding the focal length of a lens system, the first step is to make the axis of the system intersect the axis of revolution of the table. The system is placed on the table, its base-board being in contact with the graduated bar. The axis of the system is then directed towards a *distant* object, whose image is received on a ground-glass screen, placed as in Fig. 6. If the table-top be now turned through a small angle, the image will in general move. After a few trials, a position of the system on the table will be found such that the image of the object does not move from a vertical line on the ground glass, when the table is turned through a small angle. The nodal point now lies in a plane containing the axis of revolution and perpendicular to the direction of the distant object, but does not necessarily lie *on* the axis of revolution.* The table-top is turned end for end, and the system is moved on the table so as to refocus the image on the screen, which has not been moved. It will probably be found that, although the image does not move when the table-top is slightly turned, the image does not fall again upon the vertical line on the screen. The distance between the image and the vertical line is measured, and the graduated bar is moved at right angles to itself through half this distance, and is then clamped. The axis of the lens

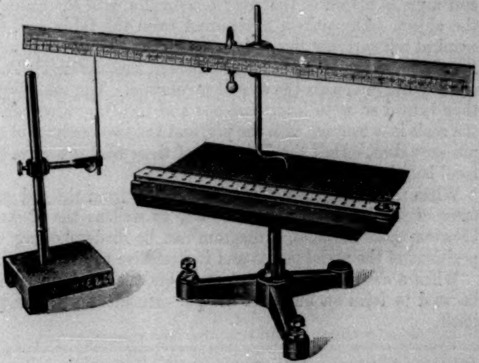

FIG. 7.

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* The image of the distant object point lies on the straight line which passes through the nodal point of emergence and is parallel to the straight line joining the distant point to the nodal point of incidence. The image is, therefore, undisturbed only when the nodal point of emergence does not move at right angles to the two parallel lines.

system will then intersect the axis of the table when its base is in contact with the graduated bar.

The system is readjusted so that the image of the distant point does not move when the table is turned, and the reading of the lens index on the graduated bar is recorded. The nodal point of emergence now lies on the axis of the table. The table-top is then turned end for end, a similar adjustment is made, and the new index reading is recorded. The difference between the readings gives the distance between the nodal points.

The focal length is now measured. The system is adjusted, as above, so that a nodal point lies on the axis of revolution, and a pin is placed so that its tip coincides with the image of the object. The system is removed from the table, and the cranked rod carrying the scale is secured to the table-top as in Fig. 7, the rod and scale being so adjusted that the divided face of the scale intersects the axis of revolution. The table top is then turned so as to bring first one and then the other end of the scale into contact with the pin, and the two readings of the pin are taken. Half the difference of these readings gives the focal length.

When the distances of the principal foci from the end surfaces of the lens system are known, as well as the distance between those surfaces, a diagram can be made showing the positions of the nodal points and foci relative to the system.

When a *distant* object is not available, a plane mirror may be used to form an image of the pin coincident with the pin itself.

IV. Determination of the Astigmatism and Curvature of Field of a Lens System.

The apparatus is arranged for the investigation of the "defects" of astigmatism and curvature of field when, as in landscape photography, the objects of which the lens is to form images are at infinite or very great distances from it.

The lens A (Fig. 8), which is to be tested, is supported, with its axis horizontal, upon a stand. A ground-glass screen B is mounted on a sliding carriage C which moves on the track D, as in Fig. 5. A horizontal line divided to half centimetres is ruled on the ground glass. For convenience, the frame supporting the ground glass is clamped to a vertical rod rising from the carriage (*see* Figs. 6 and 7). To obtain a beam of parallel light a collimator is used. A projection lens E, with its aperture diminished by a stop, is mounted with its axis horizontal on a board F. A pair of horizontal and vertical wires stretched across an opening in a metal plate intersect at G the focus of the lens E. The board is clamped to a vertical rod fixed to a sliding carriage which moves on the track H, and the axis of E is adjusted to the same height as that of A. A flame K illuminates the cross-wires.

The two tracks and the stand supporting A rest on a table. The axis of A is parallel to, and the plane of the ground glass is perpendicular to the direction of the track D.

The collimator is first placed so that its axis approximately coincides with that of the lens system A. The carriage C is then adjusted so that the cross-wires are sharply focused on the ground-glass screen B, and the index reading of the carriage on the scale of the track is taken and also the reading of the image of the vertical wire upon the scale on the screen. The collimator is then moved into an oblique position, as in Fig. 8, and the optical effect of the cross-wires on the screen is examined. Unless the lens be free from astigmatism and curvature of field, there will be no clear image of both wires. It will, however, be possible to adjust the screen so as to obtain (1) a sharp image of the vertical wire, formed by primary focal lines, and (2) a sharp image of a *part* of the horizontal line, formed by secondary focal lines. The two readings of the carriage C will probably differ from each other and from the reading found when the collimator was in the symmetrical position. The observations are repeated for a number of positions of the collimator on either side of the symmetrical position, the obliquity being increased as far as the mount of the lens A allows. The difference between the two readings for (1) and (2) for a given obliquity of the collimator is the astigmatic difference for that obliquity of the incident parallel beam.

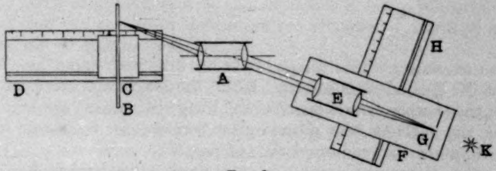

FIG. 8.

The track H is not *necessary* for the experiment, though it is convenient, since it allows the collimator to be readily adjusted so as to fill the lens A with light.

Unless the lens A is well-corrected, it will be necessary to stop it down to obtain a sharp image of the vertical wire, when the obliquity is considerable. It will not always be possible to obtain a sharp image of the *whole* of the horizontal wire, though it will be possible to get a sharp image of any particular point on the wire by suitable adjustment of the screen.

Two curves (1) and (2) are then plotted. In (1) the abscissa is the reading of the image of the vertical wire upon the scale on the ground-glass screen. In (2) the abscissa is the reading of the *sharply focused point* of the image of the horizontal line on the scale on the ground-glass screen. In each case the ordinate is the reading of the carriage C.

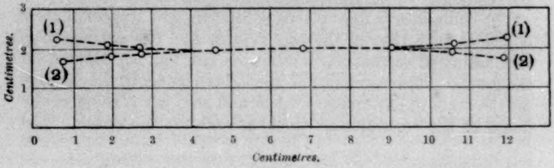

FIG. 9.

The diagram obtained in a test of a projection lens of 21·7 cm. focal length is shown in Fig. 9. The two image surfaces are slightly curved, but in opposite directions. The primary image surface, on which the image of the vertical wire comes to a focus, is concave towards the lens.

XLIX. *Measurements and Notes on the Visibility of Point Sources of Light.* *By* Clifford C. Paterson *and* B. P. Dudding, *A.R.C.Sc.* (*From the National Physical Laboratory.*)

The consideration and observation of point sources of light has generally been concerned either with

(*a*) The observation of stars ; or

(*b*) The observation of distant lights on land and on sea.

The solution of problems connected with either of these fields of work has been the object of most of the investigations into this subject which the authors have been able to trace.

In attempting to classify stars according to a scale of magnitude astronomers have devised many and various forms of apparatus, but the work is not in any way quantitative, one star being compared against and expressed in terms of another, and thus the results of their work are recorded in terms of an arbitrary standard of magnitude.

Some accounts of the most recent work on this subject can be found in the records of " The Astrophysical Journal,"* and a particularly full account of, and comparison between, German and American results has been given by Prof. Pickering.†

The observation of signal lights at sea has for many years played an important part in the safe navigation of ships, and the appreciation of distant lights in connection with railway work is a matter of great importance. The laws governing the degree of appreciation of these lights by the human eye have been the subject of several researches. In the laboratory small illuminated apertures have been used by most observers both in experiments on the visibility of lights as distinct from their colour, and in experiments devised to test the eye for colour vision. But, although so much qualitative work has been done, the authors have been unable to find any reference to work where absolute measurements have been made, in terms of a reproducible standard, of the visibility of a point source of light. From the time of Helmholtz many workers on the Continent have been interested in the visibility of periodic lights, and have experimented with a view to determining to what extent the visibility depends on the time of duration of such a periodic light. Some of the more complete investigations are those of Broca and Sulzer,* and since the completion of the authors' experiments a Paper has been published by MM. Blondel and Rey† in which is discussed the appreciation by the eye of lights of short duration.

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* Vols. II., IV., XIII. and XXXI.

† Part II., Vol. XIV., " Harvard College Observatory Annual."

This work is outside the scope of the present Paper, and deals with a totally different problem, but it is interesting to note that in this Paper reference is made to the limit of visibility or perception which is given as one-tenth candle at one kilometre distance for a dark night in the open, or 0·5 to 0·6 of this for observation in a laboratory where all stray light is excluded.

This agrees approximately with the figures obtained by the authors, but it is, of course, a variable quantity depending on the acuity of the eye of the observer.

A short account is given by MM. André Broca and Polack‡ of some experiments made on illuminated pinholes, with a specially constructed apparatus, with a view to determine how different coloured signal lights might be distinguished, even though so faint as to make it impossible to distinguish the colour. Their figures do not enable the value of limiting visibilities of the various coloured signals to be computed, but the generalisations given in this Paper have been supported by the authors' experiments, and will be further emphasised later.

The Deutsche Seewarte carried out experiments with a view to finding out the minimum candle-power for ships' signal lights, and their report was issued in 1894. Their units for green and red light were a Hefner candle with a green and red glass placed respectively in front of it. No data are given as to the percentage transmission of these glasses, and, therefore, it has been impossible to compare the results with those of the present authors. It may be deduced from a formula given in the report that for very clear weather 2·1 Hefner candles (white) are required for visibility at 2 sea miles, and 13·1 Hefner candles for 5 sea miles.

The suitability of a light for signal purposes can, of course, always be tested by the direct method of setting up the light in the desired position, and ascertaining if it is sufficiently visible at the range of distance for which it is designed. This is the method which has usually been employed to fix a standard of reference, but for the proper comparison of different lights some more universal method is wanted.

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* "Journal de Physiologie et de Pathologie Générale," No. 4, Juillet, 1902.

† Sur la perception des lumières breves à la limite de leur portée, "Journal de Physique," Juillet et Août, 1911.

‡ "Comptes Rendus," Nov. 11, 1907.

The property of light spoken of as its " visibility " needs to be defined in terms of some definite unit, and it should be possible to measure it by comparison with a standard whose value in terms of such unit is known. It has been the object of this work to ascertain if such a unit is generally applicable, and to construct an apparatus by means of which measurements of the visibility of point sources of light in terms of this unit can be made.

Definition of Point Source.—For the purpose of this Paper a point source of light is taken as one whose linear dimensions subtend an angle at the eye less than the resolving power of the eye, *i.e.*, about 30 seconds of arc for a mean wave-length $0{\cdot}5\times 10^{-3}$ mm. and pupilar aperture 4·5 mm.

The most obvious step in commencing the work was to produce point sources of light whose candle-power could be measured and yet be small enough to be used over the range of the ordinary laboratory dark room, *i.e.*, from 2 to 10 metres.

The pinholes were constructed having diameters varying from 0·01 mm. to 0·20 mm. They were pierced in circular brass plates about 2 cm. diameter and 2 mm. thick. The plate was first deeply countersunk on one side and then rubbed down on the other, until the metal at the base of the countersink was very thin. This thin metal was finally pierced by means of needles ground under the microscope, the piercing being also done under the microscope.

These pinholes were mounted in front of a flame so as to be fully illuminated. A very satisfactory flame was found in that given by a lamp sold by the Vacuum Oil Company burning a wax known as " Vaclite." This lamp will give a steady constant light for several hours without any tendency to smoke, and the wick chars very little.

The candle-power per square millimetre of the white portion of the standard flame used was obtained by measuring the candle-power of several small areas of the flame, the smallest of which was 13·5 sq. mm. area. From these measurements it was found that the candle-power per square millimetre of the flame was $6{\cdot}6\times 10^{-3}$.

In the earliest experiments the pinholes were mounted on a metal plate placed in front of the lamp and held in place by spring clips. This enabled changes of pinhole to be rapidly made. The pinholes were viewed direct, no optical arrangements being used. Later, a visibility photometer was constructed for use with one eye. The principle was the same as a later binocular photometer, although the latter is improved in technical details. The photometer for use with one eye was discarded in favour of the binocular, because the method of employment of the latter more nearly approached the ordinary method of viewing distant lights.

A photograph of the binocular visibility photometer is shown in Fig. 1; and Fig. 2 is a sketch of the general arrangements of the photometer. The instrument consists of a brass tube, A, about 2 metres long, at one end of which are placed absorption wedges, B, and the pinhole, this latter being held on the wedges by spring clips. The pinhole is illuminated by the standard vaclite lamp, C, which is contained in a box arranged with suitable ventilation. The object of the box is to enable the instrument to be used in the open without the wind causing the flame to flicker. At the end of the tube are placed two adjustable mirrors. In one, D, is seen the image of the standard point source, and the other, E, can be arranged to bring the image of the distant source into the same field of vision. To enable the observer accurately to centre his eyes, the tubes, F, are capable of a sideway adjustment to suit the distance between the eyes of the observer and of sufficient diameter to enable a full pencil of light to enter the eye.

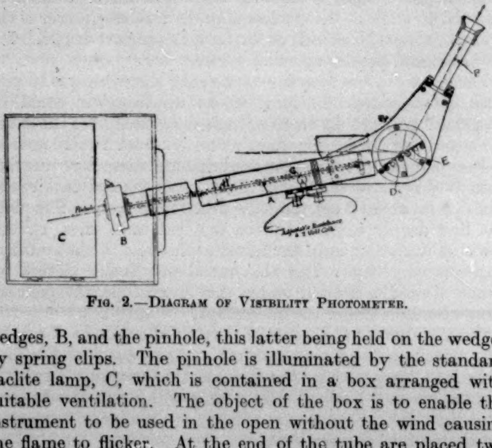

FIG. 2.—DIAGRAM OF VISIBILITY PHOTOMETER.

At G is placed a small metallic filament lamp suitably

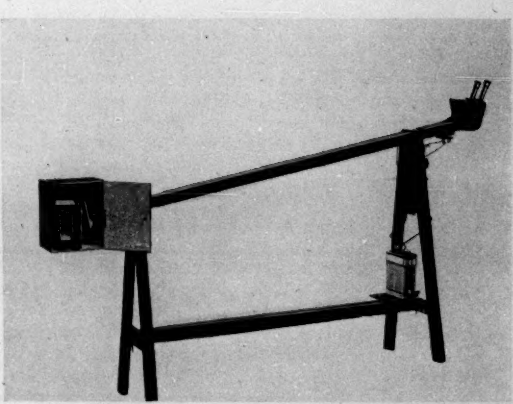

FIG. 1.—BINOCULAR VISIBILITY PHOTOMETER.

To face page 382].

screened, by which the field surrounding the standard point can be slightly illuminated. This was found necessary for outdoor work and is dealt with later.

The visibility of the standards is varied by sliding the wedges relatively to each other. The motion is given to the wedges by means of the rod H and bevel wheels.

In using the apparatus the points of light were viewed in the mirrors, these being arranged so that the images of the sources were separated by a small angular distance. The observer has to look at one point and carry in his mind the memory of it whilst looking at the other. This procedure was necessary because of the considerable brightening of a point source when it is seen obliquely, and, that is to say, when its image does not fall on the centre of the fovea centralis. This effect becomes even more marked when the colours are different; in fact, it was found to be impossible to work with the points close together, so that they were viewed at the same time.

Having arranged the mirrors so that an image of each source can be seen, the standard source is varied by means of the wedges until it matches the distant source for visibility.

Before passing on to enumerate the experiments, it may be mentioned that in all the work hereafter described two point sources of light are matched for visibility. This is most easily done by imagining the points, while matching them, to be distant lights on the horizon. It is found that different observers, provided their vision is normal, will agree within close limits as to when lights are equally visible, even though the colours are quite different.

The match made here is purely a physiological one, and when of different colours depends partly, but not entirely, on the luminosity curve* for the eye of the observer.

The principal laws which will first be demonstrated are :—

I.—That the visibility of a point source of light is independent of its intrinsic brightness and is proportional to its total candle-power.

II.—That the visibility of a given source varies inversely as the square of its distance from the eye of the observer.

I.—*Effect of Intrinsic Brightness.*

Experiment 1.—The smallest illuminated pinhole was set up at about 2 metres from the eye, and others of different sizes in turn were set up at the same distance from the eye, but with an adjustable sector disc inserted in the path of the beam.

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* Sir W. de W. Abney: "Phil. Trans.," A., 193. (1900.)

The sector opening was adjusted in each case till the visibility of the larger pinhole matched the smaller. The pinholes in this experiment subtended an angle of less than 10 seconds of arc, *i.e.*, below the resolving power of the eye. They were at the same distance from the eye, so that the measurements were not complicated by differences of accommodation. In this way a pinhole of large candle-power has its brightness reduced until it matches the smaller, whose brightness remains constant. The following tables show the results obtained, and it is seen that when the candle-powers of the two holes are equal they are also equally visible, their distances from the eye being equal.

TABLE I.

A pinhole for comparison purposes was set up at 2 metres distance. It had a candle-power $=5{\cdot}9\times10^{-6}$.

Pinhole normal candle-power $\times10^{-6}$.	Angle of disc open for equal visibility as reference point.	Intrinsic brightness (original brightness $=1{\cdot}0$).	Equivalent candle-power of dimmed pinhole.	Error. Percentage difference in candle-power from the reference source.
$162\ \times10^{-6}$	$13{\cdot}_5$	0·037	$6{\cdot}1_0\times10^{-6}$	$+3{\cdot}_5$
$84{\cdot}_5\times10^{-6}$	$25{\cdot}_5$	$0{\cdot}07_1$	$6{\cdot}0\ \times10^{-6}$	$+1{\cdot}_5$
$38{\cdot}_2\times10^{-6}$	$58{\cdot}_5$	$0{\cdot}16_2$	$6{\cdot}2\ \times10^{-6}$	$+5{\cdot}_0$
$22{\cdot}_8\times10^{-6}$	$90{\cdot}_0$	0·25	$5{\cdot}7\ \times10^{-6}$	$-3{\cdot}_5$
$12{\cdot}_2\times10^{-6}$	$168{\cdot}_5$	0·47	$5{\cdot}7\ \times10^{-6}$	$-3{\cdot}_5$

TABLE II.

Smaller pinhole used as comparison source candle-power $=3{\cdot}6\times10^{-6}$.

Pinhole normal candle-power $\times10^{-6}$.	Angle of disc open for equal visibility as reference point.	Intrinsic brightness (original brightness $=1{\cdot}0$).	Equivalent candle-power of dimmed pinhole.	Error. Percentage difference in candle-power from the reference source.
$162\ \times10^{-6}$	$7{\cdot}_5$	0·021	$3{\cdot}_4\ \times10^{-6}$	$-5{\cdot}_6$
$84{\cdot}_5\times10^{-6}$	$14{\cdot}_5$	0·04	$3{\cdot}_4\ \times10^{-6}$	$-5{\cdot}_6$
$38{\cdot}_2\times10^{-6}$	$33{\cdot}_0$	$0{\cdot}09_2$	$3{\cdot}_5\ \times10^{-6}$	$-2{\cdot}_8$
$22{\cdot}_8\times10^{-6}$	$57{\cdot}_0$	$0{\cdot}15_8$	$3{\cdot}_6\ \times10^{-6}$	0
$12{\cdot}_2\times10^{-6}$	$108{\cdot}_0$	0·3	$3{\cdot}6_5\times10^{-6}$	$+1{\cdot}_4$

The last columns in the tables indicate the order of agreement between the adjusted candle-power and the candle-power of the point of reference.

Wedges.—The sector disc, although giving the most absolute method of dimming down the intensity of the light without changing its character, is not suitable where a wide range of variation is desired, and in order to exceed the range over which the last experiment was conducted, the sector disc was replaced by absorption wedges. The wedges used were those belonging to a Féry pyrometer, and although slightly selective, the selectivity has not been found in any way to influence the accuracy of the work. These wedges slide relatively to one another and are fitted with a scale of position.

The wedges were calibrated by using them in the path of a beam of light from an arc, the candle-power of the beam being measured. The measurements were made on an ordinary photometer bench, using a Lummer-Brodhun photometer head. The wedges were removed from time to time and the candle-power of the incident beam measured. In this way a curve of transmission for the wedges was obtained, connecting the relative position of the wedges to one another, and the percentage transmission of the light.

Experiment.—To prove the validity of substituting the calibrated wedges in the place of the sector disc, as a means of reducing the intrinsic brightness of the sources of light.

Illuminated pinholes of diameters varying from 0·03 mm. to 1·00 mm. were set up one after another behind the wedges in the standard apparatus. They were adjusted by means of the absorption wedges until they matched a comparison source for visibility in the manner described earlier. The results are shown in the following tables :—

TABLE III.

Reference source candle-power $=4\cdot1\times10^{-6}$.

Area in sq. mm. of pinhole.	Pinhole candle-power $\times10^{-6}$ (for normal brightness).	Wedge transmission percentage.	Equivalent candle-power $\times10^{-6}$.	Error. Percentage difference from the reference source.
$0\cdot78_5$	4320	$0\cdot09_5$	$4\cdot1$	0
$0\cdot18_9$	1040	$0\cdot4_3$	$4\cdot4_5$	$+8\cdot_5$
$0\cdot029_5$	162	$2\cdot6$	$4\cdot2$	$-2\cdot_5$
$0\cdot015_4$	$84\cdot_5$	$4\cdot4$	$3\cdot7$	$-9\cdot_5$
$0\cdot0069_5$	$38\cdot_2$	$10\cdot_4$	$3\cdot9_5$	$-3\cdot_5$
$0\cdot0034_5$	$18\cdot_9$	$23\cdot_8$	$4\cdot4$	$+7\cdot_5$

TABLE IV.

Reference source $= 1\cdot3_5 \times 10^{-6}$ candle-power.

Area in sq. mm. of pinhole.	Pinhole candle-power $\times 10^{-6}$ (for normal brightness).	Wedge transmission percentage.	Equivalent candle-power $\times 10^{-6}$.	Error. Percentage difference from the reference source.
$0\cdot78_5$	4320	$0\cdot032_5$	$1\cdot4_0$	$+3\cdot_5$
$0\cdot18_9$	1040	$0\cdot13_5$	$1\cdot4_0$	$+3\cdot_5$
$0\cdot029_5$	162	$0\cdot7_9$	$1\cdot2_8$	$-5\cdot_0$
$0\cdot015_4$	$84\cdot_5$	$1\cdot5_2$	$1\cdot2_9$	$-4\cdot_5$
$0\cdot0069_5$	$38\cdot_2$	$3\cdot6_0$	$1\cdot3_8$	$+2\cdot_0$
$0\cdot0034_5$	$18\cdot_9$	$6\cdot_8$	$1\cdot2_9$	$-4\cdot_5$

Column 3 gives a measure of the intrinsic brightness of the source, the original brightness being represented by 100·0.

In the above test it is interesting to note that the largest pinhole subtended an angle of about 2 min., and was therefore greater than the generally accepted limit of the resolving power of the eye. But, in spite of this, it appears that no marked change is seen in the law connecting intrinsic brightness and visibility, although the area of one spot was 3,075 times the other. Lord Rayleigh* has shown that for areas subtending considerable angles this law does not hold. The corresponding areas of the largest pinhole and the reference pinhole in Table IV. are shown to scale in Fig. 3. These had equal candle-power when appearing equally visible.

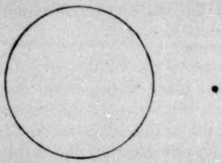

FIG. 3.

The object of giving the two tables is to illustrate the improved accuracy obtained by using rather a faint reference point.

The above experiments show conclusively that for sources subtending angles up to two minutes of arc, the intrinsic brilliancy of the source does not affect its visibility, the total candle-power being the only factor to be considered.

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* Collected Papers.

Further experiments were devised and carried out which confirmed the previous results. Lenses were used in the path of the beam from one point, about midway between the eye and the point of light. By this means the intrinsic brightness of this point was kept constant, but the apparent size of the point changed by changing the lenses. Matches made with the standard in the photometer showed that agreement to the order of the previous work was obtained between the theoretical visibility, calculated on the assumptions previously outlined, and the measured visibility.

II.—*The Inverse Square Law.*

Experiment 1.—One pinhole was set up at about 3 metres from the eye and used as a point of reference. The other pinholes in turn were mounted on a travelling carriage and taken to distances up to 20 metres, so as to be equally visible as the point of reference.

Standard of reference—$2{\cdot}5 \times 10^{-6}$ candle-power at $2{\cdot}9_5$ metres.

Visibility assumed $\propto \dfrac{2{\cdot}5_0}{(2{\cdot}9_5)^2}$, *i.e.*, $\propto 0{\cdot}287$.

Pinhole candle-power $\times 10^{-6}$.	Distance in metres (d).	Visibility (on assumption that visibility $\propto 1/d^2$).	Error. Percentage difference between column 3 & 0·287.
$1{\cdot}2_2$	$2{\cdot}1_1$	0·274	$-4{\cdot}_5$
$6{\cdot}3_5$	$4{\cdot}7_5$	0·282	$-2{\cdot}_0$
$16{\cdot}_2$	$7{\cdot}4_6$	0·291	$+1{\cdot}_5$
$25{\cdot}_2$	$9{\cdot}0_5$	0·308	$+7{\cdot}_5$
$30{\cdot}_5$	$10{\cdot}8_0$	0·262	$-8{\cdot}_6$
$52{\cdot}_2$	$13{\cdot}2_5$	0·297	$+3{\cdot}_5$
$112{\cdot}_0$	$20{\cdot}_4$	0·270	$-6{\cdot}_0$

The agreement of the values in column 3 with that of the comparison source, viz., 0·287, is to be noted.

Experiment 2.—Greater distances were not obtainable in the laboratory dark room, so that the investigation was continued at night in the 550 ft. gangway of the Ship Tank at the National Physical Laboratory. Here the investigation was more complete, and was carried out for red and green as well as white lights. The method of procedure was slightly changed when working on longer ranges.

A pinhole was set up as standard with the wedges placed in the beam from it, this giving a means of adjusting the visibility of the standard. Another pinhole was placed at various known distances and the standard source varied until it matched the distant point for visibility. The visibility of the distant point could then be measured in terms of the size of the standard, and the calibration of the absorption wedges.

It is slightly more difficult to match lights of different colours for visibility than lights of the same colour, the principal reason for this being the unequal behaviour of lights of different colour with regard to oblique vision. The authors found the same effects as those mentioned by Broca and Polack ("Comptes Rendus," ref. cit.). A green point viewed slightly obliquely appears to have an intensity of four or five times that by direct vision and a white point about twice. The green loses its characteristic colour and each point becomes less well defined. A red point becomes much less visible when viewed obliquely, and its colour is also indistinguishable when the light is near the vanishing point.

With regard, therefore, to matching green and white lights for visibility, the unequal brightening seen when looking from one point to another bothers the observer considerably and his settings become more erratic. It is for this reason that the authors kept the points always at a considerable angular distance from each other, but this does not entirely avoid the above effect. The red is troublesome for a different reason. Very few people seem to see a red point source as a point unless it is very dull, the image always spreading out somewhat, and appearing to have dimensions, causing the matching of white and red points to be more difficult than matching two spots of the same colour.

As previously mentioned, the authors tried viewing the two points to be matched simultaneously, but finally gave it up as being less accurate than the method of viewing each point separately, and carrying the memory of its visibility when looking at the other.

A method sometimes used was to shut out the point under observation by means of a screen before looking away from it, and to unscreen the other while the eye is focused on the place where it will appear, thus avoiding any oblique sight of the points. When working over these long ranges it was necessary to change the size of the pinhole from time to time, owing to the great difficulty of making efficient matches with very bright points, and also to ensure keeping below the limit of the resolving power of the eye. When changes were made, two readings at that station were taken, one with the larger pinhole which served for greater distances, and one with a much smaller pinhole which was to serve for the shorter distances. In general, the visibility at a change like this would be reduced or increased three to eight times, but the readings obtained always checked off within the errors of the experiments. This, of course, should follow in view of the proof that visibility is proportional to the candle-power alone.

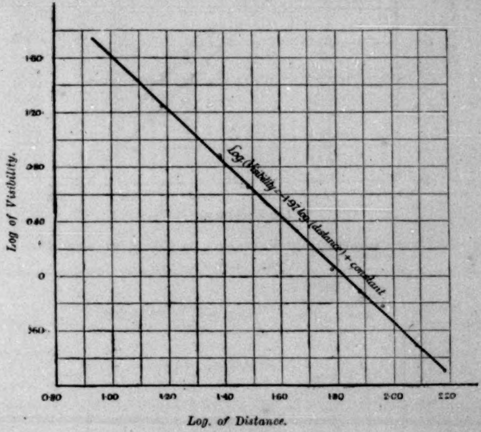

Fig. 4.—Curve showing the Variation of Visibility with Distance (White Light).

(This indicates the close agreement with the inverse square law.)

The results of the tests are shown in Tables V., VI. and VII., where the visibility obtained by assuming the inverse square law is shown in column three and the percentage error in the last column. These results are also plotted in Figs. 4, 5 and 6; but in order to show the results effectually, the candle-power of each pinhole is multiplied up to equal the value of candle-power of the largest hole, and its visibility at the various ranges at

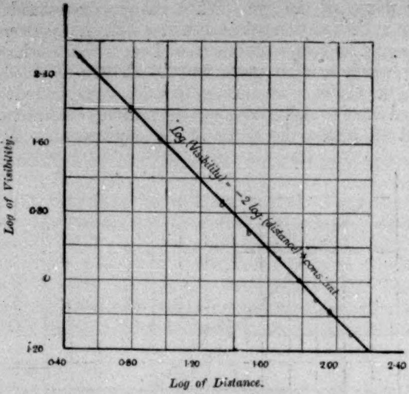

FIG. 5.—CURVE SHOWING THE VARIATION OF VISIBILITY WITH DISTANCE (GREEN LIGHT).

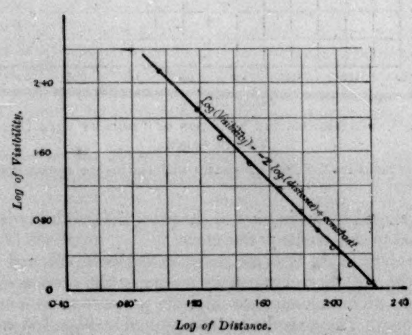

FIG. 6.—CURVE SHOWING THE VARIATION OF VISIBILITY WITH DISTANCE (RED LIGHT).

TABLE V.

White light as distant sources ; white light standard.

Pinhole candle-power $\times 10^{-6}$.	Distance in metres (approximately).	Measured visibility in terms of the standard.	Visibility in terms of 10^{-6} candle-power at 1 metre. Calculated assuming inverse square law.	Error. Percentage between columns 3 and 4.
$101\cdot_5$	$9\cdot1_5$	$1\cdot1_6$	0·120	$-\ 4\cdot_0$
$101\cdot_5$	$15\cdot_2$	$0\cdot41_5$	0·44	$-\ 6\cdot_5$
$101\cdot_5$	$24\cdot_4$	0·18	$0\cdot17_0$	$+\ 5\cdot_0$
1040	$24\cdot_4$	$1\cdot8_0$	$1\cdot7_5$	$+\ 3\cdot_0$
1040	$30\cdot_5$	$1\cdot1_0$	$1\cdot1_5$	$-\ 4\cdot_5$
1040	$45\cdot_7$	$0\cdot51_5$	$0\cdot51_0$	$+\ 1\cdot_0$
1040	$60\cdot_9$	0·27	$0\cdot28_0$	$-\ 3\cdot_5$
1040	$76\cdot_2$	$0\cdot18_7$	$0\cdot18_0$	$+\ 4\cdot_0$
4320	$76\cdot_2$	$0\cdot76_5$	$0\cdot74_5$	$+\ 3\cdot_0$
4320	$91\cdot_4$	0·60	$0\cdot52_0$	$+15\cdot_5$
4320	$122\cdot_0$	$0\cdot31_5$	$0\cdot29_0$	$+\ 7\cdot_5$
4320	$152\cdot_4$	$0\cdot19_8$	$0\cdot18_6$	$+\ 6\cdot_5$

TABLE VI.

Green light distant source ; white light standard.

Candle-power $\times 10^{-6}$ Green light.*	Distance in metres (approximately).	Measured visibility in terms of the standard.	Visibility in terms of 10^{-6} candle-power at 1 metre. Calculated assuming inverse square law.	Error. Percentage between columns 3 and 4.
8·1	$3\cdot0_5$	0·91	$0\cdot8_7$	+4·5
8·1	$6\cdot1_0$	$0\cdot22_5$	0·22	+2·
$51\cdot_8$	$6\cdot1_0$	$1\cdot3_6$	$1\cdot4_0$	$-3\cdot^{6}$
$51\cdot_8$	$9\cdot1_5$	$0\cdot6_1$	$0\cdot61_8$	$-1\cdot^{0}_{6}$
$51\cdot_8$	21·3	$0\cdot11_9$	$0\cdot11_4$	$+4\cdot_5$
216	21·3	$0\cdot4_5$	$0\cdot47_5$	$-5\cdot_9$
216	$30\cdot_5$	$0\cdot2_8$	0·23	0
581	$30\cdot_5$	$0\cdot58_5$	$0\cdot62_5$	$-6\cdot_5$
581	$45\cdot_7$	$0\cdot28_5$	$0\cdot28_0$	$+2\cdot_0$
581	$60\cdot_9$	0·16	$0\cdot15_7$	$+2\cdot_0$
1215	$60\cdot_9$	0·36	0·33	$+9\cdot_0$
1215	$76\cdot_2$	$0\cdot20_5$	0·21	$-2\cdot_5$
1215	$91\cdot_4$	$0\cdot15_3$	$0\cdot14_5$	$+6\cdot_5$
3725	$91\cdot_4$	0·44	$0\cdot44_5$	0
3725	$122\cdot_0$	$0\cdot24^{6}_{5}$	0·25	$-2\cdot_0$
3725	$152\cdot_4$	$0\cdot16_4$	0·16	$+2\cdot_5$

* The method of obtaining the percentage of light transmitted by a coloured glass is described later.

TABLE VII.

Red light distant source; white light standard.

Candle-power × 10^{-6} Red light.*	Distance in metres (approximately).	Measured visibility in terms of the standard.	Visibility calculated in terms of 10^{-6} candle-power at 1 metre.	Error. Percentage between columns 3 and 4.
125	$9{\cdot}1_5$	1·54	1·5	$+2{\cdot}_5$
125	$15{\cdot}2_0$	$0{\cdot}54_5$	0·54	+1·0
125	21·3	0·26	$0{\cdot}27_5$	$-4{\cdot}_0$
125	$30{\cdot}_5$	$0{\cdot}13_5$	$0{\cdot}13_4$	0
519	$30{\cdot}_5$	0·54	0·56	$-3{\cdot}_5$
519	$45{\cdot}_7$	0·26	0·25	$+4{\cdot}_0$
519	$60{\cdot}_9$	$0{\cdot}14_5$	0·14	$+3{\cdot}_5$
2915	$60{\cdot}_9$	0·76	$0{\cdot}78_5$	$-3{\cdot}_0$
2915	$76{\cdot}_2$	$0{\cdot}48_5$	0·50	$-3{\cdot}_0$
2915	$91{\cdot}_4$	0·33	0·35	$-5{\cdot}_5$
2915	$122{\cdot}_0$	$0{\cdot}19_2$	0·20	$-4{\cdot}_0$
2915	$152{\cdot}_4$	$0{\cdot}12_2$	$0{\cdot}12_5$	$-2{\cdot}_4$

which it was observed is multiplied in the same ratio. The log of this derived visibility is plotted against the log of the distance. The line drawn indicates the close agreement to the inverse square law.

The slight variation from the square law in the case of the white light (Fig. 4) is probably due merely to errors of experiment and, in virtue of the results in Figs. 5 and 6, must not be given weight.

Experiment 3.—The result obtained in the last experiment has been confirmed by tests extending over a mile. Two small electric lamps, one a carbon filament and the other a tungsten, calibrated for candle-power and current, were set up at various distances and their visibility measured by means of the visibility photometer.

The following table gives the values obtained, using the carbon-filament lamp:—

Candle-power of lamp.	Distance in metres.	Calculated visibility on inverse square law.	Measured visibility.	Error. Percentage.
0·18	483	$0{\cdot}7_6$	$0{\cdot}8_0$	+5
0·18	644	$0{\cdot}4_8$	$0{\cdot}41_5$	−4
0·69	1098	$0{\cdot}5_7$	$0{\cdot}5_7$	0
0·69	1610	0·26	$0{\cdot}24_5$	$-7{\cdot}_5$

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* The method of obtaining the percentage of light transmitted by a coloured glass is described later.

The following table gives the values obtained, using a metallic-filament lamp :—

Candle-power of lamp.	Distance in metres.	Calculated visibility on inverse square law.	Measured visibility.	Error. Percentage.
$0{\cdot}18_5$	483	$0{\cdot}7_8$	$0{\cdot}7_4$	-5
$0{\cdot}18_5$	644	$0{\cdot}4_4$	$0{\cdot}4_6$	$+4{\cdot}_5$
$0{\cdot}7_0$	1093	$0{\cdot}57_5$	$0{\cdot}5_7$	$-1{\cdot}_9$
$0{\cdot}7_0$	1610	$0{\cdot}27$	$0{\cdot}2_5$	$-7{\cdot}_5$

These readings were obtained on dark, clear nights in April, with the moon behind dense clouds.

The experiments 1 to 3 under this section confirm what would be expected from theory : that the visibility of a point source of light falls off proportionally to the inverse square of its distance from the eye of the observer. Incidentally, it should be observed that Experiment 3 shows that at least, up to ranges of 1 mile, atmospheric absorption is not appreciable on a night which would ordinarily be called clear and dark. Similar results were obtained on a 1-mile range for green and red lights. The two different types of lamp were used, in order to see if the light which had the preponderance of short wave-lengths was more absorbed than the redder light. Under the conditions of the experiment the measurements do not indicate any such difference. Several attempts have been made to work on longer ranges, but up to the present have been unsuccessful, due, among other reasons, to the prevalence of varying local mists.

Having established the fundamental theorems that the visibility of a point source is proportional to : (1) the total candle-power of the source, and is independent of intrinsic brightness ; (2) the inverse square of the distance ; it was possible to choose a suitable unit in terms of which visibility could be expressed.

The unit decided upon as being of a convenient order of magnitude was one-millionth of a candle of 1 metre distance. Thus the visibility of a point of 1 candle-power at 1 kilometre, neglecting absorption, would be equal to this unit, and would, by the eye, be considered equally visible. It is not suggested that a point whose visibility is equal to 2 is twice as bright to the eye as one of visibility equal to 1 ; but the former could have its candle-power halved or its distance increased $\sqrt{2}$ and be made equally visible.

Effect of Slight Illumination of the Field of Vision.—When observing in the open, the field of view on which the distant source is seen is rarely as black as that of the standard source, and thus in relation to the standard its visibility is diminished. In order to eliminate this disturbing factor, it was found necessary to produce equality of illumination of the two fields of vision. This was achieved by illuminating the background surrounding the standard source by means of a high efficiency electric lamp, fitted into the photometer and suitably screened. The effect of observing a point of light against a faintly illuminated background was more fully investigated by arranging a point of light in the centre of a white screen, which could be illuminated to the same degree as the field of vision in the open.

A very bright moonlight night, atmosphere clear, gave a field in the photometer whose illumination was equal to $\frac{5}{1000}$ of a metre candle. Slight mist increased the value of the illumination considerably, a value of $\frac{1}{100}$ of a metre candle having been recorded. Various pinholes were measured in this way for visibility, when viewed in a black field and in fields of different illuminations.

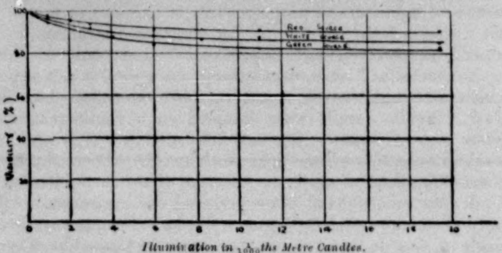

FIG. 7.—CURVES SHOWING THE LOSS OF VISIBILITY OF A POINT SOURCE OF LIGHT WHEN OBSERVED ON A SLIGHTLY ILLUMINATED BACKGROUND.

Red and green lights were also observed, the results being plotted in Fig. 7. The fact that the visibility appears to reach a constant value is rather unexpected, for undoubtedly, when working at a field illumination equal to 1 metre candle the point becomes nearly invisible. The above diagram, therefore, must only be considered as applying to very faint illuminations.

The dimming in each case was about 10 per cent., the green appearing slightly dimmer and the red brighter than the white source, when the field had an illumination equal to $\frac{4}{1000}$ metre candle. This may be considered about an average value for practical observation of lights on clear nights.

Coloured Glasses used in Signal Lanterns.—In connection with signal lights, in which coloured glass screens are placed before a white flame, it is often required to determine by how much the visibility of a point source is reduced by placing a coloured glass screen in front of it. As the visibility of a point source varies directly as the candle-power, this is equivalent to finding the percentage of total light transmitted by the coloured screen. This measurement can be very conveniently and comparatively easily made by means of the visibility photometer. The visibility of a white point source is measured; the coloured screen is then placed before the white source and the reduced visibility measured by the apparatus as previously described. Some green glasses used for signal purposes have been found to transmit only 5 per cent. of the light, whilst others transmit as much as 15 per cent.

The results obtained by different observers are in close agreement so long as they have normal vision, and the figures obtained may be taken as representing the fraction of the light transmitted through the coloured glass, as appreciated by an average eye when observing a point source. Cases of defective vision are discussed later. The application of the laboratory tests to predict the visibility of lights in the open over long ranges has been generally confirmed by such tests as the following made by different observers.

A visibility which was considered as low as could be regarded as satisfactory for practical purposes was fixed in the laboratory in terms of a white point source of known visibility. A green light was then set up in the open, and observers, moving away from the lamp, judged when it could be regarded as equally visible. The candle-power of the lamp without its green glass screen was measured simultaneously by means of a portable photometer.

Calculating the visibility of this light from the value of the transmission of the green glass determined in the manner described, it was found to agree within narrow limits with the visibility previously determined in the laboratory. The visibility thus found as the lowest which was considered desirable for practical purposes was $0{\cdot}12 \times 10^{-6}$ candle-power at 1 metre distance. Using this figure, the candle-power of white light necessary for visibility at any range can be calculated, and by determining the transmission of a glass in the way described earlier, the required candle-power of the light behind the glass can be also calculated for visibility at any range.

It is interesting to note that the authors' results agree fairly closely with those calculated from the report of the Deutsche Seewarte of 1894 (ref. cit.) :—

Range.	Candle-power (British) of white light required in clear air (from report of Deutsche Seewarte).	Candle-power of white light (British units) calculated from results of the authors' experiments.*
1 sea-mile=1,855 metres approximately	0·47	0·41
2 sea-miles=3,710 metres approximately	1·9$_0$	1·6$_0$
5 sea-miles=9,275 metres approximately	11·$_8$	10·$_0$

Effects of Using Spectacles in the Observation of Point Sources.

In the course of the work previously described, unexpected results were obtained when observations were made by people using spectacles. The anomalous results usually occurred when making visibility measurements of green point sources.

Experiments were undertaken with a view to determine the cause of such results, but they are not in any way exhaustive. The Paper contains descriptions of the experiments† together with diagrams showing the results and also what appears to be an explanation of the phenomena. These are summarised as follows :—

(*a*) Experiments showing change in the visibility of a point source when viewed through lenses used as spectacles.

For distances greater than 3 metres, positive lenses produced a marked loss of visibility amounting to 60 per cent. in the case of a 1 diopter lens. The negative lenses for these same distances of observation produced much less dimming ; and in the case of a green point source a lens of 1 diopter (negative) produces a brightening of about 20 per cent.

When used to view a point at distances less than 2 metres, the effects of negative and positive lenses become more nearly equal. These experiments give weight to the supposition that the dimming is due to the image not being focused on the retina, owing to the eye reaching the limit of its accommodation in a negative direction. This was not, however, supported by the appearance of the points of light. They seemed quite well defined, even when viewed through a relatively powerful lens. It was subsequently found, however, that the dimming *was* actually due to the spreading of the light, owing to want of focus, but that with the faint sources employed, the diffused light was not apparent. The above conclusion was arrived at as the result of the following :—

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* No allowance has been made in column 3 for atmospheric absorption, a factor which probably comes in on the 5-mile range, even in very clear weather.

† For complete account of this section of the Paper, *see* "Proc." of Optical Convention, &c.

(*b*) Spherical aberration.

In order to investigate the effect of spherical aberration an experiment is described in which the aperture of the eye was reduced by means of stops varying from 2 mm. to 5 mm. in diameter, but although slightly less dimming occurred for the 2 mm. aperture than for that of 5 mm., yet in both cases the loss of visibility was about 50 per cent. when using a positive 1 diopter lens. The experiment showed a somewhat improved definition of the image of the point when viewed through a smaller stop, but it had very little effect in lessening the relative loss of visibility of the green light when viewed through a positive lens.

(*c*) Chromatic aberration.

Further experiments were made in which the relative dimming of green and red point sources was investigated. The diagrams show how greatly the green light is dimmed, relatively to the red light, when viewed through any positive lens, and indicate conclusively that the chromatic aberration of the eye is mainly responsible for the apparent relative dimming of the green light. It therefore follows that unless the observer using positive spectacles has sufficient negative accommodation available to properly focus a green light at infinity, the latter will appear to him dimmed in proportion to the amount the image is out of focus. This difficulty is not so likely to occur with red light, since objects of this colour do not require so much accommodation in the negative direction to focus them on the retina. The authors do not suggest that the notes they publish are in any way exhaustive. The portions of the subject investigated are merely those which were immediately concerned with the visibility of steady signal lights at sea.

In conclusion the authors desire to express their acknowledgments to Dr. Glazebrook, director of the National Physical Laboratory.

DISCUSSION.

Dr. **Schuster**, in opening the discussion, remarked that the subject was one of considerable practical importance, and the authors seemed to have gone into the question of visibility with great thoroughness.

Dr. W. **Rosenhain** considered that the point of most interest in the Paper was the fact that the authors had found that visibility depended only on candle-power and not on intrinsic brightness. The French lighthouse engineers had always held the reverse to be the case, but there seemed no gainsaying the authors' results, and he thought that some designers would have to modify their present theories.

Mr. **Chalmers** disagreed with Dr. Rosenhain in his view, and thought that there was nothing in the Paper which entailed a modification of present ideas connected with lighthouse design. The Paper did not deal with the use of lens and prism systems.

Dr. **Rosenhain** differed. Quite apart from the question of relation between the size of the source and the lens system in a lighthouse, the French designers had always kept the whole lens system relatively small, on the assumption that for the same candle-power the more concentrated the source, the greater would be the visibility.

Mr. **Rayner** remarked that Mr. Paterson had shown what a difference there was in looking at a small hole (3 mm. in diameter at 3 M.) illuminated by a flame in a dark room, according as a green or a red glass was placed over the hole. In the case of a red glass it appeared sharp and round, but in the case of the green one the hole appeared to be surrounded by brilliant green rays, no doubt produced by some imperfection in the eye.

A further investigation of this phenomenon, in order to see what part of the spectrum gave the effect most strongly in various individuals, would be of interest; and it might be found that by using a coloured screen to reduce the intensity of the rays producing the effect most strongly, a distinct gain might result when the finest definition in a telescope or microscope was required. Such a screen might be of considerable assistance in studying lunar detail or in bacteriology.

Mr. **Trotter** raised the question of what happened when an image of a point of light fell between a rod and a cone on the retina.

Dr. **John Milne** referred to the fatigue of different parts of the retina and to oblique vision.

Dr. S. P. **Thompson** mentioned that in their account of the Paper the authors had not touched on the question of oblique vision, which greatly intensified the visibility of most point sources of light. The question of rods and cones and that of oblique vision had been considered some years ago when the N-rays were discussed. The effects were probably due to indirect vision or fatigue effects.

Dr. R. S. **Clay** stated that better seeing resulted from oblique vision when the illumination was faint. He did not quite understand Mr. Paterson's remarks on astigmatism and the use of spectacles, and whether an observer actually saw less when he used his spectacles. He asked whether any results were obtained on misty nights.

Dr. C. E. K. **Mees** asked what precautions were taken to rest the eye; five or 10 minutes of rest were not nearly sufficient, he had sometimes needed more than an hour. It depended upon the colour, and the recovery of the eye sensitiveness for red was much quicker than for green.

Mr. **Paterson**, in reply to Dr. Rosenhain and Mr. Chalmers, stated that there were two questions involved. High intrinsic brilliancy might in many cases be desirable from the point of view of the design of the refracting and reflecting systems of a lighthouse, but in the authors' view there appeared to be no object in seeking high intrinsic brilliancy for the light as a whole when viewed from a distance without lens systems. They hesitated to say whether a misty condition of the atmosphere would affect this conclusion. In reply to Mr. Trotter, they thought that the way in which the rods and cones acted was very obscure, but it would be safe to say that the image of the point would hardly remain stationary during observation, but would be in a condition of rapid movement over an appreciable area of the retina. Answering Dr. Thompson, they explained that the question of oblique vision was considered in considerable detail in the Paper and was a factor of great importance. The green light was seen better by oblique than by direct vision, and the red was seen better by direct than by oblique vision. The question raised by Dr. Clay as to the use of spectacles was entirely a matter as to whether a spot of light was really focused properly on the retina. Often a light appeared to the observer to be focused, but, in reality, it was surrounded by circles of diffusion too faint to be noticed, yet sufficient to cause serious diminution of visibility. Hence, if an observer's spectacles were correct for nearer vision and were slightly too positive for distant vision, he would find great diminution of visibility of the green light. The point raised by Dr. Mees was of importance. All the authors' measurements were taken either after long periods in the open at night or in the dark room of the laboratory, and discrepancies were not noticed which could be attributed to a differential fatiguing effect.

L. *Some Points in the Use and Design of Refractometers.* *By* THOMAS MARTIN LOWRY, *D.Sc., F.C.S.*

1. *Choice of Wave-lengths.*—During many years it has been customary in measuring the refractive indices of solids and liquids to select as the standard wave-lengths the D line of sodium and the C, F and G lines of hydrogen. This choice was probably based solely upon expediency, the sodium flame and the hydrogen vacuum tube forming part of the most commonplace laboratory equipment; the four lines also appear in the form of absorption bands as Fraunhofer lines in the solar spectrum, and served as convenient points of reference at a time when sunlight was still relied upon as one of the main illuminants in optical work. On other grounds the choice would be difficult to justify, but the force of tradition and the necessity for keeping in touch with previous measurements have up to the present sufficed to compel the majority of experimenters to adhere to the traditional standards.

2. *Disadvantages of the Present Standards.*—The disadvantages of the existing standards are obvious. The principal standard is a doublet, and, of the three hydrogen lines used to express the dispersion, only the first (the red C line) is of any merit, whilst the third (the violet G line) is so weak as to be difficult to use even under favourable conditions. The best proof of the inadequacy of the existing standards is to be found in the fact that they have been so widely abandoned in all work outside the ordinary routine—*e.g.*, in Mr. Gifford's work on *optical glasses*, in the work of C. and M. Cuthbertson on the *dispersive power of gases*, in *interferometry*, as in Tutton's wave-length comparator, where he has followed the example of Michelson, who selected as the basis of his *standard measurements of wave-length* the lines

Cd 6438·4722, Cd 5085·8240, Cd 4799·9107;

finally, Dorn and Lohmann, in their measurements of the *refractive indices of liquid crystals*, by a total reflexion method, have used the series

Li 6708, Na 5893, Hg 5461, Hg 4359,

identical with that which the writer, after several years' experience with nearly 30 wave-lengths, has found to be the only satisfactory series for routine work in the measurements of *optical and magnetic rotatory dispersion*.

3. *Suggestions for New Standards.*—The ideal series of standard wave-lengths would undoubtedly include Michelson's three cadmium lines, but, unfortunately, the enclosed cadmium arc is still a curiosity which has been seen and handled by very few workers; the open silver-cadmium arc, described by the writer in 1909, has rendered the cadmium spectrum more accessible, but is not altogether suitable for general laboratory use. At the present stage of development there can be little doubt that the mercury spectrum must figure largely in any new standard series of wave-lengths. The green mercury line Hg 5461 has, indeed, already displaced the sodium doublet as principal standard in all Cuthbertson's later work on the dispersive power of gases, and is rapidly displacing sodium for everyday use in polarimetry and saccharimetry; it has proved to be the only possible standard in the author's experiments on the optical rotation of long columns of quartz. Its spectral purity is little inferior to that of the cadmium lines, whilst its steadiness and ease of production render it capable of widespread acceptance. The violet line Hg 4359 is the only line in the blue or violet region of the spectrum with which polarimetric measurements can be satisfactorily made; in refractometry it has the advantage that it lies so close to the hydrogen line H 4341 that it is scarcely possible to distinguish the two lines; but in a vacuum tube containing both hydrogen and mercury, the mercury line, being slightly less refrangible, dominates the hydrogen and gives very satisfactory readings. In my own practice I have used the green and violet mercury lines for all essential measurements; auxiliary measurements have been made with the sodium doublet, in order to keep in touch with earlier measurements, and with Li 6708 as the best and deepest of the red lines at present available. The series Li, Na, Hg, Hg is identical with that arrived at independently by Dorn and Lohmann, and appears to be the most suitable for general use at the present time.

4. *Form of the Dispersion Curves.*—The difficulties outlined above would, to a large extent, disappear if the form of the dispersion curves were accurately known. The writer has found that, in the case of a large number of simple organic compounds, the optical and magnetic rotatory dispersions may be expressed by simple formulæ of the type, $\alpha = \frac{a_0}{\lambda^2 - \lambda_0^2}$. In this equation the absolute optical or magnetic rotatory power of the substance is expressed by a single constant a_0, whilst the

rotatory dispersive power is expressed by a second constant λ_0. When a formula of this sort has been established, the actual wave-lengths employed in determining the constants are of very little importance, as any suitable pair should give the same results. Refractive dispersion is obviously a more complex property than rotatory dispersion, and must involve at least three constants and measurements with three wave-lengths. But in the opinion of the writer it is almost a waste of time (to which he himself must plead guilty) to multiply observations of $n_C n_D n_F n_G$, or any analogous series, until serious work has been undertaken on the dispersion law for typical organic compounds in the infra-red, visible and ultra-violet regions of the spectrum. Work in this direction has been carried out in the case of crystals, but no serviceable range of data is yet available for the organic compounds which form the chief subject of refractometric investigations.

5. *Calibration of a Refractometer.*—The experiments of Mr. Gifford have shown that marked variations of refractive index occur in prisms of the same glass, even when taken from different parts of the same melting. It is, therefore, unwise to accept the tabulated data for a refractometer without checking the actual performance of the instrument. The most convenient method of checking the instrument is by means of a piece of quartz, cut so as to give both the ordinary and the extraordinary ray; fluorite, calcite and silica may also be used. Mr. Gifford, who had already published measurements of the refractive indices of these substances for 26 wave-lengths, kindly undertook, at the author's request, to add values for those lines in the spectra of lithium, cadmium and mercury, which promise to be of greatest importance as standards in future measurements; the values obtained are reproduced in the following table to five places of decimals :—

Table of Refractive Indices.

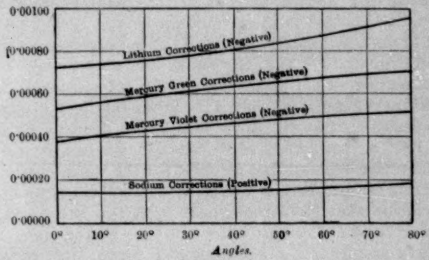
Wave length.	Fluorite.	Quartz (ordinary ray).	Quartz (extra-ordinary ray).	Calcite (ordinary ray).	Calcite (extra-ordinary ray).	Vitreous silica.
6708 Li	1·43226	1·54146	1·55047	1·65367	1·48427	1·45607
6438 Cd....	1·43272	1·54231	1·55135	1·65503	1·48468	1·45677
5461 Hg ...	1·43499	1·54617	1·55534	1·66165	1·48789	1·46015
5086 Cd....	1·43619	1·54824	1·55748	1·66527	1·48954	1·46190
4800 Cd....	1·43691	1·55013	1·55945	1·66861	1·49110	1·46357
4359 Hg ...	1·43952	1·55380	1·56323	1·67518	1·49410	1·46674
4046 Hg ...	1·44153	1·55715	1·56671	1·68134	1·49691	1·46968

The actual calibration of a Pulfrich refractometer with quartz gave the following figures :—

	Li 6708	Na 5889	Hg 5461	Hg 4357
Ord. ray.	1·61687	1·62210	1·62635	1·64454
Ext. ray.	1·61687	1·62212	1·62634	1·64450
Mean	1·61687	1·62211	1·62634	1·64452

The nominal refractive power of the prism for sodium light was 1·62197, an error of 0·00014.

6. *Adaptation of Refractometer Tables to New Series of Wave-lengths.*—From the data given above it is possible to deduce the refractive power of a liquid from the angular readings of the refractometer for the new series of wave-lengths. The trouble of calculation may, however, be avoided by combining the existing tables with a series of correction curves (*see* Fig.).



Thus, for

Lithium, *take* μ for Na+H red corr. (neg.)+curve corr. (pos.).

Sodium, *take* μ for Na+curve corr. (pos.).

Hg green, *take* μ for Na+H red corr. (pos.)*+curve corr. (neg.).

Hg violet, *take* μ for Na+H violet corr. (pos.)+curve corr. (neg.).

For these curves, and for the data on which they are based, I am indebted to Mr. F. Tinker, B.Sc.

7. *Temperature Regulation.*—The water-jacketing usually supplied with the Pulfrich refractometer is very inefficient and far below the standard of modern requirements. All the leads are of such narrow bore that it is impossible to drag any

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* Note the reversal of the sign.

adequate flow of water through them even with the help of a powerful suction pump. It is therefore impossible to keep the temperature of the flow even approximately the same as that of the water in the thermostat, unless the room be heated practically to the same temperature and the bath kept cool by a flow of cold water acting in opposition to the usual regulated heater. The necessity of working at room temperature is also obvious from the fact that there is no external jacketing to the liquid, which is merely surrounded by a piece of wood. The inadequacy of this arrangement was proved by actual tests. With the room at 20°C. and the water circulation at 20° a micrometer reading of 1° 26·3′ was obtained for one of the octyl alcohols; with the whole apparatus at room temperature at 16·1°C. the reading was 1° 7·6′, a change of 4·8′ per deg. C. With the room at 16·1° and the bath at 20°, the highest temperature that could be reached in the refractometer was 19·7°; but the reading of the instrument was then 1° 22·9, corresponding with a temperature of 19·3°, *i.e.*, the liquid was actually 0·4° below the temperature of the water-flow in the refractometer, for a temperature difference of only 4°. From this test it appears very probable that at 60° there would be a discrepancy of 4° and at 100° a discrepancy of 8° between the temperature of the liquid under test and that recorded for the water circulation by which its temperature is nominally controlled; certainly all measurements that have been made with this instrument in the higher ranges of temperature must be looked upon with very grave suspicion. A refractometer prism, with a jacket designed to overcome these defects, has been constructed; but, at the time of writing, the instrument has not yet been completed, and it is not possible to describe any actual tests of the efficiency of the jacketing.

LI. *Some Recent Advances in the Measurement of Light and Illumination.* *By* J. S. Dow *and* V. H. Mackinney.

Perhaps the most interesting feature of the progress of the past 10 years has been, not so much the actual advance in the construction of apparatus, as the change in the view-point from which measurements of light have been regarded. Special refinements have been introduced in the laboratory where photometry has been carried to a high stage of precision and accuracy. But at the same time there has been a growing recognition that the measurement of light and illumination is not only of considerable industrial consequence, but a fairly simple and feasible process in the hands of others besides the expert. For practical purposes the actual illumination in a room, on a table, desk or paper, is often of much greater importance for the layman than a knowledge of the amount of light given by the lamps employed.

Many instances could be mentioned of the value of such measurements. Determinations of the illumination on a table or desk, for example, enables us to judge whether the light is sufficient for reading, in a school or library, or for some industrial process in the factory. They also afford a ready means of comparing the concentrating power of different kinds of shades and reflectors, and, as will be seen later, have many other subsidiary uses in connection with the measurement of reflecting power, absorption of glasses, &c. In practice, the chief requirements in an illumination photometer are that it should be compact and portable, and capable of being carried about with the ease of small hand cameras, that it should be easy and quick to manipulate, sufficiently accurate for practical purposes, and readily set right should the reading, for any reason, vary in course of time.

In order to secure the twin qualities of simplicity and portability some concession as regards accuracy may well be made. For the great majority of practical purposes it may safely be said that an accuracy of 5 or 10 per cent. is ample, while one can imagine circumstances in which an even greater scope might be permissible.

Indeed, it might, perhaps, be suggested that a somewhat excessive stress has sometimes been laid on accuracy, essential as this doubtless is in the laboratory. For the complexity which this extreme accuracy entails prevents the ordinary man from making use of photometry, and limits its practical applications. Since the authors became interested in this subject, they have been struck by the great variety of uses to which photometry can be put, and many most interesting and unsuspected applications have come to light.

During the last few years the Illuminating Engineering Society has been instrumental in showing the great value of measurements of illumination in the school, library and workshop, and this latter question has recently received close attention from the Home Office. Without some means of measurement it would be difficult to specify the exact conditions of illumination required for certain practical processes. But once we have available simple instruments for the purpose the possibility of doing so becomes much less remote.

A Simple Apparatus for the Measurement of Illumination.

In 1909 the authors had the pleasure of describing a compact and portable instrument for measuring illumination before the Optical Society. This has since been improved and remodelled and is now known as the "Holophane Lumeter." It is hardly necessary to describe this instrument again, but a few salient points in its construction may be noted. The most novel feature is that it can be used not only to measure illumination, but also surface brightness. The general appearance of the instrument (which is only 5¾ in. by 4½ in. by 1¾ in.) is shown in Fig. 1.

In order to increase the range of the instrument, recourse is had to two dark glasses reducing the light by $\frac{1}{10}$ and $\frac{1}{100}$ respectively. These can be placed in the path of the rays from the object studied. By this means the reading can be multiplied by 10, 100 or 1,000, so that values up to 2,000 ft.-candles can be measured. The choice of a suitable glass, which is sufficiently opaque, uniform and neutral for the purpose, naturally demands some care, but the most recent variety adopted has proved to have very good qualities in this respect.

The small lamp used in the instrument is fed from a 4-volt accumulator packed in the case of the instrument, and the whole can be carried about as easily as a small hand camera. The instrument can be set to read correctly, by reference to a given illumination, by simply adjusting the position of the lamp slightly. In determining surface brightness one merely points the instrument at the object to be viewed and compares its brightness with that of the illuminated plate inside. When measurements of illumination, as distinct from surface brightness, are to be made, all that is necessary is to observe the brightness of a standard mat celluloid screen supplied with the instrument, and this is also used in calibration.

A useful addition for daylight measurements is a small cap provided with a special yellow Wratten gelatine screen, which makes the transmitted light identical to the eye with daylight, and so removes the difficulty experienced by many observers in comparing sources of light which differ in colour. For relative measurements (such as mostly occur in daylight work) the amount of light absorbed by this screen may be ignored. It is, however, quite easy to determine the approximate absorption and allow for it when absolute measurements are desired.

Some Practical Applications of Illumination Measurements.

Allusion has already been made to the value of measurements of illumination in practice. Not only do such measurements constitute a valuable record, but they enable the lighting engineer to verify his calculations and to demonstrate to consumers the effect of a certain change in their installation. It is not uncommon to meet consumers whose personal impression leads them quite astray in judging an alteration of this kind, their views being based on examination of the apparent brightness of the lighting units rather than the actual working illumination provided. By the aid of a simple measuring instrument the engineer has little difficulty in showing that the recommended form of shade and reflector leads to an increase in the illumination in desirable directions. In cases in which the present installation clearly gives too low an illumination a measurement tells him how much it must be increased, and what types of lamps and fixtures ought, therefore, to be introduced. A study of the transactions of the Illuminating Engineering Society during the last two years will serve to show how large a part actual measurements of illumination in schools railways, libraries, &c., has played. But it may be observed that in many of the problems in illumination that have recently come to the front, measurements of surface brightness are demanded to an equally great extent. For example, in considering the comparative advantage of direct and indirect lighting, the respective brightness of the walls and ceiling, and the distribution of this brightness are all-important. In preserving a record of the conditions of illumination in a room it is often desirable to map out, not only the illumination on the working level, but the brightness of surrounding objects. Again, another point which is now exciting much attention is the desirable intrinsic brilliancy of the sources themselves.

In connection with industrial processes, again, it is a great advantage to be able to measure illumination in any plane, and even in places which are only accessible with difficulty. In many cases this can be done easily with an apparatus utilising a separate test-plate. The standard screen can be readily placed at any point where it is desired to take a measurement and observed from a distance. When it is only desired to obtain relative values and to determine surface brightness, it is not necessary to do even this, since one has only to point the instrument at the object to be examined.

The facility to measure illumination and surface brightness carries with it the possibility of determining the relative reflecting power of a surface. All that is necessary is to measure the surface brightness at a given spot and then to place the white celulloid test-card in position and measure the illumination. A simple calculation shows the relative reflecting power of the surface as compared with the test-card. This method can be readily applied to the wall-papers in any room lighted by natural or artificial light.

It may be observed that the reflecting power of coloured materials depends to some extent on the quality of light by which they are illuminated, and the same results will not be secured by natural and artificial light. This fact has been recently utilised by Mr. T. E. Ritchie,* of the Union Electric Co., as a means of testing the resemblance of various artificial illuminants to daylight. A series of ribbons of delicate shades of colour was prepared, and the reflecting power of the series tested with the holophane lumeter by good average daylight. The tests were subsequently repeated with a series of artificial illuminants, and it was found that the closest approximation to the daylight values occurred in the case of indirect arc lighting.

One other quality of reflecting surfaces, which is not only of consequence to the lighting engineer, but plays a part in many physical problems, is their "matness." An interesting illustration is afforded by the silvered or powdered aluminium screens sometimes used for cinematograph screens. When such screens are used, the majority of the light is reflected directly back to the audience, so that the method is only applicable in fairly long and narrow rooms. From some tests which one of the authors recently carried out on materials of this kind, it would appear that the brightness of the image on a cinematographic screen (usually of the order of ·1 ft.-candle) might be increased as much as 10 times under favourable circumstances.

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* "Colour Discrimination by Artificial Light," "Illum. Eng." (Lond., Feb. 1912.)

The same appliances which serve for the measurement of reflecting power can also be used to measure the absorption of glasses, gelatines and solutions. This is a point of interest to the optical industry, for it is well known that much remains to be done in the matter of producing glass of specified absorbing power.

It may be suggested that photometry might often be applied by meteorologists and others interested in climatic variations. Many of the investigations of the variations in daylight from day to day, which are now tolerably complete, have been carried out by men like Weber, of Kiel, and Basquin, of Chicago, with reference to daylight illumination in schools. Such researches are also interesting to the meteorologist and the photographer. In addition to measurements of illuminations, there is no difficulty with our instrument in studying the surface brightness of external objects and even (when the sky is not too bright) making observations on clouds. There is no inherent difficulty in measuring the very low orders of brightness such as are met with by night, and the authors have suggested at a recent meeting of the Royal Astronomical Association that simple photometric methods might readily be applied to obtain useful information regarding the brightness of the night sky, comets and stars, &c., with much greater precision than at present.

As an illustration of the use of the instrument in studying daylight conditions, it occurred to the authors that it would be interesting to take measurements of the illumination during the recent eclipse of the sun on April 17. The measurements were made indoors in shadow, a position being selected where the initial illumination was of the order of 100 ft.-candles. Photographs of the solar disc were taken simultaneously with the measurements. It was interesting to notice that the illumination at totality at about 12:10 p.m. was about 8 per cent. of the illumination when the eclipse commenced, almost exactly what the percentage area of the disc obscured (92·7 per cent.) would suggest. One would suppose that records of this kind would be of considerable service to astronomers. The process is extremely simple and could be applied in exactly the same way to an eclipse of the moon.

There remains one very interesting application of measurements of surface brightness which deserves mention. In April, 1911, the authors read a Paper before the Royal Photographic Society in which the value of such data in connection with photography was pointed out. It is surely of interest to photographers to study the variations in the brightness of objects both by natural and artificial light, and the corresponding allowance that should be made in determining the exposure. The fluctuations in daylight are so enormous, and the range of tone in such subjects as clouds, landscapes and the interiors of churches so extreme, that it must often require long experience and skilled judgment to get a successful result.

In the case of artificial light the difficulty of judging the exposure is often considerable and the actinometer cannot readily be applied. The authors' interest in this application of the holophane lumeter was aroused by the difficulty experienced in getting successful photographs of artificially lighted interiors for purposes of illuminating engineering. In many such cases it is desired, not only to make a pretty picture, but to reproduce the play of light and shadow and the distribution of illumination with exactitude, and to show the actual sources of light without halation. In many cases, too, retouching is not permissible.

In work of this kind photometry and photography go hand in hand, the same instrument conveniently serving both to take measurements of illumination from an engineering standpoint and as a means of judging the exposure.

It may be urged, however, that the value of measurements of surface brightness in photography is especially great in research; for example, in studying the complicated questions connected with the appearance of coloured objects to the eye and their reproduction on the photographic plate.

A New Apparatus for Studying the Distribution of Light.

The determination of polar curves of light distribution, and the somewhat elaborate calculations by which the mean spherical candle-power is determined, form possibly the most laborious part of commercial photometry. A great deal of ingenuity has recently been expended on the simplification of these calculations. But there seems room for more to be done in the direction of facilitating the experimental part of the work, and the authors have recently devised an apparatus which seems to have merits in this respect. It may be mentioned that a similar method has been worked out by Dr. Clayton Sharp, and was described in a lecture before the John Hopkins University, Baltimore, U.S.A., in 1910. The device employed by the authors was, however, worked out independently as a result of experience with the holophane lumeter.

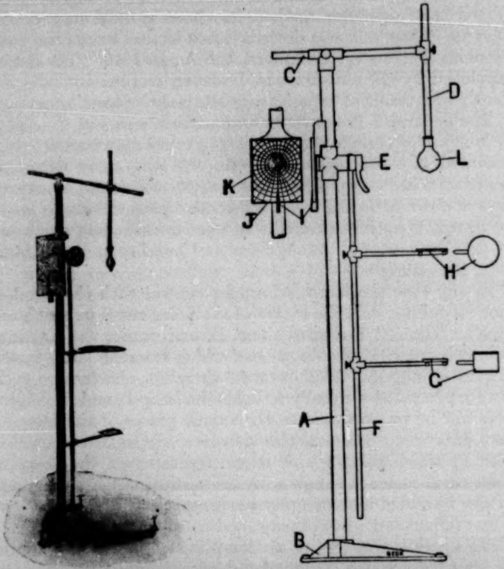
The apparatus is shown diagrammatically in Fig. 2. Fig. 3 is from a photograph illustrating its general appearance. It is mounted on a substantial base, B; the lamp to be tested, L, being attached to an adjustable rod, D, which can be moved up and down or to and fro. When more powerful lamps are to be tested, it may be desirable to remove this arm (which can be done by means of the sliding piece C) and to hang the lamp from the ceiling.

In any case the lamp is brought central with the point E (corresponding with the centre of the polar curve paper shown at K). The rod F rotates about E, and carries the standard celluloid surface G, the distance of which from the lamp, tested can be adjudged. As F rotates, therefore, the face of G is always presented vertically towards the lamp L, and its brightness will be proportional to the candle-power of the lamp in that direction. There is also a pointer, I, rigidly connected with F, which rotates with it and indicates on the sheet of polar curve paper the angle with the vertical which F takes up. It may be added that this paper is arranged to be a convenient height (about 5 ft.) from the ground.

The operation of testing the lamp is as follows: A lamp of known candle-power is first placed in position. The standard surface G is observed through the holophane lumeter, and its distance on the rod is adjusted until the reading of the instrument is some convenient sub-multiple of the known candle-power in that direction. (For example, if the candle-power were known to be 16, the reading on the instrument might conveniently be 1·6 ft.-candles.) G is then clamped in position, and the lamp to be tested is constituted for the standard lamp.

The arm F is then rotated, the brightness of G being noted in each case, and the corresponding candle-power marked on the polar curve paper opposite the pointer. The curve is thus traced out at once, while the experiment is in progress, and there is no need for subsequent calculations.

When great accuracy is not necessary it might be possible to use this apparatus even in rooms which have not black walls, and in which there is a certain amount of stray light.



FIGS. 2 AND 3.—NEW FORM OF POLAR CURVE APPARATUS.

A, main support; B, pedestal; C, adjustable horizontal support; D, adjustable vertical rod carrying lamp to be tested; E, pivot about which F rotates; F, rotating rod, carrying celluloid screen; G, celluloid screen; H, screen for cutting off direct light; I, pointer; J, plate on which Polar Curve Paper is placed; K, Polar Curve Paper; L, lamp to be tested.

Under these conditions we make use of H, which is an adjustable black screen, and which can be inserted in any convenient position between the lamp and G. If, therefore, there is any question of the readings being affected by stray light, a reading is taken first without H in position. The brightness of G is

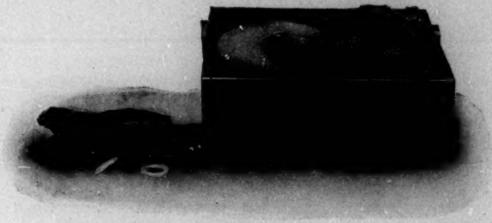


FIG. 1.

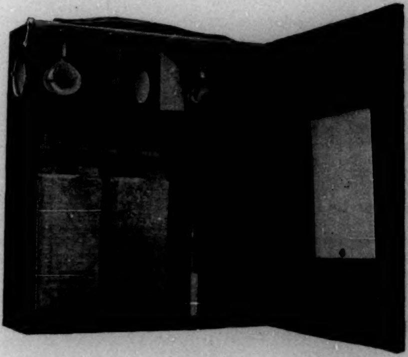


FIG. 4.

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then proportional to the illumination due to the lamp tested plus the stray light (if any). The screen H is then inserted, blocking out the direct light from the lamp tested. The brightness of G is now due to stray light only. Hence, by subtracting the latter reading from the former, we get the true value, *i.e.*, reading without dark screen—reading with dark screen = true candle-power.

The advantages of this method may be summarised as follow :—

1. The quickness and convenience with which the results are obtained.

2. The polar curve is worked out automatically while the experiment is in progress ; there is no calculation necessary.

3. The apparatus is simple and portable, and can readily be moved from room to room, which an elaborate photometric bench, equipped with heavy and large mirrors, cannot.

4. Only one observer is needed for the photometric manipulations, and he can make all the observations in one position, there is no necessity to be continually crossing the room to adjust mirrors, &c.

5. By the use of the special screens provided, the apparatus can be used in an emergency in an ordinary room without darkened walls, and allowance can be made for stray light as described above. It is expected that the apparatus will be most useful for lamps of moderate candle-power (although the fact of the holophane lumeter reading up to 2,000 ft.-candles obviously enables very powerful sources to be tested).

The Holophane Portable Standard of Light.

Some experiments have also been made on a new form of portable standard of light, and Fig. 4 is a photograph of the first rough model constructed by Messrs. Beck. It consists essentially in a low-voltage tungsten lamp which is fed by a portable accumulator packed away in a convenient form of case. A mounted celluloid screen attached to a graduated metal tape is also provided in the case and the screen can thus be set up any desired distance from the small tungsten lamp, which acts as a secondary standard, and any desired illumination produced. It is, of course, also possible to illuminate any other photometric surface by this lamp, and thus use the apparatus for checking comparison lamps used in photometry in the usual way.

The most novel part of the apparatus is the means employed of maintaining the voltage across the standard tungsten lamp constant. This is accomplished as follows: A small carbon filament lamp is run in parallel with the tungsten lamp and in series with both is placed a suitable adjustable rheostat. (This can be seen on the right at the back of the box in the illustration.) Between the two lamps is placed a specially designed form of photometric screen by the aid of which their intensities can be compared. The two photometric surfaces are inspected through an aperture on the side of the box furthest from the observer. The exact nature of this screen need not be described in detail. It is sufficient to say that the observer sees three slots. When the centre one is in balance that on the left hand is too bright and that on the right hand too dark; the side slots thus act as "gauge glasses." In order to facilitate accurate judgment a yellow glass may be placed in front of the aperture so as to make the colour of the light from the two lamps appear the same.

The action of this part of the apparatus depends on the fact that any change in the P.D. supplied by the cell affects the two lamps unequally, and therefore puts the screen out of balance. Suppose, for example, that the voltage of the cell falls by 1 per cent. The candle-power of the carbon-filament lamp will be reduced by 6 to 7 per cent., and that of the tungsten lamp by 3 to 4 per cent. This difference is immediately revealed on the photometric screen, and it will be observed that the resulting change in brightness is almost the same as that of the standard, so that the device is a very sensitive means of detecting any change in P.D.

It will be observed that this portable standard acts as a temporary custodian of the standard of light in a somewhat similar way to the holophane lumeter, but over longer periods of time and with greater accuracy. It is first set from a suitable laboratory standard, and is always capable of readjustment to the correct value should it vary. Having once been set, however, it should be suitable for use for a long time and owing to its portable nature can be readily carried about from place to place or sent away to be re-checked very easily.

Work in Technical Institutions and Colleges.

Simple photometric instruments should be specially appreciated in technical institutions and colleges in which space is often limited and it is often desirable to transfer experiments from one room to another.

It may safely be said that the experiments carried on to-day are often somewhat out of touch with practice and are needlessly laborious to the student. In a modern laboratory labour-saving devices avoid much of this routine work, and there is no reason why the student, once he has mastered the fundamental principles, should not make use of these simplifications. The psychological effect of this is unfortunate, because the student receives an unjustified impression of the difficulties and uncertainties of photometry.

So much time is often expended on the study of his tools that he does not arrive at the stage of using them. Yet, as a matter of fact, there are numberless important applications of photometry which are both useful and interesting as laboratory experiments.

What is demanded is the simplification and popularisation of methods of measuring light and illumination ; and in this Paper the results of some experiments in this direction have been described. But much still remains to be done, and we may hope to see remarkable progress in this direction during the next few years.

Mr. A. P. TROTTER said that several kinds of photometers had been invented for measuring surface brightness, but none had come into practical use until the authors had produced this excellent apparatus. They had not only produced a very portable instrument, but had done a large amount of practical work with it. The result was that it seemed likely that some branches of industrial photometry would be revolutionised.

The primary measurement in photometry was that of candle-power. The next was that of illumination, with the unit of 1 ft.-candle which, for all practical purposes, was equal to 1 candle-metre. The third was surface-brightness—not a good term—the unit of which was the brightness of a perfectly white surface illuminated with 1 ft.-candle. The authors used a test-plate of white celluloid. This could be used without correction for measurement of illumination, but in expressing surface-brightness, the albedo of the test-plate should be allowed for. It was probably about 0·85. There was no advantage in trying to get nearer to unity ; but measurements of surface brightness had no meaning unless they were referred to an albedo of unity, as well as to an arbitrary unit of illumination.

It was not surprising that others were adopting photometers for the measurement of surface brightness. Mr. Edgcumbe was showing one in the Exhibition, and he (Mr. Trotter) had been so impressed with the importance of measurements of surface brightness that he had designed an attachment to his portable photometer to enable such measurements to be made.

The new development in practical photometry which was now possible was the measurement of illumination in inaccessible places or on a plane, such, for example, as the front of a telephone switchboard, where an illumination photometer could not be placed.

Mr. R. S. WHIPPLE referred to the point concerning the constancy of the light intensity of the portable standard of light. Waidner and Burgess had dealt with this problem in their examination of Holborn pyrometers. If the lamps were run for eight hours at an overload, and then always at an under-load, the candle-power would not vary for years. That might be advisable in this case.

Mr. C. C. **Paterson** thought the standard exceedingly convenient; it might take the place of the moving-coil amperemeter. The holophane lumeter had successfully been tried on his suggestion in coal mines.

Mr. **Chalmers** had also used the holophane lumeter for measurement in connection with the obtaining of even illumination for vertical surfaces and had found it possible to make measurements very rapidly and sufficiently accurate for his purpose, an accuracy of from 5 to 10 per cent. being obtained.

Prof. S. P. **Thompson** dwelt upon the importance of the instruments for legal disputes as to the amount of light; surveyors had no longer any excuse for being inaccurate now they were supplied with numerical data.

Dr. **Schuster** added that architects should likewise study the problem; so far all they could tell the inquirer was that so many square feet of flooring required so many square feet of windows; but they could not give any explanation as to the distribution of the windows.

LII. *The Measurement of Colour.* *By* A. C. Jolley *and* A. J. Bull.

This Paper deals with a series of experiments made with a view to ascertaining the form of optical instrument most suitable for the measurement of colour. All forms of colour-mixing arrangements in which intermittent light is required were omitted from these experiments, because the authors have found that intermittent illumination produces a somewhat rapid fatigue of the eye and the readings soon become of little value.

The standard colours employed may be derived either from the spectrum of white light or from colours produced by selective absorption. The latter suffer from the disadvantage that abrupt endings to the chosen regions cannot be secured nor a uniformity of transmission over the selected region, so that such colours must be somewhat arbitrarily chosen.

Methods most likely to be useful are those based upon the Young-Helmholtz's hypothesis. The central fact of this hypothesis is that it is possible to imitate any colour by the admixture in suitable quantities of the lights of three narrow regions of the spectrum, provided these are suitably chosen, and we have found that the only three with which we could match all shades were, for the blue primary a narrow region from 0·463 to 0·466μ, for the green the region from 0·529 to 0·535μ, and for the red from 0·634 to 0·644μ. Instead of adding these coloured lights broader primaries may be subtracted from the white light by the selective absorption of suitably chosen dyes, but in this case the difficulty of expression becomes a serious one, owing to the impossibility of securing dyes which absorb one primary region and transmit the remainder on the white light in equal proportions. This is indicated by the absorption curves in Fig. 1, which are those of a set of dyes carefully selected for the purpose.

The authors found the following experimental arrangements answer well for the matching of colours. Fig. 2 shows a method of employing monochromatic primaries. The colour under test is placed half-way over a transmission grating, the coloured patch and a portion of the grating being surrounded with black velvet. The lamp L provides the illumination for the colour-patch, whose image half fills the field of the lens R, when the observer's eye is at the slit S. At a distance of 6 or 8 metres three straight filament lamps are placed in such positions relative to the grating that each filament and the observing slit are conjugate for some particular wave-length.

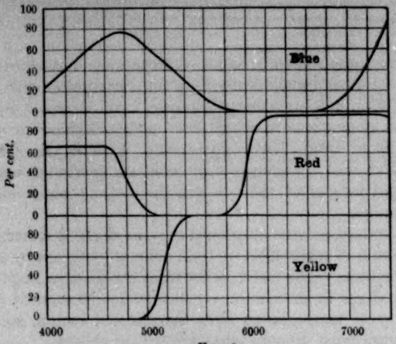

FIG. 1.

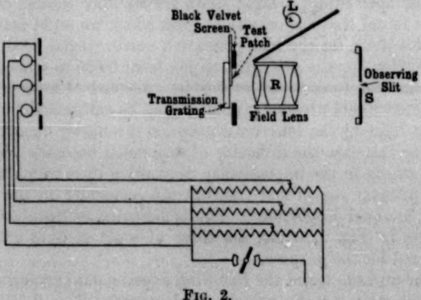
L
Black Velvet Screen
Test Patch
R
Observing Slit
S
Transmission Grating
Field Lens

FIG. 2.

As diffraction effects interfered with the use of a very narrow slit a short range of wave-lengths, comprising some 50 tenth-metres was usually employed. The voltage at the terminals of each lamp was made variable by connecting it across a potentiometer resistance placed under the observer's hand. In this way the intensity of each luminant was completely under control. The wave-lengths of the transmitted primaries were determined by means of a Hilger wave-length spectroscope placed at the observing slit. With this arrangement it was found that all pigment colours could be accurately matched, but that different eyes, sometimes the two eyes of the same individual, required different amounts of the primaries. So that, although the matchings were perfect, the arrangement did not lend itself to the specification of a given hue. It was found that with broader bands the proportions of the primaries were the same for different eyes, and Fig. 3 shows the plan of an arrangement for using these broader bands. It consists of a reversed spectroscope. The light from a white diffusing surface, A, which is lit by the lamp L, passes through three broad slits in the screen B. The grating C was half covered by the patch of colour under observation and was illuminated by light reflected from L by a surface-silvered mirror. The variation of the amounts of the primary lights was effected by sliding across the apertures in B three long-graded wedges of lamp-black in gelatine. This arrangement, although not as perfect as the previous one, was sufficiently good to show that the matches obtained were correct for eyes differing considerably in their colour sensitiveness, and, therefore, indicated that instruments employing broad-banded primaries are more likely to yield comparative results than those employing narrow ones.

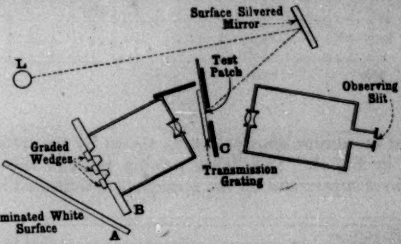

FIG 3.

Red, green and blue primaries produced by selective absorption were also tried, although these suffer from the defects that, except in the case of the red, it is not possible to obtain colour filters which transmit a large proportion of a given region terminated by abrupt absorptions, as shown by the transmission curves in Fig. 4. The lights passing through these colour filters were superposed by the arrangement indicated in Fig. 5.

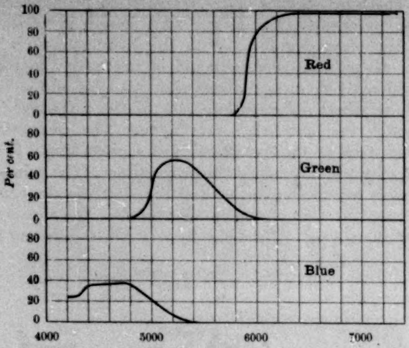

FIG. 4.

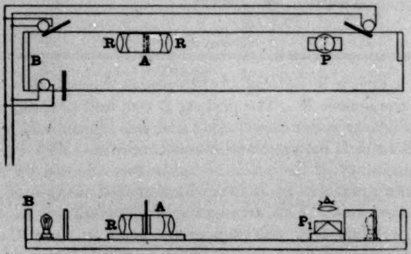

FIG. 5.

The two components of a large rectilinear lens were mounted in a framework and three variable apertures in the screen A covered by selected red, green and blue broad-banded filters.

A photometer, P, devised by Dr. C. V. Drysdale, and consisting of two right-angled prisms mounted with their edges in contact, was placed in such a position that the lens focuses the illuminated surface B on the photometer, therefore, one-half of the photometer is filled with light transmitted by the apertures in A, while the other receives light from the test patch which is illuminated by a lamp connected in parallel with those illuminating B. The matches were made by varing the apertures in A. This is the simplest arrangement tried, and the colours could be specified in terms of the apertures.

LIII. *Diffraction Images.*[*] *By* J. W. Gordon.

In this Paper an attempt is made to exhibit the theory of diffraction patterns in a comprehensive form and by means of an elementary mode of treatment. The object aimed at is to put the theory of diffraction upon the same text-book level as the theories of refraction and reflection.

The mode by which this simplification in the treatment of the subject is effected consists essentially in substituting as the surface of resolution the envelope which forms the boundary of an aplanatic pencil in place of the wave front which occupies its aperture. By this alteration in the mode of analysis a great simplification in the processes of the analysis is effected, the analogies between different forms of aperture which have little resemblance to one another become apparent, and an elementary form of exposition is rendered possible.

By reason of the limited space within which this Paper is restricted the subject is treated only illustratively and in outline, the mathematics involved being for the most part reserved for discussion in another Paper.

The general doctrine is developed in 17 propositions, as follows:—

(i.) The diffraction pattern produced by an aplanatic pencil of polygonal cross-section when that pattern is formed and observed in an apertural plane remote from the focus is a series of Fresnel bands bordering the edge of the aperture of the pencil in that plane.

(ii.) The diffraction pattern appearing in the focal plane is essentially a radiate pattern, and when the aplanatic pencil is of polygonal cross-section the pattern takes the form of a star, in which every side of the polygon is represented by two opposed rays in the star.

(iii.) The rays of the radiate pattern are broken lines of light showing maxima and minima of brightness in regular succession and having a definite centre-to-centre distance for the segments of the line. This centre-to-centre distance is proportional to the reciprocal $\frac{1}{\sin u_1}$; u_1 being the angle subtended at the focal point by the edge of one face of the polygon.

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[*] This Paper appears here in an abriged form and consequently certain diagrams have been omitted. It has, however, been thought convenient to retain the original numbering of the figures which remain.

(iv.) When, by the multiplication of sides, an aplanatic pencil of polygonal cross-section becomes indistinguishable from one of which the cross-section is a curve, the radiate pattern passes into an annular form, and from that cause a quasi-annular, or oval (circular) diffraction pattern may arise in the focal plane as, for example, when the image of a star is observed in the focal plane of a telescope.

(v.) Whatever the shape which the diffraction pattern in a focal plane may assume, that shape is always bilaterally symmetrical.

(vi.) The diffraction pattern in the focal plane formed by a concave radiant surface is identical with that so formed by the convex surface which fits into the concave.

(vii.) A pencil of aplanatic light is perfectly transparent to an impulse of diffracted light in tune with it; and, therefore, is transparent to the diffracted light from any other segment of the same homocentric pencil.

(viii.) Successive segments of a diffraction pattern, taken radially, are in opposite phase.

(ix.) In the case of an oval (circular) pattern in the focal plane the maximum amplitude in various rings varies approximately in inverse proportion to the square of the radius of the ring.

(x.) In the radiate pattern the maximum amplitude varies approximately in inverse proportion to the radial distance of the segment.

(xi.) The ray velocity of diffracted light is less than the ray velocity of aplanatic light in the region beyond the boundary of the aplanatic pencil.

(xii.) Within the boundary of a homocentric pencil the ray velocity of diffracted light varies with the angle of the ray to the wave front through which it passes and is equal to $\frac{1}{\cos\theta}V$ if $\frac{\pi}{2}-\theta$ be that angle and V equal to the velocity of aplanatic light.

(xiii.) The diffraction pattern due to the periodic structure of a regular compound aperture tends to settle down to the pattern of a double aperture, although the pattern of a compound aperture, comprising only a small number of components, may show marked deviations from the double-aperture type of pattern.

(xiv.) In the diffraction patterns of compound apertures of regular structure the diffraction pattern of the simple component aperture is found as a bright field upon which the secondary diffraction pattern, due to the periodic structure of the compound aperture, is displayed.

(xv.) Every compound aperture exhibits three diffraction patterns, that is to say—

1. A primary pattern, which results from the form and dimensions of the simple component aperture.

2. A secondary pattern, which results from mutual interference of a number of primary patterns, and

3. A tertiary pattern, which is determined by the form and dimensions of the entire compound aperture considered as an undivided whole.

(xvi.) The diffraction pattern visible within the aperture of an aplanatic pencil near the focal point is produced by visual projection upon the plane examined of the diffraction pattern formed by optical projection in the focal plane.

(xvii.) The diffraction pattern external to the aperture of an aplanatic pencil which has its interval ϱ proportional to $\frac{m}{\sin u}$ is formed upon a sheet disposed at right angles to the surface of the aplanatic pencil.

The bearings of the theory thus outlined are illustrated by reference to the optical problems connected with the formation of a spectrum by a diffraction grating: also by reference to the effect of stopping out the centre of an object glass and again by the problem presented by the ultimate limit of resolving power in optical instruments.

A few words by way of preface may be addressed to the subject of Fresnel's method of investigating diffraction phenomena.

It is well known that Fresnel's method of investigation consists in tracing down the optical projection of a light impulse from its source—its primary source in the original focus, or its secondary source in some selected wave front or image of the focus, as the case may be—to an object point in the diffraction pattern which is under investigation. If this deduction of the light impulse through its successive stages has been successfully carried out, it yields in the result a numerical value which expresses the light intensity at that object point.

Now it is obvious without discussion that this method of inquiry can yield no information directly concerning virtual diffraction images. It is concerned with real images only, and this is probably why the virtual images, which appear as diffraction patterns in the telescope, although they have attracted much notice and have been largely utilised by telescope makers in the testing of their lenses, have hitherto remained without scientific explanation.

The expedient which has been adopted as the basis of the system unfolded in this Paper is the substitution of the enveloping surface of the aplanatic pencil for the wave front as the surface of resolution. Fig. 3 is a diagram which serves to illustrate the change so made. In this diagram A_1–A_2 is the aperture through which the focused pencil of aplanatic light is received. G is the object point at which the amplitude of the diffracted light is to be computed. By the Fresnel method this is done

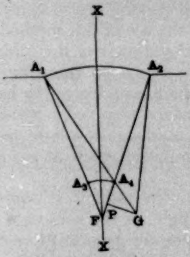


FIG. 3.

by taking a wave front in the aperture A_1–A_2 as the surface of resolution, and assuming that the resultant impulse is transmitted to the point G by rays included in the angle A_1GA_2. It can be shown that the result so arrived at must be subject to very considerable correction. For instance, it is quite certain that the wave front marked A_3–A_4 in the diagram would send diffracted light to the point G. But for the purpose of this calculation its contribution to the whole result is ignored. Again, the ray velocity of the diffracted light must vary, not only from one ray to another, but at different points along the same ray, for the angle θ, as above defined, which determines the ray velocity, is constantly varying along the length of any given ray. In fact, this method of calculation is never applied to such a case as is illustrated by Fig. 3. Only when the points

F and G lie so close together, that these sources of error can be neglected, is the Fresnel method applicable.

The alternative method here adopted is to substitute, as above stated, for the wave front in A_1–A_2, the enveloping surface of the aplanatic cone between the points A_2 and P. The line GP meets the edge ray A–F at right angles, and, therefore, none of the wave fronts between P and F can send any diffracted light to G, since that would involve their radiating a light impulse backwards—a manifest impossibility. Upon this method, however, all that part of the aplanatic cone which can radiate any light upon G is accounted for, and, furthermore, a proper allowance is made for varying ray velocity within the aplanatic pencil, because the data from which we calculate, being derived from the state of phase distribution known to subsist in the envelope, are themselves duly modified by the action of the varying ray velocity. An incidental advantage arises from the use of this method in the circumstance that the required integration over the conical surface is, in fact, a very much simpler operation than the integration over the spherical surface which constitutes the base of the cone, and, therefore, the calculations involved in what is the most ordinary case—the case, that is to say, of a light pencil of circular aperture—are greatly simplified, while at the same time the result is corrected.

Postulating, then, the radiant surface which envelops an aplanatic pencil of light as the source of diffraction, we may proceed to deduce certain general properties of the resulting diffraction patterns. It will be assumed, in the first place, that a bright star is the object of which the diffraction fringe is the optical image, and that the image-forming pencil is transmitted by a simple—that is to say, an undivided—aperture. Fig. 4 will serve the purposes of discussion.

Let N_1 be the nodal plane of an objective system; N_2 the nodal plane of the eye-lens, F the focal point and F–G the aplanatic focal plane. $A_1A_2A_3$ is the aperture, having one straight side A_1–A_2, and bounded otherwise by the circular segment $A_1A_3A_2$.

It is obvious, upon inspection of the diagram, that the envelope $A_1A_2A_3F$ must necessarily be of the nature (1) of a cone, or (2) of a pyramid; or (3), as shown in the figure, a compound of elements derived from both; that is to say, it must be either a faceted or a convex surface, if we disregard the alternative that the surface might be concave. This last is a case which is not necessary to distinguish, for it will presently appear that in the focal plane the pattern thrown by a concave surface is the same as that thrown by a convex. In Fig. 4, then, we have all the possible varieties of radiant surfaces illustrated, since every aplanatic ray must pass through the

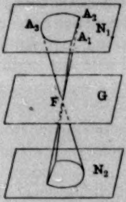


Fig. 4.

focal point F. The problem, therefore, is to determine how many varieties of diffraction patterns can arise from a triangular flat surface and a conical segment, respectively.

To deal first with the triangular flat radiant surface: Let Fig. 5 represent such a surface—a facet of a pyramid enveloping the aplanatic pencil transmitted by a polygonal aperture,

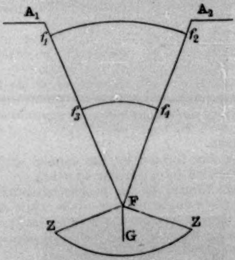


Fig. 5.

upon which certain isophasal lines f, f have been traced. Let it be assumed, in the first place, that the spectator's eye is focused on a surface perpendicular to the facet where it is traversed by the isophasal f_1f_2, and that his axis of vision passes through the line F–G, traced upon the complementary pyramid

ZFZ, and disposed symmetrically with reference to the radiant surface A_1FA_2. The pyramid ZFZ is a pyramid placed, as shown, so as to be coaxial and covertical with the pyramid of which A_1FA_2 is one face. The angles A_1FZ and A_2FZ are right angles.

It is obvious upon inspection that, under these conditions, the radiant surface—taking that to be the enveloping surface—which produces a diffraction pattern in the isophasal surface through f_1–f_2 is identical with the surface-producing diffraction in the well-known case of a pencil of unfocused light passing a straight edge, for the superficial rays are in all parts of the region in question perpendicular to the isophasal f_1–f_2 and are disposed within the boundary of a straight edge in the plane A_1FA_2. We ought, therefore, under these conditions, to see the well-known Fresnel diffraction pattern produced by a straight edge. In fact, we do see such a pattern. Fig. A in the photographic plate is a photograph obtained under the conditions specified. Next, let the focus of the eye be drawn back towards the focal plane, and suppose that a halt is made at the isophasal f_3–f_4. The conditions of image formation are

DESCRIPTION OF PLATE.

A. Diffraction pattern formed within the aperture of the aplanatic pencil and remote from the focal plane. This is a Fresnel pattern.

B. The like pattern formed in a pencil of rectangular section and exhibiting four symmetrically disposed Fresnel patterns.

C. Diffraction pattern formed within the aperture of the aplanatic pencil and near to the focal plane. This exhibits the projection upon the apertural plane of the Fraunhofer pattern of the focal plane.

D. Focal plane pattern of six rays produced by the three-sided prismatic pencil of aplanatic light transmitted by a triangular aperture.

E. The like pattern of four rays yielded by a pencil of square section.

F. The like pattern of ten rays yielded by a pencil having the form of a regular five-sided prism.

G. Focal plane pattern formed by the conical surface of the pencil of aplanatic light transmitted by a circular aperture.

H. The like pattern formed by the pencil transmitted by a semicircular aperture exhibiting the radiate pattern—two-rayed star—of the flat face in combination with the ring system of the hemicone.

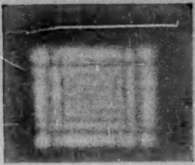
I. The like pattern formed by the two hemicones resulting from the division of a circular aperture by a cross wire.

J. Focal patterns formed by a compound aperture comprising two parallel slots. The upper figure is the image of a star, the lower the image of a bright object line.

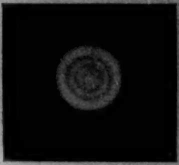
K. The like patterns formed by a compound aperture comprising three parallel slots.

L. Apertural pattern seen near the focal point in the prismatic pencil transmitted by a triangular aperture. The photograph is defective. If perfect it would show strongly curved segments of those rays which originate in the internal faces of the prism, thus illustrating the increased ray velocity of diffracted light within the aperture of the aplanatic pencil.

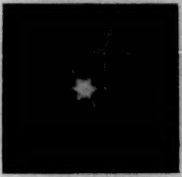
A



B



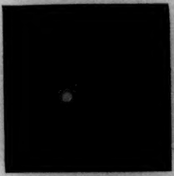
C



D

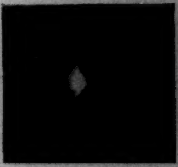
E

F



G

H



I

J

K

L

To face p. 428.

obviously unchanged, save in one respect, namely this, that the light amplitude is increased in the proportion of radius of (f_1-f_2) $\div$ radius of (f_3-f_4). We find, accordingly, that the breadth of the image produced by diffraction in this place is less, and its brightness greater, but its characteristic features are unaltered.

If we suppose the aperture of the aplanatic pencil to be square in section, we shall obviously obtain four such Fresnel patterns arranged as sides of the square (*see* Fig. B in the photographic plate); and since the characteristic feature of such a pattern is a group of parallel bands arranged about a centre of figure and returning into themselves, I propose to speak of this as a pattern of the annular type. If the polygon be one having very numerous sides, then the pattern becomes indistinguishable from a system of oval bands, if the word "oval" is used in a sense large enough to include the circle. As the pencil of circular section is the most important which we have to consider, the term "annular" is appropriate enough to be used with the extensive meaning which will make it apt when the perimeter of a polygon is rarely meant.

The first result, then, of this discussion of the subject is that:

1. The diffraction pattern produced by an aplanatic pencil of polygonal cross-section, when that pattern is formed within the aperture of the pencil in an apertural plane remote from the focal point, is a series of Fresnel bands bordering the edge of the aperture of the pencil in that plane.

But this rule is obviously inapplicable to a focal plane, for in the focal plane the aperture of the aplanatic pencil contracts to a point, and there is no edge about which the Fresnel bands can be formed. A point, however, although it cannot have sides, or an edge, can have different aspects. Thus, we may define the line F–G of Fig. 5 as being a line of symmetry with reference to the surface A_1FA_2 and with reference to the isophasals drawn upon that surface. We have seen that the Fresnel bands contract gradually as the isophasals contract towards the point F, and therefore, if we suppose these bands to continue to contract within the contour of the face A_1FA_2, until the line F–G is reached, we shall obviously have a broken line of diffracted light formed here. What is so seen is the condensed light of the Fresnel bands. In fact, a phenomenon very much like this does occur, but it is not accurately so described, for the pattern seen in the focal plane is the optical projection, not of those bands, internal to the aperture, which are shown in the photographs above referred to, but of the external diffraction bands

—too feebly luminous to appear in the photographs—which edge the aperture outside and lie in the external shadow. The periodic distance of the bands apart is the same in both series. But the brilliancy of the internal fringe is enormously greater than that of the external.

There is another distinction between the patterns of the apertural and focal planes—equally characteristic, but not equally pronounced—namely, this, that the focal-plane pattern is always and essentially bilaterally symmetrical. In a sense, the same thing may be said of the diffraction fringe of the apertural plane. But these two halves of that fringe are developed in separate apertural planes—the two halves of the focal-plane pattern must, on the contrary, be observed in the same focal plane. The explanation is very simple. The radiating line on one side of a focal-plane pattern is formed by optical projection, taking its rise in the radiations given off from the face of the envelope of the convergent pencil on the objective side of the focal point. A radiating line, which is the image and continuation of that line, is seen by visual projection when the diffracted light from the diverging pencil on the ocular side of the focal point is received into the eye and projected by a focusing effort upon the focal plane. The geometry of the optical projection which produces the real radiate pattern is identical with the geometry of the visual projection which produces the virtual radiate pattern. The two elements, therefore, are inseparable, and the radiate pattern is always bilaterally symmetrical.

The radiating bright line now under consideration differs from Fresnel bands, not only by reason of its direction in relation to the focal point, but also by the circumstance, already referred to, that it is a segmented line—whereas they are continuous.

Let the centre-to-centre distance of the segments of the radiate line be equal to ρ. The question is, what influence determines the magnitude of ρ? The discussion of this problem upon the present line of investigation need not be followed in detail here, since it leads only to the known result that ρ is proportional to $\frac{1}{\sin u_1}$, if $2u_1$ be written for the solid angle subtended by the breadth of the radiant face in question.

The form of the diffraction pattern has, so far, been determined only with reference to the line of symmetry F–G, and it is obvious that any polygonal aperture must yield a radiate pattern in which the various radiant faces give origin to as many optically projected radiate lines in the pattern. As every such optically projected line is balanced by a visually projected line, it follows that the number of radii in the pattern is potentially double the number of sides in the polygon. Thus, a single straight edge produces two radii, which lie in the same straight line and meet in the focal point. A three-sided pyramid yields a six-rayed star, and so on. But in the case of a square pyramid only four rays appear in the radiate pattern, because the four visually projected rays coincide with the four optically projected, the opposite sides of a square being parallel to one another. For the same reason a regular six-sided pyramid yields a six-rayed star, precisely like that yielded by the pencil which passes an aperture having the cross-section of an equilateral triangle. (*See* Figs. C, D and E of the photographic plate.) Moreover, it is quite immaterial that the aperture is eccentric in the apertural plane, however great its eccentricity may be. Thus, all the apertures shown in Fig. 7, A, yield the same diffraction pattern in the focal plane; that, namely, which is illustrated in Fig. 7, B. The effects produced by the varying eccentricities of the component apertures are so very subordinate and inconspicuous as to be hardly noticeable. The typical pattern remains quite unchanged.

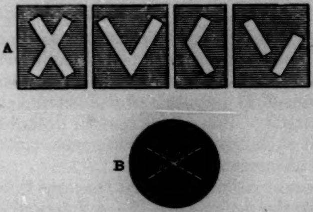


FIG. 7.

When, however, the polygon passes into a curve, we find a line of symmetry in every azimuth. Thus the rays of the radiate pattern become so numerous that they constitute a new type of annular pattern.

Summarising the facts so far ascertained concerning diffraction patterns in the focal plane, we have the following result:—

2. The diffraction pattern appearing in the focal plane is essentially a radiate pattern, and when the aplanatic pencil is

of polygonal cross-section the pattern takes the form of a star, in which every side of the polygon is represented by two opposed rays in the star.

3. The rays of the radiate pattern are broken lines of light, showing maxima and minima of brightness in regular succession and having a definite centre-to-centre distance for the segments of the line. This centre-to-centre distance is proportional to the reciprocal $\frac{1}{\sin u_1}$.

4. When, by the multiplication of sides, an aplanatic pencil of polygonal cross-section becomes indistinguishable from one of which the cross-section is a curve, the radiate pattern passes into an annular form, and from that cause a quasi-annular or oval (circular) diffraction pattern may arise in the focal plane, as, for example, when the image of a star is observed in the focal plane of a telescope.

5. Whatever shape the diffraction pattern in a focal plane may assume, that shape is always bilaterally symmetrical.

It will be convenient now to take up again the consideration of diffraction in apertural planes; and in this connection the most important is the law that an aplanatic pencil is perfectly transparent to impulses of diffracted light which are "in time" with it. This, which may be described as a particular example of the great law of resonance, gives rise to some very striking phenomena.

Among the most striking—and, at the same time, most easily intelligible—is the following: The diffraction pattern exhibited in the focal plane by a pencil of light, transmitted by a semicircular aperture, is of the form shown by Fig. G of the photographic plate. It comprises, in accordance with the laws above explained, the quasi-annular pattern due to the curved surface and the radiate pattern due to the flat surface of the aplanatic pencil. If we put two hemicones together, so as to form one homogeneous cone, the flat face, of course, disappears and with it disappears the radiate pattern. We have only the circular pattern left of Fig. F. But now let the cone be divided into two hemicones by a fine wire drawn across the aperture. The shadow of this wire will, of course, give rise to two flat surfaces facing one another and very nearly adjacent to one another, but still separated by the short space corresponding to the thickness of the wire. It is found that under these conditions the radiate pattern reappears as shown in

Fig. H. The diffraction pattern so produced by two hemiconical pencils, complementary to one another, is indistinguishable from the pattern due to one of them alone.

It obviously follows from this observation that the one hemicone is perfectly transparent to the diffracted light given off by the other hemicone. And this is by no means hard to be understood. The ray velocity of the diffracted light is, as we have seen, greater than that of aplanatic light so long as the impulse of diffracted light is transmitted within the aplanatic system. But it is not so when it escapes from that system. It follows that in crossing the shadow of the wire the diffracted light from the one hemicone must get out of phase with the aplanatic light of the other hemicone. It therefore produces interference and, since both hemicones are parts of one homocentric pencil, the phase difference between the two halves sets up the phenomena of regular interference. Thus, the recipient half becomes endowed with the power of radiating from its curved surface the diffracted impulses proper to a flat surface; in other words, it is transparent—and, observation shows, very perfectly transparent—to the diffracted light of the active half. The pencil being symmetrical, each half is, of course, active and receptive at the same time. It will be convenient to summarise in two rules the result of the last three paragraphs.

6. The diffraction pattern in the focal plane formed by a concave radiant surface is identical with that so formed by the convex surface which fits into the concave.

7. A pencil of aplanatic light is perfectly transparent to an impulse of diffracted light in tune with it; and, therefore, is transparent to the diffracted light from any other segment of the same homocentric pencil.

It is well known, and has been pointed out in the earlier part of this Paper, that the varying light-intensity in a diffraction pattern, which results in its periodic design, is due to the varying phase range of the radiation given off by the radiant surface along different axes. The periodic distance ρ may therefore be connected with this change of phase range and, taking the known function $\frac{m \sin \varphi}{\varphi}$ to represent the light amplitude at various points in the diffraction pattern, we may say that the zero value corresponding to the division between bright segments of the pattern must represent those values of the phase range angle φ for which $\sin\varphi$ equals zero; that is to say, that in these cases φ must equal π or some integral multiple of π. If φ equals zero, we have $\frac{\sin\varphi}{\varphi}=1$, the commonly employed expression for the amplitude of focal light.

The expression $\frac{m\sin\varphi}{\varphi}$ serves to express the light amplitude in the diffraction fringe produced by an aplanatic pencil of square section. In that case the diffracted light is condensed, as has above been pointed out, into four lines of symmetry, whereas, in the case of the circular fringe, the diffracted light is distributed in all azimuths. It is manifest to the eye that a pencil of square section yields a pattern in which the amplitude falls off much less rapidly than in the case of a pattern produced by a circular aperture.

It will be convenient now to give a graphic representation of the light intensity calculated by means of this expression for a pencil of square, and the well-known diagram for a pencil of circular, section having the same vertical angle. These are given in Fig. 8.

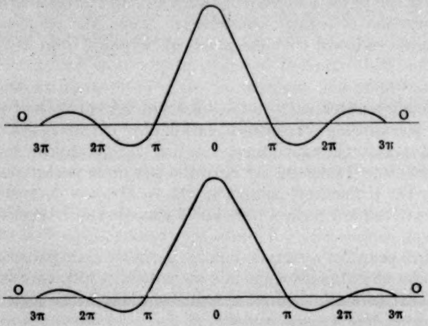


FIG. 8.

The curves of the last diagram may for practical purposes be simplified. It will be found that in the following discussion nothing turns upon the precise form of the sine curve, and as it is now proposed to indicate a practically serviceable method of graphically compounding diffraction patterns of compound apertures, a convenient diagram for use for this purpose may here be introduced.

Fig. 9 is a simplified form of Fig. 8, in which the curves are represented by angular forms, the angular forms being near enough for our present purpose to the more accurate curved forms. The upper curve in Fig. 8 is drawn to the formula $y=\frac{\sin\varphi}{\varphi}$ where $\varphi=\frac{2\pi x}{\lambda}$. In Fig. 9 alternative forms of the curves are shown, one in which the aplanatic light is, as in Fig. 8, equal to 1 and the other form in which it is equal to 0. The latter form represents, as nearly as it can be represented in this way, the diffracted light pure and simple; apart, that is to say, from the aplanatic light. It represents, therefore, the whole of the light from a non-luminiferous segment of a homocentric pencil, and the whole also of that portion of the light from a luminiferous pencil which can be affected by interference. It is in this latter form, therefore, that the curve will enter into the following computations. The first-mentioned curve, that into which the aplanatic light enters, is drawn approximately to the formula $y=\frac{\sin\varphi}{\varphi}=\frac{2\sin\varphi}{2\varphi}$; the latter curve to the formula

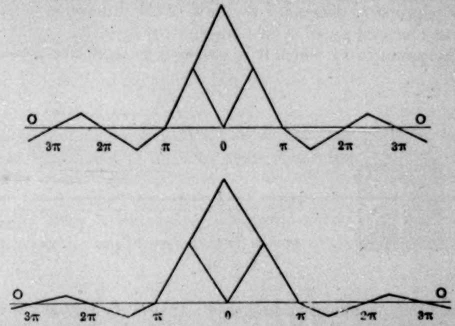


FIG. 9.

$$y=\frac{2\sin\left\{(n+m)\frac{\pi}{2}\right\}}{(n+1)\pi}.$$

Here $(n+m)\frac{\pi}{2}=\varphi$, and n is the integral, m the fractional, part of the coefficient $(n+m)$. It will be observed that the essential features of these curves are these :—

8. Successive segments of the pattern taken radially are in opposite phase.

9. In the case of the circular pattern the maximum amplitude in various rings varies approximately in inverse proportion to the square of the radius of the ring.

10. In the radiate pattern the maximum amplitude varies approximately in inverse proportion to the radial distance of the segment.

We come now to the case of an aperture comprising several divisions, and will first consider the case of a compound aperture in which there are several parallel rectangular openings. It is required to determine the form of the diffraction pattern due to a focused pencil of light transmitted by such an aperture.

The principle by which it is proposed to solve this problem is the principle already established : that every segment of the homocentric pencil will be transparent to the diffracted light from the surface of every other segment (above, 7).

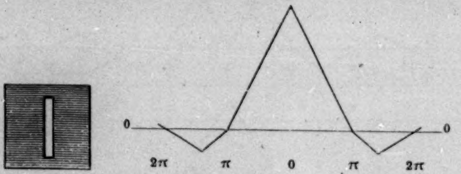


FIG. 10.

To illustrate the application of this principle, let Figs. 10, 11 and 12 be considered.

Fig. 10 represents a simple aperture and, side by side with it, the amplitude curve of its diffraction pattern. Here there is no question of a compound image, and therefore the form of curve is shown into which the aplanatic light enters as a component.

Fig. 11 shows a double aperture, comprising two equal and similar simple apertures separated by a non-luminiferous segment of equal angular magnitude. Thus, the whole pencil is bounded by four surfaces, two inner, which form the boundaries of the interior non-luminiferous segment, and two outer, which form part of the envelope of the whole pencil. The end walls, which complete the envelope, will be passed over without discussion in the present connection.

Here we have two diffraction patterns to be combined; namely, that yielded by the interior segment and that yielded by the whole pencil. The angles being small, we may take the ratio between the sines to be the same as a ratio between the angles; that is to say, 1 : 3. We have therefore to combine two patterns similar in type, but having, the one a ρ interval three times as great as that of the other. It is to be noted in connection with the composition of these two patterns, that it takes place, not in the focal plane itself, but before the focal plane is reached; that is to say, the component impulses are combined where first they meet within the aplanatic pencil itself. This circumstance has an important bearing on the form of the component curves, which must be employed for finding the form of the resultant curve. For it is to be noted that $\frac{\sin \varphi}{\varphi}$ is the expression for a resultant amplitude; that is to say, it is a mean value. The component amplitudes of which it is the mean are individually equal to $\cos \theta L$, if L equals the amplitude of the aplanatic light. If, then, we wish to give a graphic representation of the impulses which, under the con-

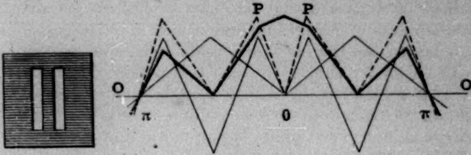


FIG. 11.

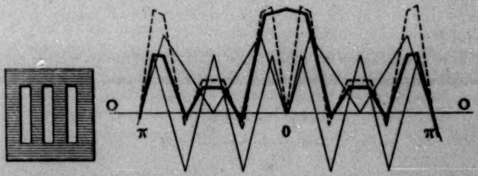


FIG. 12.

dition specified, enter into combination with one another, we must modify the curve of Fig. 11 by equating it to $2 \cos\theta \sin \left\{(n+m)\frac{\pi}{2}\right\}$ simply, that is to say, by giving to all the components substantially the same amplitude (since $\cos\theta$ may be taken to be equal to 1), so that the only difference between them is in the ρ interval. The two components are drawn accordingly in Fig. 11 to the same vertical scale.

It may be remarked that, in order to prevent the diagram from assuming unmanageable dimensions, a reduced scale has been adopted for the ordinates, and for greater clearness an increased scale for abscissæ in the construction of this curve.

By adding together the ordinates of these component curves we obtain the resultant curve shown by a broken line in Fig. 11.

This resultant curve shows the resultant impulse, not as it should be plotted in the focal plane, but as it may be thought of as potentially existing within the envelope of the aplanatic pencil itself. In order to pass from this form to the form which it assumes when in the focal plane, we must perform upon this curve the same operation which, under similar conditions, we perform upon the simple sine curve of the single aperture; that is to say, we must divide it by φ, or what is approximately equal to φ, *i.e.*, $(n+1)\frac{\pi}{2}$. Carrying out this operation we obtain, as a final resultant, that part of the curve shown by the heavy line in Fig. 11 which lies outside the two points marked P and P.

There remains to be added the central segment which represents the aplanatic light, for if we put $n=0$ and $m=0$ we shall have

$$\frac{2 \sin \left\{(n+m)\frac{\pi}{2}\right\}}{(n+1)\pi}=0\,;$$

that is to say, the ordinate representing aplanatic light has disappeared. Its measure is, however, easily found. For it is to be noted that, compared with the amplitude of the new component, the radiation from the outer pair of surfaces, the amplitude of the unreduced resultant is in the proportion 4 : 3. Now, this new component has an amplitude proportional to the sine of the whole divergence angle, equal, that is to say, to three times that of the radiation from the surface of the simple aperture. Thus, the unreduced resultant has a maximum amplitude of four times that of the simple aperture amplitude;

equal, that is to say, to twice the amplitude of the aplanatic light transmitted by the simple aperture. But twice the amplitude of the aplanatic light transmitted by the simple aperture is the amplitude of the aplanatic light transmitted by the double aperture. It follows that the aplanatic light in the new diagram must be represented by an ordinate having the same height as that of the ordinate representing the maximum of diffracted light in the unreduced resultant curve.

Passing now to Fig. 12, we have a compound aperture containing three simple apertures, and, accordingly, we have three radiant surfaces to be considered. The first and second apertures are bounded by the same enveloping surfaces as those already discussed in connection with Fig. 11; and therefore the diffraction pattern, so far as it is due to them, may be taken to be represented by the unreduced resultant—the broken line curve—of Fig. 11. We must in this connection make use of the unreduced resultant for two reasons. First of all, because the quantity of aplanatic light will obviously be different in this case from what it is in Fig. 11, since we have now to deal with three simple apertures instead of with two. In the next place we must recur to the unreduced form, and not to the reduced form of resultant curve, because it is required now to combine with the two patterns already considered a third pattern; that, namely, which is due to the outermost envelope having a divergence angle five times as great as the divergence angle of the single aperture, and therefore a ρ interval equal to one-fifth only of the original ρ interval. The components, consisting of these two curves, are shown by thin lines in Fig. 12, the unreduced resultant by a broken line, the reduced resultant by a heavy line, in which last named has been incorporated an ordinate representing the aplanatic light; the magnitude of which has been determined by the application of the rule already explained.

It will now be evident that a general type of diffraction pattern can be identified as belonging to the diffraction-grating type of aperture. Thus, let it be supposed that a number of similar apertures entering into the compound aperture is very large. It will then be evident that the first maximum of diffracted light tends to become equal to the aplanatic light and that the second maximum tends to a magnitude equal to half of the foregoing. Between these two maximum points there will be a region of depressed illumination approximating to zero. Beyond the second maximum there will be a minimum point at which the illumination reaches zero, and this point will correspond to the first minimum point in the pattern of the simple aperture. Beyond this point we get a repetition of the pattern on reduced amplitude and of opposite phase. In this description the reader will recognise the well-known form of diffraction pattern thrown by a diffraction grating.

It is not possible, within the limits of this Paper, to investigate with any completeness the application of this method of plotting down diffraction patterns. Enough has been said to illustrate the matter, and we may now formulate the following results :—

11. The ray velocity of diffracted light is less than the ray velocity of aplanatic light in the region beyond the boundary of the aplanatic pencil.*

12. Within the boundary of a homocentric pencil, the ray velocity of diffracted light varies with the angle of the ray to the wave front through which it passes, and is equal to $\frac{1}{\cos\theta}$ V, if $\frac{\pi}{2}-\theta$ be that angle, and V=the ray velocity of aplanatic light.

13. The diffraction pattern due to the periodic structure of a regular compound aperture tends to settle down to the pattern of a double aperture, although the pattern of a compound aperture comprising only a small number of components may show marked deviations from the double-aperture type of pattern.

14. In the diffraction patterns of compound apertures of regular structure the diffraction pattern of the simple component aperture is found as a bright field upon which the secondary diffraction pattern, due to the periodic structure of the compound aperture, is displayed.

The ideas of primary and secondary diffraction patterns being thus introduced, it may be well to interpose the observation that—

15. Every compound aperture exhibits three diffraction patterns; that is to say, (1) the primary pattern, which

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* Owing to limitations of space this proposition is not fully proved here. The foregoing proof goes only to show that the diffracted light gets out of phase in passing through the shadow of a wire. Fuller investigation shows this loss of phase to be due to loss of ray velocity and justifies the rule as here formulated.

results from the form and dimensions of the simple component aperture; (2) a secondary pattern, which results from the mutual interference of a number of primary patterns; and (3) a tertiary pattern, which is determined by the form and dimensions of the entire compound aperture, considered as an undivided whole.

These three elements in the diffraction pattern of a grating determine the three principal properties of the resulting spectrum; that is to say—

(1) The diffraction pattern of the aperture occupied by the whole grating (the tertiary pattern) determines the definition;

(2) The smallness of the periodic distance of the ruling (the secondary pattern) determines the dispersive power; and

(3) The breadth of the opening, in comparison with the periodic distance (the primary pattern), determines the relative brilliancy of different parts of the spectrum.

We may now turn to another matter involving the theory of diffraction patterns, and consider a question which, though long since settled by the practical method of trial, can hardly yet be said to have passed out of the region of discussion.

It has from time to time been proposed to attempt improvement in the resolving power of optical instruments by introducing a stop into the middle of an object glass; the notion being that, inasmuch as the diffraction pattern of an annular opening has a smaller ρ interval than that of the corresponding unstopped aperture, it should be possible by means of a stop to obtain a diminished antipoint, and, therefore, better definition. Many attempts have been made from time to time to put this idea into practice, but always without success. It can now be shown that the idea is radically unsound, and that it is in the nature of things impossible that improved definition can be arrived at in that way. It is generally said that the difficulty of realising this programme arises from the circumstance that the outer rings produced by an annular aperture are brighter than those produced by an unstopped aperture, and that the brightness of these remoter rings is the cause of failure.

That explanation, however, leaves out of account the main source of the difficulty, the real nature of which will at once appear if we examine the diffraction pattern of a stopped aperture. Fig. 13 will serve the purpose. In Fig. 13, A, the aperture is shown side by side with its curve, and in B the same aperture is shown almost entirely stopped down. Side by side with B is the diffraction curve produced by the interior faces, which, having an angle differing only by a negligibly small quantity from that of the external pair of faces, yield what is substantially the same diffraction curve, subject only to this, that the aplanatic light is reduced to zero. Of course, when the aperture is thus very largely obstructed, only a small quantity of aplanatic light can get through, and hence in the curve of B the aplanatic light is represented by a very small quantity. Side by side with this new component the old component from A is shown, and the resultant curve is shown by the heavy line. It will here be seen that the resultant pattern shows rings of approximately double brightness, because diffraction comes from four radiant surfaces instead of two, while at the focal point there is now minimum instead of maximum brightness.

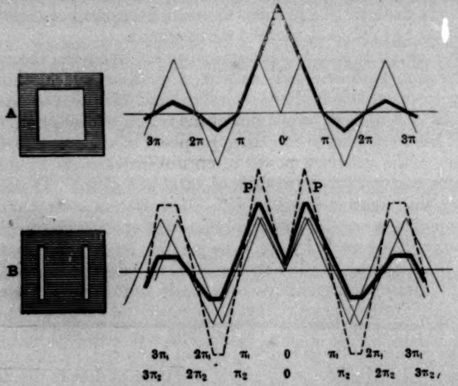
A
3π 2π π 0 π 2π 3π
P P
B
$3\pi_1$ $2\pi_1$ π_1 0 π_1 $2\pi_1$ $3\pi_1$
$3\pi_2$ $2\pi_2$ π_2 0 π_2 $2\pi_2$ $3\pi_2$

FIG. 13.

Compared with A, the resultant curve of B does, indeed, appear to have reduced radial dimensions ; but only because, by the disappearance of the aplanatic maximum, the two first maxima of diffracted light, marked P in the diagram, have become conspicuous, whereas they were unconspicuous before, being outshone by the aplanatic light. It is quite unnecessary, with these two diagrams under the eye, to enter into a discussion as to why the apparently diminished diffraction pattern of a stopped aperture, such as B, should yield an antipoint inferior for the purposes of definition to the apparently broader diffraction pattern of diagram A.

The case which has so far been discussed is that of a number of segments which have a common centre. There remains to be considered the alternative case of a number of segments which are not concentrically situated in the aplanatic pencil. We may, for example, have a case such as that of the triple aperture of Fig. 14, where we have three similar apertures, one placed upon the axis of collimation, the other two disposed eccentrically to it. These apertures are assumed to be all circular and all of the same diameter. It follows that every one of them will yield the same diffraction pattern, and the whole result is that we have the ring system corresponding to any one of these simple apertures reinforced threefold. A very simple way of obtaining a strong diffraction pattern in the focal plane of a small aperture is to multiply apertures in this way. If the small apertures are carefully arranged in azimuth, so as to avoid the formation of a radiate pattern, we may get in this way a strong image of the pattern of the small component aperture.

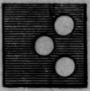

FIG. 14.

It is not, however, possible to arrange so small a number as the three apertures shown in Fig. 14 so as to produce this effect. Being unevenly distributed in azimuth, they will necessarily produce a radiate pattern, and the question arises: What will that radiate pattern be?

There is a very simple way of solving that problem. Radiate patterns, as we know, are yielded by flat radiant surfaces. We must therefore consider how the segmental pencils transmitted by these three component apertures can be arranged as component parts of flat faces. In other words, we must draw as many straight lines as possible connecting the three apertures. The secondary radiate pattern then will contain a pair of rays lying at right angles across every one of those lines.

In the case shown there are three such lines; and, therefore, the secondary pattern is of the radiate type, the same that would be produced by a pencil of the three-sided prism type.

It is, however, not to be supposed that the ray structure is as pronounced as it would be if the aperture were undivided. The primary diffraction pattern controls the illumination; and, consequently, the secondary pattern is cut up by it in such a way as to present to the eye, not its own general contour, but that of the primary pattern. The general effect is that the secondary pattern appears to be a diapered pattern wrought over the primary.

There is one remaining class of diffraction patterns still to be considered: those, namely, which are seen when the aperture of the aplanatic pencil is examined near the focal point. This pattern is of the annular type, and when formed in a pencil of circular section it takes the form shown in the photograph (Fig. C). This pattern is very familiar to the makers of astronomical objectives, as it affords one the means of testing the corrections of their glasses. This pattern is the result of visual projection, and for its explanation we must bear in mind that the region now in question can only be examined by looking at it through the diffraction pattern developed in the focal plane. Whatever is seen must, of course, be the visual projection, upon the focal plane of the eye, of that part of the diffraction pattern which constitutes the optical projection of the point on which the eye is focused.

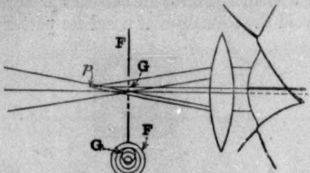

FIG. 15.

It does not, of course, follow that what is seen by visual projection is not also actually constituted by optical projection, and in this case it has been shown that at any great distance from the focal point the internal diffraction pattern conforms to the Fresnel type and can be explained by Fresnel's method of investigation. It is, however, more practically useful when we are discussing the patterns formed in the near neighbourhood of the focal plane to treat them as virtual than as real images, a mode of treatment which is illustrated by Fig. 15.

Here the observer's eye is taken to be focused on a point *p*, which is seen by looking through the focal plane F–G, occupied, of course, by the diffraction pattern of the focal plane. Let F of Fig. 16 be a plan of that diffraction pattern on an enlarged scale, and let G be the projection on that pattern of the observer's eye when focused on the point *p*. It will be seen that the projection covers two complete zones, and takes in part of a third. The central and next adjacent zones, being opposite in phase, will exactly counteract one another, and the point *p* will appear to be illuminated by the light received through the partially admitted third zone. It will therefore appear bright, but not of full brightness. If the observer selected a point slightly higher than *p*, so that his projection circle coincided with the circle traced in broken lines, he would cut out some of the negative second zone and admit a little more of the positive third zone. The new point would consequently appear more brightly luminous than the first. If his projection circle were made to coincide with one of the dividing rings between adjacent zones, the spot in which he then focused would appear bright with full illumination, or completely dark according as the number of included zones was odd or even. It is obvious that, as the focal point travels over the apertural plane, the projection circle will travel correspondingly over the diffraction pattern in the focal plane, and will yield focal points of varying illumination in such an arrangement that we shall have a ring system on the apertural plane, with the same ρ interval as the ring system of the focal plane.

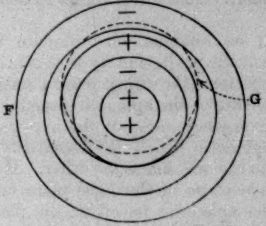

FIG. 16.

To this rule of the equivalence of the ρ interval in the two systems there must obviously be one marked exception. For it is to be observed that, as the projection circle traverses the diffraction pattern, it passes from maximum to minimum illuminating capacity and back to maximum in the space of one ρ interval measured in the focal plane. This fact it is which secures the repetition of the ρ interval in the projected image in the apertural plane. But this proposition does not hold good when the edge of the projection circle is traversing the central zone, for in the central zone we have two adjacent ρ intervals which transmit light in the same phase. Moreover, when that central zone is passed, a marked change occurs in the light-giving capacity of the projection circle. It is then intersected by zone segments, which, being alternately positive and negative, and all very limited in angular magnitude, come very near to cancelling one another, so that when the central zone is excluded from the aperture of the projection circle there can be no bright illumination, and scarcely any illumination at all, of the object point. The whole region so defined is a region of darkness.

Now it will be obvious, on reference to Fig. 15, that when the edge of the projection circle stands upon the centre of the diffraction pattern, the object point p must stand upon the edge of the aperture of the aplanatic pencil. This must be so, because the divergence angle of the eye and the divergence angle of the aplanatic pencil are necessarily the same angle. Whatever limits the one, limits the other. Hence the diameter of the projection circle in the focal plane is the same as the diameter of the aplanatic pencil in the plane of the object point p. It follows, therefore, that in carrying the eye from a point within the aperture to a point without the aperture the observer must pass across a broad bright ring, the outermost edge of which lies outside the boundary of the aplanatic pencil itself and constitutes the innermost of the external diffraction rings. This broad, bright edge is, in fact, the most striking feature of these internal diffraction patterns.

It will now be evident why the internal diffraction pattern of the aplanatic pencil near its focal point does not present the features of a Fresnel system. It stands in striking contrast with the Fresnel system of the remoter apertural planes in respect (1) of the uniformity of its ρ interval, and (2) in the uniform intensity of both its bright and its dark rings. In fact, as seen, these patterns near the focal point to all apperance are not Fresnel patterns at all, but projections of the Fraunhofer pattern in the focal plane itself. We thus arrive at a further rule.

16. The diffraction pattern internal to the aperture of the aplanatic pencil, in the near neighbourhood of the focal plane, is the image formed by visual projection of the Fraunhofer pattern in the focal plane.

In the discussion of the visually projected diffraction patterns it has been convenient to speak of them as derived from the Fraunhofer pattern in the focal plane. But here it is necessary to take note that this is not the identical pattern which has been made the subject of discussion in the earlier part of this Paper. That pattern was formed, it will be remembered, along lines of symmetry situated at right angles to the surface of the aplanatic pencil. The focal plane, on the contrary, lies at right angles to the optical axis. The one pattern may, however, be very simply derived from the other. The process of derivation is illustrated by Fig. 17. Here x-x is the optical axis; F–G the line of symmetry on which the radiate pattern is formed; F_1–G_1 and F_2–G_2 are lines of symmetry parallel to F–G; and Fg_1–g_2 is a line traced in the focal plane. The rules above given enable us to plot down the diffraction pattern along any one of the lines F–G, F_1–G_1, and F_2–G_2. We find the diffraction pattern in the focal plane by selecting the point g_1 from the F_1–G_1 pattern; the point g_2 from the F_2–G_2 pattern, and by selecting, on the same principle, the remaining points from the remaining patterns which intersect the focal plane. In the case of a conical aplanatic pencil of circular section—the common case presented by the telescope and microscope—this

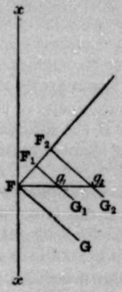

FIG. 17.

mode of proceeding leads to a very simple rule, for all the patterns in question are alike, except that they expand as they travel away from the focal point in the direction of the light source. This expansion, however, does not affect the ρ interval, which is the same in all. Consequently, in this case, the diffraction pattern in the focal plane has the same form as that already discussed, but a ρ interval greater in the proportion $\frac{1}{\cos u} : 1$.

It is now evident that it is important to distinguish between the focal plane strictly so called and the focal surface for diffracted light which lies everywhere at right angles to the edge ray of the aplanatic pencil. We may therefore formulate a new rule, thus :—

17. The diffraction pattern external to the aperture of the aplanatic pencil is formed upon a sheet which stands in every azimuth perpendicular to the surface of the aplanatic pencil.

This last formulated law suggests a doubt as to whether the aplanatic focal plane is the plane in which the visible image is most sharply defined. If the diffraction fringe is more compressed in a plane lying slightly beyond that theoretical focal plane than in that focal plane itself, then there will hardly be room for doubt that this lower plane is that in which our empirical focusing is done. If this be so, then it is clear that calculations as to the ultimate limit of resolving power, based upon the assumption that the aplanatic focal plane is that in which the picture is developed, are quite untrustworthy, and this whole subject stands in need of thorough reinvestigation.

LIV. *The Shape of Scales Required for Reflecting Instruments with Concave Mirrors.* By E. H. Rayner, *M.A.* (*From the National Physical Laboratory.*)

Summary.

1. The accuracy of reading certain types of deflecting instruments, which operate without any static friction, can be greatly improved by observing their deflections by the reflection of a beam of light from a concave mirror having a radius of curvature of several feet.

2. When the angular deflection is large the shape of the scale for such an instrument is somewhat similar to a cardioide, varying according to the relative values of the radius of curvature of the mirror and its distance of the source.

3. A special case is discussed when the curve approximates (over a large arc) exceedingly closely to a circle of definite radius.

4. In general the radius of a similar circle may be calculated, which will approximate sufficiently accurately to the true curve.

5. Some suggestions are given as to the use of mica for the window in the covers of such instruments.

Many types of scientific instruments, especially electrical, operate by the forces to be measured causing rotation about a vertical axis, such rotation being opposed and balanced by some elastic control, such as the bending of a spring or the torsion of a wire or flattened strip of metal. The pointer may be a few inches long, and the support of the moving part is often of the nature of pivots such as are used in watch and clock mechanism. The accuracy of reading can be increased by lengthening the pointer, and so increasing the scale length; but a limit is soon reached in practice by the friction of the pivots assisted by the increase in weight and inertia of the pointer. In instruments, however, in which the whole weight of the moving part is supported by a wire or strip of metal, the friction limit practically vanishes; and if a mirror be attached to the moving part and light from a fixed source be projected on to the mirror, the full advantage of a "pointer" of many feet in length may be obtained, and the accuracy of reading multiplied fifty-fold as compared with the same instrument provided with a pointer moving over a scale in the usual way. Such instruments have intrinsically very little damping, and in practice either air, oil or magnetic damping must be employed to bring the moving part to rest in a convenient time.

The experience which emphasized the importance of the subject dealt with in this Paper has been gained by the arrangements adopted at the National Physical Laboratory for the precision apparatus for alternating-current measurements. These have to be of the deflecting type, and to enable readings to be made to an accuracy of one or two parts in ten thousand it is necessary to have a scale length of some 20 ft. or more, the instrument being at about 10 ft. from the centre of the scale.

Of the instruments in use the two most important are a Kelvin electrostatic voltmeter of the single quadrant type, and a specially designed quadrant electrometer used as a standard wattmeter.

The other instrument is specially designed with sufficient air friction to bring it to rest in the most suitable time.

There are two methods used for measuring the angular deflection of such instruments: the first is to observe (in a telescope) the apparent motion of a scale reflected from a plane mirror fixed to the moving part of the instrument. For small angular displacements and special research this method is valuable, as the telescope partakes of the nature of a microscope and a very finely divided scale may be used with advantage.

One of its drawbacks is that only one person can make observations at a time; and for routine work, often involving observation for several hours a day, the strain renders the method quite impracticable. In such cases a concave mirror is employed and a real image of an artificial source of light is projected on to a translucent or opaque white scale, on which suitable marks are drawn. A Nernst lamp and a condensing lens with a fine vertical wire in front of it form a suitable source of light, producing on the scale a circular bright spot with a fine line across the vertical diameter.

This Paper deals with the shape of the scale required in order to have the vertical wire always sharply in focus, while the concave mirror rotates about a vertical axis.

If the angular deflection be only a few degrees a straight scale is commonly used; but it becomes unsatisfactory if the deflection is considerable. Neither does a circular scale with the mirror at the centre sufficiently approximate to the proper curve for instruments reading by a reflected ray of light whose working deflection may reach 120°.

From the diagrams given it will be seen that the scale has to continually approach the mirror as the angle of incidence increases, and that for deflections of 90° from the incident ray the distance may be reduced to one-half.

Nomenclature.—

u=distance from object to mirror.

v_0=distance from mirror to scale when the angle of incidence in a horizontal plane is zero. The angle in a vertical plane is assumed to be zero. In practice it should be kept as small as convenient; v_0 is measured along the axis of symmetry of the curve and intersects the scale at its "centre."

v_θ=distance from mirror to scale when the reflected beam makes an angle θ with the incident beam. The angle of incidence$=\frac{\theta}{2}$.

m=magnification$=\frac{v_0}{u}$.

a=radius of curvature of the mirror.

Then $\frac{1}{v_\theta}+\frac{1}{u}=\frac{2}{a\cos\frac{\theta}{2}}$ which, when $\theta=0$, reduces to the ordinary equation of the concave mirror.

For any permanent installation this is worth while, drawing out the curve full size and bending the scale to it. The scale varies somewhat in shape, according to the relative values of u, v_0 and a.

For ordinary purposes, however, a circle may be drawn which will agree sufficiently accurately with the proper curve. The radius of this circle may be taken as the perpendicular to the axis of symmetry from the point where the curve is parallel to that axis, the centre of the circle being on the axis of symmetry at a distance from the intersection of the scale and the axis equal to the above radius.

For about 60° on each side of the axis of symmetry a circle so drawn will lie within or without the true curve according as the magnification is less or greater than about 1·37. When, however, the curve is parallel to the axis of symmetry at the point where $\theta=60°$, the coincidence between the circle so drawn and the true curve is exceedingly close. In this case the perpendicular to the axis of symmetry, from the point where the curve is parallel to that axis, is exactly equal to the distance from the foot of the perpendicular to the point where the axis of symmetry cuts the scale at right angles. We thus have the circle touching the true curve at $\theta=0$, and also at two

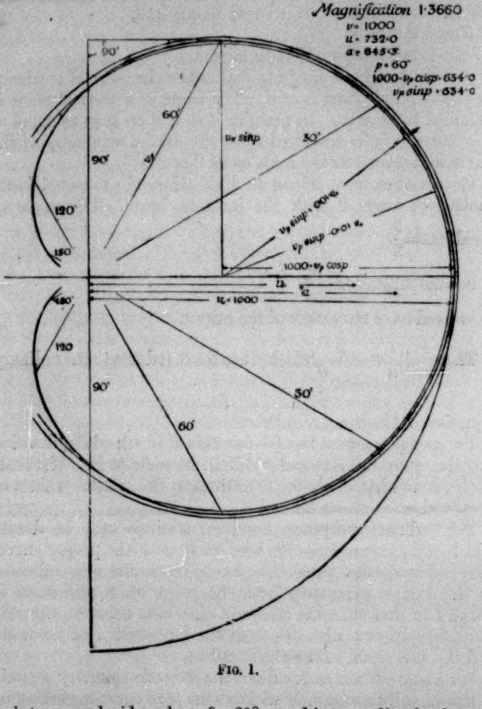
Magnification 1·3660
v = 1000
u = 732·0
a = 845·5
p = 60°
1000-v_p cos p = 634·0
v_p sin p = 634·0
90°
60°
90°
v = sin p
30°
120°
150°
v_p sin p + 0·01 v_0
v_p sin p − 0·01 v_0
1000 − v_p cos p
u
a
v_0 = 1000
150°
120
90°
30°
60°

FIG. 1.

points on each side, where $\theta=60^\circ$; and intermediately the two curves do not diverge from one another by two parts in a thousand.

Throughout the Paper the distance v_0, between the scale

($\theta=0$) and the mirror, has been taken as 1,000. In this special case u, the distance between source and mirror is $1{,}000\sqrt{3}-1 = 732{\cdot}05$ and $a=1{,}000\times\frac{2}{3}\,(3-\sqrt{3})=845{\cdot}298$.

The magnification $\dfrac{v_0}{u}=\dfrac{3}{2(3-\sqrt{3})}=1{\cdot}36603$.

Fig. 1 gives the curve for this case, and the agreement with

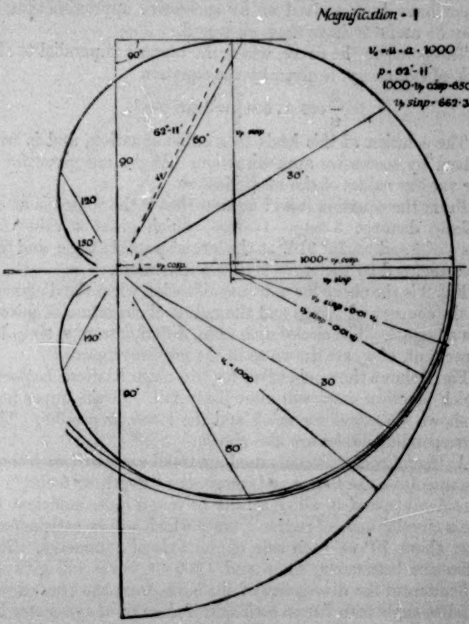
Magnification = 1
v₀ = u = a = 1000
p = 62°-11'
1000 - v_p cos p = 650·7
v_p sin p = 662·3
90°
62°-11'
60°
v_p sin p
90
45°
30°
120
150°
p
v_p cos p
1000 - v_p cos p
v_p sin p
v_p cos p + 0·01 v₀
v_p sin p - 0·01 v₀
150°
u = 1000
30
90°
60°

FIG. 2.

a circle drawn as described is so near that it is quite impossible to distinguish between them for about 70° on each side of the axis of symmetry, except when drawn on a very large scale. Two other circles have been drawn instead, one longer by

1 per cent. of v_0 and the other shorter by the same amount, to show how close the coincidence is. It is therefore desirable to choose, if convenient, the values for v_0, u and a in the ratio of 1,000, 732 and 845, as described above, using a radius of 634 for the curvature of the scale.

The values of many of the quantities given in the table at the end are carried to a much higher accuracy than is necessary, as the numbers may be used for interpolating; and as many of them have been arrived at by successive approximation, it may be useful to have them on record.

The point on the curve, where the tangent is parallel to the axis of symmetry, is given by the equation

$$\frac{a}{u}\cos p \,.\, \cos\frac{p}{2}=3\cos p-1.$$

The solution of this leads to a cubic equation, and is best solved by successive approximation. Angles are given for p for various values of the magnification.

From the equation it will be seen that if the source is at an infinite distance $3\cos p-1=$zero, which gives a value for $\cos p$ of $\frac{1}{3}$ and $p=70^\circ\ 31{\cdot}7'$ as the largest possible angle, and the area enclosed by the curve is then a maximum.

Fig. 2 is the curve for unit magnification when the distances of the source and image and the radius of curvature of mirror are all equal. The circles with radii, differing from $v_p \sin p$ by 1 per cent. of v_0, are drawn as in the previous figure.

Fig. 3 shows the semi-curves for two magnifications between which practical cases will most likely fall. In the upper half is shown the curve for $m=3$ and the lower or $m=0{\cdot}5$. The corresponding circles are also shown.

As the mirrors commonly used are small compared with their distance from the source and image, the "depth of focus" is considerable, and it will generally be found quite sufficient to use a circular scale of radius $v_p \sin p$ which will be satisfactory over about 70° on each side of the axis of symmetry. The difference between $v_p \sin p$ and $1{,}000-v_p \cos p$ will give an indication of the divergence of the circle from the true curve. If a less angle than 70° on each side of the axis of symmetry be sufficient a slightly different circle will fit the curve somewhat better.

It is generally essential to completely enclose instruments to avoid entrance of dust and disturbance by air currents. Some transparent window must therefore be provided, and for moderate deflections plane parallel glass will probably be found most satisfactory. For large deflections a curved window is necessary and mica is the most convenient material.

This should be split into as thin a sheet as possible (thin paper seems to be the best tool to use) and it should not be bent to a small radius. If the material is thick or the curvature sharp the definition will be seriously impaired. In any case curvature will cause the mica to act as a lens, and modify appreciably the focal length of the system as calculated from the mirror alone, especially when the latter has a radius of curvature of 5 or 10 ft. The focal length of the combination should, therefore, be determined and used in any calculations.

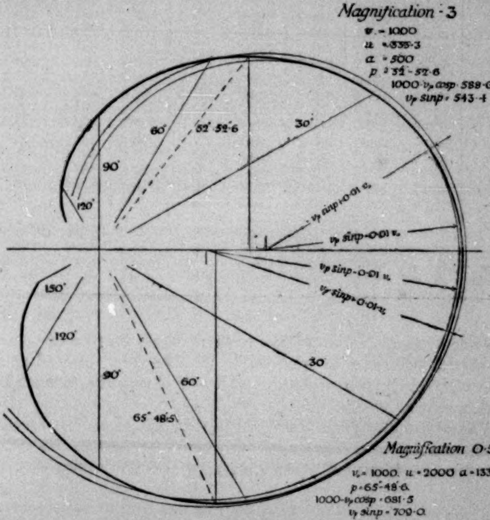
Magnification · 3
v. = 1000
u = 333·3
a = 500
p = 52°-52·6
1000-v_p cos p · 588·6
v_p sin p = 543·4
60°
52° 52·6
30°
90°
120°
v_p sin p + 0·01 v_o
v_p sin p - 0·01 v_o
v_p sin p - 0·01 v_o
v_p sin p + 0·01 v_o
150°
120°
30
90°
60°
65° 48·5
Magnification 0·5
v_o = 1000. u = 2000 a = 1333
p = 65°-48·6.
1000-v_p cos p = 681·5
v_p sin p = 709·0

FIG. 3.

The following table gives the distances from the mirror to the scale for every 10° of deflection of the reflected beam from its original direction. The values for various magnifications (m) are given which will probably cover most of the cases occurring in practice. It will be seen that though to obtain a large length

m.	0	0·5	1·0	1·36603	1·6	1·8	2·0	2·5	3·0
v_0	1000	1000	1000	1000	1000	1000	1000	1000	1000
u	∞	2000	1000	732·05	625	555·5	500	400	333·3
a	2000	1333·3	1000	845·30	769·2	714·3	666·6	571·4	500·0
θ	v_θ	v_θ	v_θ	v_θ	v_θ	v_θ	v_θ	v_θ	v_θ
0	1000·0	1000·0	1000·0	1000·0	1000·0	1000·0	1000·0	1000·0	1000·0
10	996·2	994·3	992·5	991·1	990·2	989·4	988·7	986·9	984·9
20	984·8	978·4	970·1	964·8	961·4	958·6	955·8	948·9	941·9
30	965·9	949·8	934·1	922·9	916·0	910·1	904·2	890·1	876·3
40	939·7	912·2	886·2	868·1	857·0	847·7	838·5	816·5	795·7
50	906·3	865·8	828·6	805·3	788·1	775·5	763·2	734·3	707·5
60	866·0	811·7	764·3	732·1	713·1	698·2	683·0	648·8	617·7
70	819·2	751·2	693·7	656·9	635·3	618·0	601·5	564·1	531·0
80	766·0	685·8	620·8	580·5	557·4	539·0	521·9	483·3	450·1
90	707·1	617·0	547·2	505·0	481·5	463·0	445·9	408·2	376·4
100	642·8	545·4	473·7	432·0	409·0	391·2	374·9	339·6	310·3
110	573·6	472·8	402·0	362·4	341·0	324·5	309·6	277·6	251·6
120	500·0	400·0	333·3	297·1	277·8	263·2	250·0	222·2	200·0
130	422·6	327·9	268·0	236·3	220·0	207·2	196·2	173·0	154·7
140	342·0	257·3	206·3	180·1	166·6	156·6	147·7	129·3	115·0
150	258·8	188·8	148·6	128·6	118·4	110·9	104·3	90·7	80·3
160	173·6	123·0	95·0	81·6	74·8	70·0	65·5	56·6	49·9
p	70°31·7′	65°48·5′	62°11·6′	60°0′	58°44·8′	57°45·2′	56°49·2′	54°43·0′	52°52·6′
v_p	816·5	777·2	748·7	732·1	722·7	715·5	708·8	694·1	681·6
$v_p \sin p$	769·8	709·0	662·3	634·0	617·8	605·2	593·3	566·6	543·4
$1000 - v_p \cos p$	727·8	681·5	650·7	634·0	625·4	618·2	612·2	599·1	588·6

of scale a magnification of two or three times may be desirable, it would generally be preferable to lengthen the radius of curvature of the mirror, as the scale will then not approach the instrument so much.

Values are also given for $v_p \sin p$, this being half the breadth of the curve; and this length may often be used as the radius of a circular curve sufficiently accurate for many purposes.

DISCUSSION.

Dr. **Schuster** said that the Paper was of considerable interest to all who had to measure.

Mr. S. D. **Chalmers** called attention to the fact that coma of the system might be even more important than the choice of best focus for the system, and therefore it was most desirable to pay attention to this in selecting the magnification; for unit magnification the coma was practically nil, and it might, therefore, be desirable to give up something with regard to the proper curvature for the purpose of eliminating this coma error which might cause a displacement of the line image.

Dr. **John Milne** (Edinburgh) asked how the reductions of the observations were performed.

Mr. **Rayner** replied that the question was what the actual deflections meant. For their measurements of alternating currents within 1 part in

10,000, the scale divisions were originally fixed by means of continuous currents, and the scale had only the significance of a temporary standard.

Mr. A. P. TROTTER said that he had found a difficulty in using curved scales. In a Paper communicated to the Physical Society on February 21, 1890 (see *The Electrician*, XXIV., 428), he had described how, with any reflecting instrument in which the deflections were not proportional to the quantities to be measured, a uniformly divided scale could be bent so that it gave direct readings. Such a scale was rather difficult to set up, and if the relative position of the scale and the instrument were altered, it was difficult to readjust it. Mr. Rayner got over half the difficulty by graduating the scale empirically. This having been done, it was important not to disturb the apparatus, or else to take precautions that the exact position should be reproduced.

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